Guaranteed Characterization of the Explored Space of a Mobile Robot by using Subpavings

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Characterization of the Explored area

Mission of the robot

Explore a given zone, and ensure that it has been entirely covered by its

- sensor: mapping, mine hunting, search, ...
- tool: lawn-mowing, cleaning, ...

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Computing the area explored by the robot, prior to processing sensor data enables to

- assess mission before long transfer and processing time of sensor data
- focus first data processing on problematic parts of the mission
- plan a new mission to fill the gaps

Characterization of the Explored area

Mission of the robot

Explore a given zone, and ensure that it has been entirely covered

- mapping, mine hunting, search, ...
- lawn-mowing, cleaning

Robot positioning is uncertain

Characterize the explored area w.r.t localization uncertainty

Characterization of the Explored area

Guaranteed

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- mapping, mine hunting, search, ...
- lawn-mowing, cleaning

Robot positioning is uncertain

Characterize the explored area w.r.t localization uncertainty

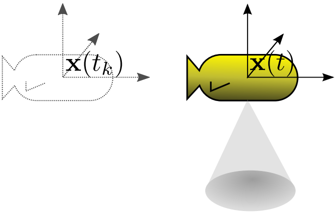
Use interval analysis to compute a guaranteed bracketing of the area explored by the robot

Outline

1. Problem statement
   - Explored area
2. Characterization of the explored area in presence of uncertainties
   - Explored area with an uncertain trajectory
   - Explored area characterization by Set Inversion
3. Application
   - Underwater exploration simulation
   - Guaranteed explored area computation
4. Strangle method
   - Improve guaranteed explored area computation

Outline

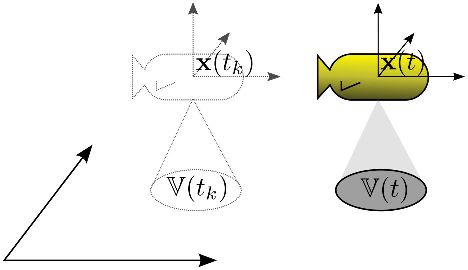
Exploration robot



$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}\left(\mathbf{x}(t), \mathbf{u}(t)\right) \\ \mathbf{y}(t) & = & \mathbf{g}\left(\mathbf{x}(t)\right) \end{cases}$$

- evolution
- observation

Visible area

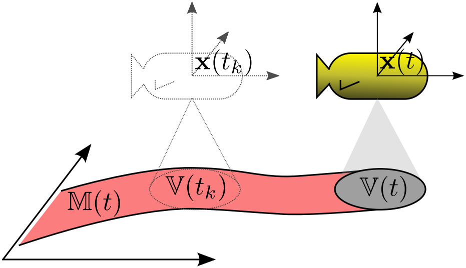


The visible area at time t is represented by the set-valued function $\mathbb{V}(t) = \left\{\mathbf{z} \in \mathbb{R}^2 : v(\mathbf{z}, \mathbf{x}(t)) \leq 0\right\}$ where $v(\mathbf{z}, \mathbf{x}(t))$ is the visibility function

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t)) \\ \mathbb{V}(t) &= \left\{\mathbf{z} \in \mathbb{R}^2 : v(\mathbf{z}, \mathbf{x}(t)) \leq 0\right\} \end{cases}$$

- evolution
- observation
- visible area

Explored area



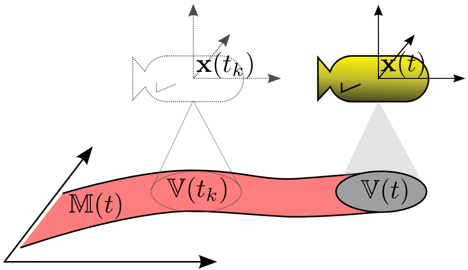
The explored area is the union of the visible areas over the whole trajectory

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{y}(t) & = & \mathbf{g}(\mathbf{x}(t)) \\ \mathbb{V}(t) & = & \left\{\mathbf{z} \in \mathbb{R}^2 : v(\mathbf{z}, \mathbf{x}(t)) \leq 0\right\} \\ \mathbb{M}(t) & = & \bigcup_{\tau \in [0,t]} \mathbb{V}(\tau) \end{cases}$$

