Image Shape Extraction using Interval Methods

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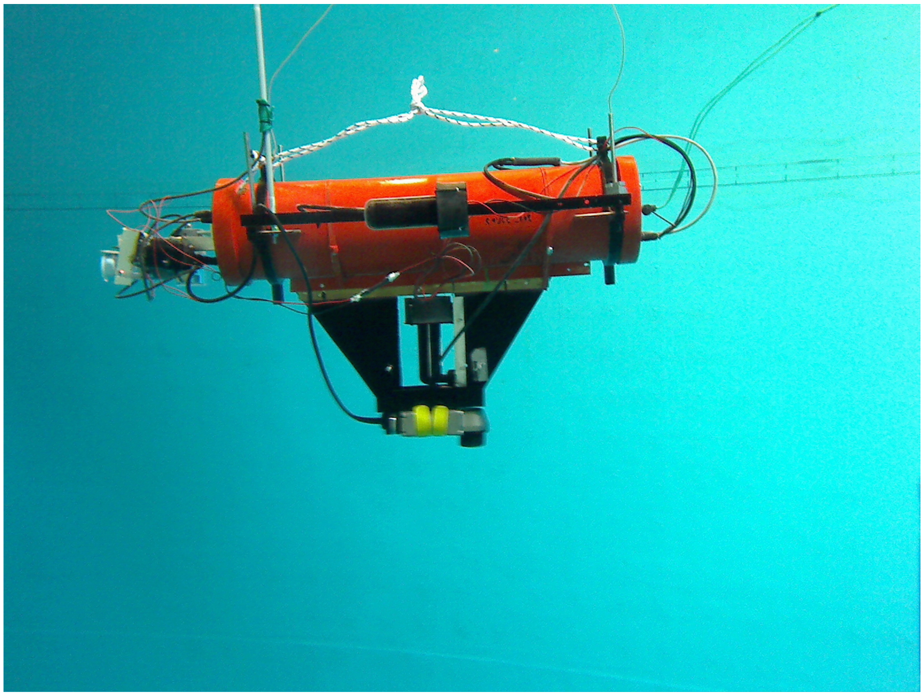