- evolution
- observation
- visible area
- explored area

Outline

Explored area with an uncertain trajectory



$$\begin{cases} \mathbf{x}(t) & \in & [\mathbf{x}](t) \\ \mathbb{V}(t) & = & \left\{ \mathbf{z} \in \mathbb{R}^2 : v\left(\mathbf{z}, \mathbf{x}(t)\right) \leq 0 \right\} \\ \mathbb{M}(t) & = & \bigcup_{\tau \in [0,t]} \mathbb{V}(\tau) \end{cases}$$

- uncertain trajectory
- visibility
- explored map

Bracketing of the visible area: guaranteed and possible

Guaranteed visible area $\mathbb{V}^{\forall}$: set of points that have necessarily been observed, regardless of the state uncertainty

$$\mathbb{V}^{\forall}_{[\mathbf{x}]}(t) = \left\{\mathbf{z} \in \mathbb{R}^2 : \forall \mathbf{x}(t) \in [\mathbf{x}](t), v\left(\mathbf{z}, \mathbf{x}(t)\right) \leq 0\right\} \tag{1}$$

Possible visible area $\mathbb{V}^{\exists}$: set of points that may have been in the robot's field of view:

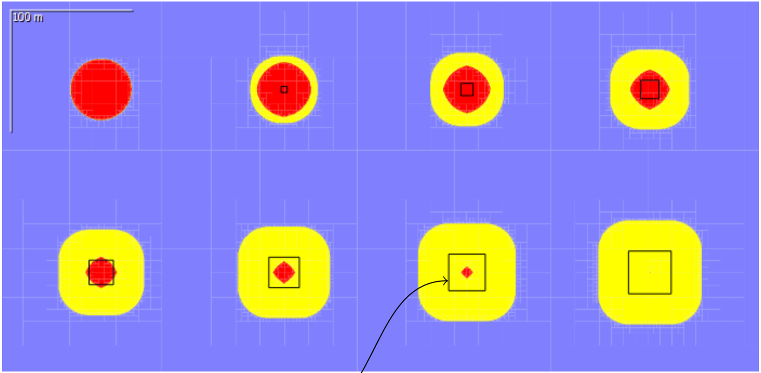
$$\mathbb{V}^{\exists}_{[\mathbf{x}]}(t) = \left\{\mathbf{z} \in \mathbb{R}^2 : \exists \mathbf{x}(t) \in [\mathbf{x}](t), v\left(\mathbf{z}, \mathbf{x}(t)\right) \leq 0\right\} \tag{2}$$

$\mathbb{V}^{\forall}_{[\mathbf{x}]}(t)$ and $\mathbb{V}^{\exists}_{[\mathbf{x}]}(t)$ form a bracketing of the actual visible area $\mathbb{V}(t)$:

$$\forall t \in [t], \mathbb{V}^{\forall}_{[\mathbf{x}]}(t) \subset \mathbb{V}(t) \subset \mathbb{V}^{\exists}_{[\mathbf{x}]}(t)$$

Guaranteed visible area depends on position accuracy

Robot is located inside a box. It observes a circular region: $v(\mathbf{z}, \mathbf{x}) = ||\mathbf{z} - \mathbf{x}||^2 - r^2$

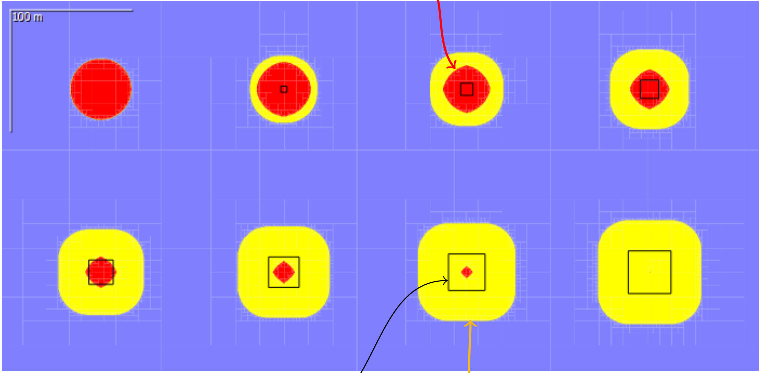


Position uncertainty box $[\mathbf{x}]$

Guaranteed visible area depends on position accuracy

Robot is located inside a box. It observes a circular region: $v(\mathbf{z},\mathbf{x}) = ||\mathbf{z}-\mathbf{x}||^2 - r^2$

Guaranteed visible area $\mathbb{V}^{\forall}$



Position uncertainty box $[\mathbf{x}]$

Possible visible area $\mathbb{V}^{\exists}$

Guaranteed and possible explored area

Guaranteed explored area $\mathbb{M}^{\forall}$: union of all the guaranteed visible areas during the mission

$$\mathbb{M}^{\forall}_{[\mathbf{x}]} = \bigcup_{t\in[t]} \mathbb{V}^{\forall}_{[\mathbf{x}]}(t), \tag{3}$$

Possible explored area $\mathbb{M}^{\exists}$: union of all the possible visible areas over time

$$\mathbb{M}^{\exists}_{[\mathbf{x}]} = \bigcup_{t\in[t]} \mathbb{V}^{\exists}_{[\mathbf{x}]}(t). \tag{4}$$

A bracketing of the actual explored area $\mathbb{M}$ is given by

$$\mathbb{M}^{\forall}_{[\mathbf{x}]} \subset \mathbb{M} \subset \mathbb{M}^{\exists}_{[\mathbf{x}]}.$$

Outline

Interval analysis (Moore)

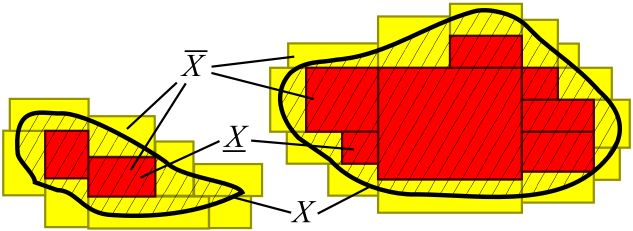
- Interval $[x] = [\underline{x}, \overline{x}]$. $\underline{x}$ is the lower bound and $\overline{x}$ is the upper bound.
- Box $[\mathbf{x}] = [\underline{\mathbf{x}}, \overline{\mathbf{x}}]$. The vectors $\underline{\mathbf{x}}$ and $\overline{\mathbf{x}}$ are respectively the lower and upper bounds.
- Interval extension of real arithmetic operators $+, -, \cdot$ and $\div$, and elementary functions such as *tan*, *sin* and *exp*...

$$[x] + [y] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}],$$
$$[x] \cdot [y] = [\min\left(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}\right), \max\left(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}\right)].$$

- The interval function $[\mathbf{f}]$ is an *inclusion function* for $\mathbf{f}$ if $\forall [\mathbf{x}] \in \mathbb{IR}^n, \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{x}])$.
- The *natural inclusion function* is obtained by replacing each operator in the expression of the function by its interval counterpart.

Set inversion via interval analysis (SIVIA)

Find $X = \mathbf{f}^{-1}([\mathbf{y}])$ such as $X = \{\mathbf{x} \mid \mathbf{f}(\mathbf{x}) \in [\mathbf{y}])\}$, where $[\mathbf{y}]$ is a known interval vector.



SIVIA is a branch-and-bound algorithm. Starting from an arbitrarily big box, it computes an inner subpaving $\underline{X}$ and an outer subpaving $\overline{X}$ such that $\underline{X} \subseteq X \subseteq \overline{X}$.

Quantifier elimination

$\forall$ and $\exists$ quantifiers appear in the expressions of $\mathbb{V}^{\forall}(t)$ and $\mathbb{V}^{\exists}(t)$. Let us remove them to simplify set computations.
Let $[v](\mathbf{z},[\mathbf{x}])$ be the minimal inclusion function for v with respect to $\mathbf{x}$.
$[v](\mathbf{z},[\mathbf{x}]) = \{v(\mathbf{z},\mathbf{x}), \mathbf{x} \in [\mathbf{x}]\}$

$$z \in \mathbb{V}^{\forall}(t) \Leftrightarrow \forall \mathbf{x} \in [\mathbf{x}], v(\mathbf{z},\mathbf{x}) \leq 0 \Leftrightarrow \overline{v}(\mathbf{z},[\mathbf{x}]) \leq 0$$
$$z \in \mathbb{V}^{\exists}(t) \Leftrightarrow \exists \mathbf{x} \in [\mathbf{x}], v(\mathbf{z},\mathbf{x}) \leq 0 \Leftrightarrow \underline{v}(\mathbf{z},[\mathbf{x}]) \leq 0$$

Expressions of the upper bound $\overline{v}$ and of the lower bound $\underline{v}$ can be derived by using symbolic interval arithmetic (Jaulin and Chabert, 2010)

$$\underline{v}(\mathbf{z},[\mathbf{x}]) = H\left((z_1-\overline{x_1})(z_1-\underline{x_1})\right)\min\left((z_1-\overline{x_1})^2,\left(z_1-\underline{x_1}\right)^2\right) + H\left((z_2-\overline{x_2})(z_2-\underline{x_2})\right)\min\left((z_2-\overline{x_2})^2,\left(z_2-\underline{x_2}\right)^2\right) - r^2$$
$$\overline{v}(\mathbf{z},[\mathbf{x}]) = \max\left((z_1-\overline{x_1})^2,\left(z_1-\underline{x_1}\right)^2\right) + \max\left((z_2-\overline{x_2})^2,\left(z_2-\underline{x_2}\right)^2\right) - r^2$$

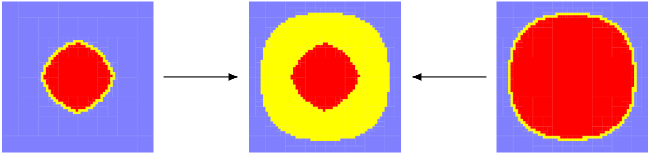
Explored area computation: visible area

Use SIVIA to compute $\mathbb{V}^{\forall}(t)$ and $\mathbb{V}^{\exists}(t)$:

$$\underline{\mathbb{V}^{\forall}(t)} \subset \mathbb{V}^{\forall}(t) \subset \overline{\mathbb{V}^{\forall}(t)} \quad \text{and} \quad \underline{\mathbb{V}^{\exists}(t)} \subset \mathbb{V}^{\exists}(t) \subset \overline{\mathbb{V}^{\exists}(t)}$$

-> Bracketing of $\mathbb{V}(t)$ between the two subpavings $\underline{\mathbb{V}^{\forall}(t)}$ and $\overline{\mathbb{V}^{\exists}(t)}$ such that

$$\underline{\mathbb{V}^{\forall}(t)} \subset \mathbb{V}(t) \subset \overline{\mathbb{V}^{\exists}(t)}.$$



$[\mathbb{V}^{\forall}(t)]$ $[\mathbb{V}(t)]$ $[\mathbb{V}^{\exists}(t)]$

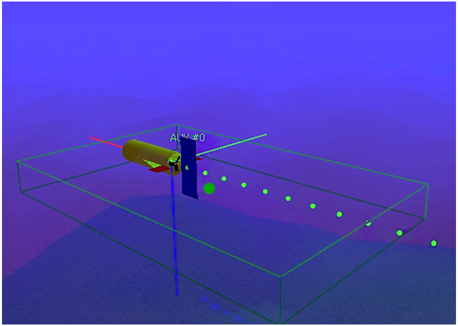
Explored area computation

Let us define $\underline{\mathbb{M}^{\forall}} = \bigcup_{t\in[t]} \underline{\mathbb{V}^{\forall}(t)}$ and $\overline{\mathbb{M}^{\exists}} = \bigcup_{t\in[t]} \overline{\mathbb{V}^{\exists}(t)}$.
Since $\underline{\mathbb{V}^{\forall}(t)} \subset \mathbb{V}^{\forall}(t)$, by applying the union operation, we obtain $\underline{\mathbb{M}^{\forall}} \subset \mathbb{M}^{\forall}$.
Similarly, we have $\mathbb{M}^{\exists} \subset \overline{\mathbb{M}^{\exists}}$.

$$\underline{\mathbb{M}^{\forall}} \subset \mathbb{M}^{\forall} \subset \mathbb{M} \subset \mathbb{M}^{\exists} \subset \overline{\mathbb{M}^{\exists}}.$$

Outline

Underwater exploration simulation



Simulate an AUV with

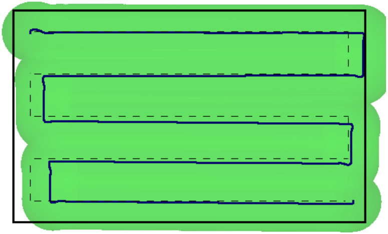
- GPS (works on surface only)
- Speed and depth sensors
- Inertial Measurement Unit
- Acoustic ranging and two beacon buoys

Mission: exploration and covering of a 500 m x 300 m area

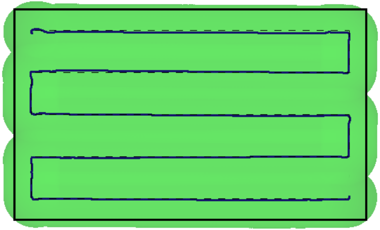
GPS only at the start and at the end

Simulated covered area

Black = target. Green = explored



GPS + dead reckoning



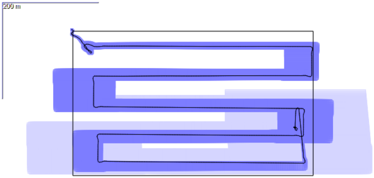
GPS + inertial + acoustic

Outline

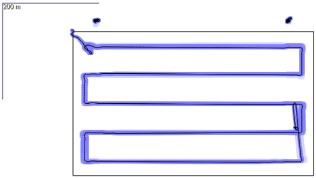
Position refining

Light blue = initial. Blue = contracted.

- Constraint propagation with distance measurements
- Forward-backward constraint propagation over trajectory with evolution equation



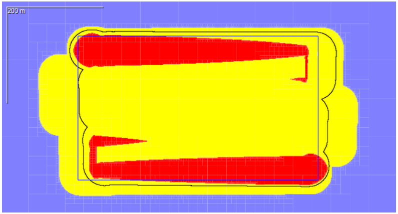
GPS + dead reckoning



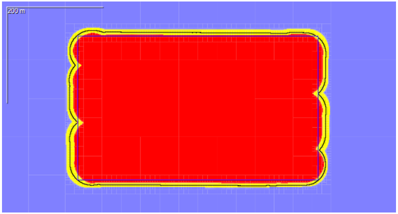
GPS + inertial + acoustic

Explored area computation.

Red=guaranteed ($\mathbb{M}^{\forall}$), Yellow=possible ($\mathbb{M}^{\exists}$), Black=truth



GPS + dead reckoning



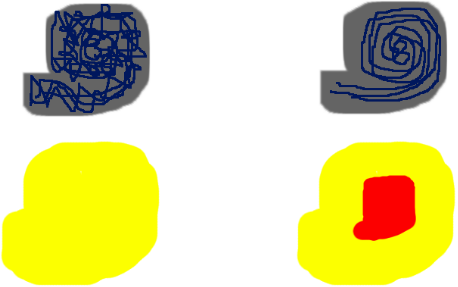
GPS + inertial + acoustic

- $\underline{\mathbb{M}^{\forall}} \subset \mathbb{M} \subset \overline{\mathbb{M}^{\exists}}$ is verified.
- $\mathbb{M}^{\forall}$ is pessimistic wrt to the real explored area, since we only use position information without taking robot evolution into account.

Outline

Taking robot evolution into account

Large position uncertainty does not prevent a robot to guaranteedly explore a zone (e.g. a lawnmower running a spiral trajectory)



We need to take robot evolution model into account to improve the guaranteed explored area computation.

Taking robot evolution into account

Let $\mathbf{x} : \mathbb{R} \to \mathbb{R}^n$ be a trajectory. $\mathbb{M}(\mathbf{x})$ is the associated explored area

$$\mathbb{M}(\mathbf{x}) = \left\{ \mathbf{z} \in \mathbb{R}^2 \mid \exists t,\ v(\mathbf{z}, \mathbf{x}(t)) \leq 0 \right\}$$

Let $\mathcal{T}$ be the set of admissible trajectories given a tube and an equation:

$$\mathcal{T} = \{ \mathbf{x} : \mathbb{R} \to \mathbb{R}^n \mid \forall t,\ \mathbf{x}(t) \in [\mathbf{x}](t),\ \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \}$$

The guaranteed explored area can be defined as

$$\mathbb{M}_{\mathcal{T}}^{\forall} = \left\{ \mathbf{z} \in \mathbb{R}^2 \mid \forall \mathbf{x} \in \mathcal{T},\ \exists t,\ v(\mathbf{z}, \mathbf{x}(t)) \leq 0 \right\} = \bigcap_{\mathbf{x} \in \mathcal{T}} \mathbb{M}(\mathbf{x})$$

The possibly explored area can be defined as

$$\mathbb{M}_{\mathcal{T}}^{\exists} = \left\{ \mathbf{z} \in \mathbb{R}^2 \mid \exists \mathbf{x} \in \mathcal{T},\ \exists t,\ v(\mathbf{z}, \mathbf{x}(t)) \leq 0 \right\} = \bigcup_{\mathbf{x} \in \mathcal{T}} \mathbb{M}(\mathbf{x})$$

Taking robot evolution into account

Let $\{[\mathbf{x_1}],\dots,[\mathbf{x_N}]\}$ be a partition of the tube $[\mathbf{x}]$ (strangle at t_s):

$$[\mathbf{x_i}](t) = \begin{cases} [\mathbf{x}](t) & t \neq t_s \\ part([\mathbf{x}](t),\ i) & t = t_s, \end{cases} \text{ where } part([\mathbf{x}](t), i) \text{ make a partition of } [\mathbf{x}](t)$$

Let $\mathcal{T}_i,\ i \in \{1 \dots N\}$ be the sets of admissible trajectories for each part:

$$\mathcal{T}_i = \{\mathbf{x} : \mathbb{R} \to \mathbb{R}^n \mid \forall t,\, \mathbf{x}(t) \in [\mathbf{x_i}](t),\, \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))\}$$

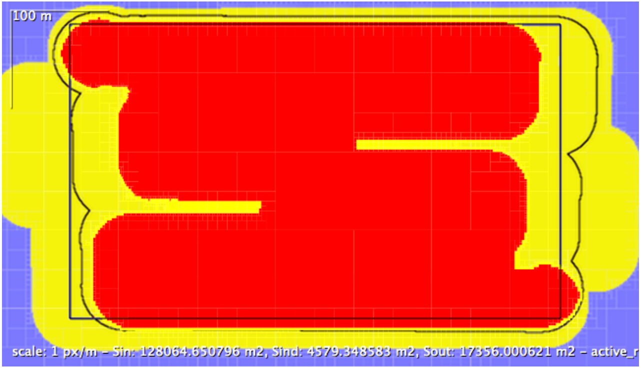
Using constraint propagation, the $\{[\mathbf{x_1}],\dots,[\mathbf{x_N}]\}$ parts can be refined to $\{[\mathbf{x_1^*}],\dots,[\mathbf{x_N^*}]\}$ such that $[\mathbf{x_i}] \supseteq [\mathbf{x_i^*}] \supseteq \mathcal{T}_i$

$$\bigcap_{i \in \{1\dots N\}} \mathbb{M}^{\forall}_{[\mathbf{x_i^*}]} \underset{\mathcal{T}_i \subseteq [\mathbf{x_i^*}]}{\subseteq} \bigcap_{i \in \{1\dots N\}} \mathbb{M}^{\forall}(\mathcal{T}_i) = \bigcap_{i \in \{1\dots N\}} \bigcap_{\mathbf{x} \in \mathcal{T}_i} \mathbb{M}(\mathbf{x}) = \mathbb{M}^{\forall}$$

$$\bigcup_{i \in \{1\dots N\}} \mathbb{M}^{\exists}_{[\mathbf{x_i^*}]} \underset{\mathcal{T}_i \subseteq [\mathbf{x_i^*}]}{\supseteq} \bigcup_{i \in \{1\dots N\}} \mathbb{M}^{\exists}(\mathcal{T}_i) = \bigcup_{i \in \{1\dots N\}} \bigcup_{\mathbf{x} \in \mathcal{T}_i} \mathbb{M}(\mathbf{x}) = \mathbb{M}^{\exists}$$

Results (GPS + dead reckoning)

Less pessimistic bracketing, by using the robot evolution equation.

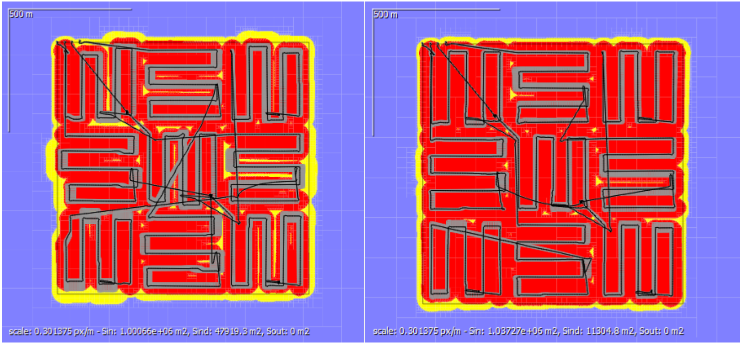


Summary

- Interval-based method to characterize the area explored by a robot.
- Position uncertainties lead to explored area uncertainty -> bracketing of the explored area between a guaranteed and a possible areas.
- The computed set-interval of the explored area can be used to
  - ensure target as been fully covered
  - focus manual checks on possible but not guaranteed areas
  - plan a complementary mission to improve coverage

Outlook

- Robot squads



Thank you!

Questions?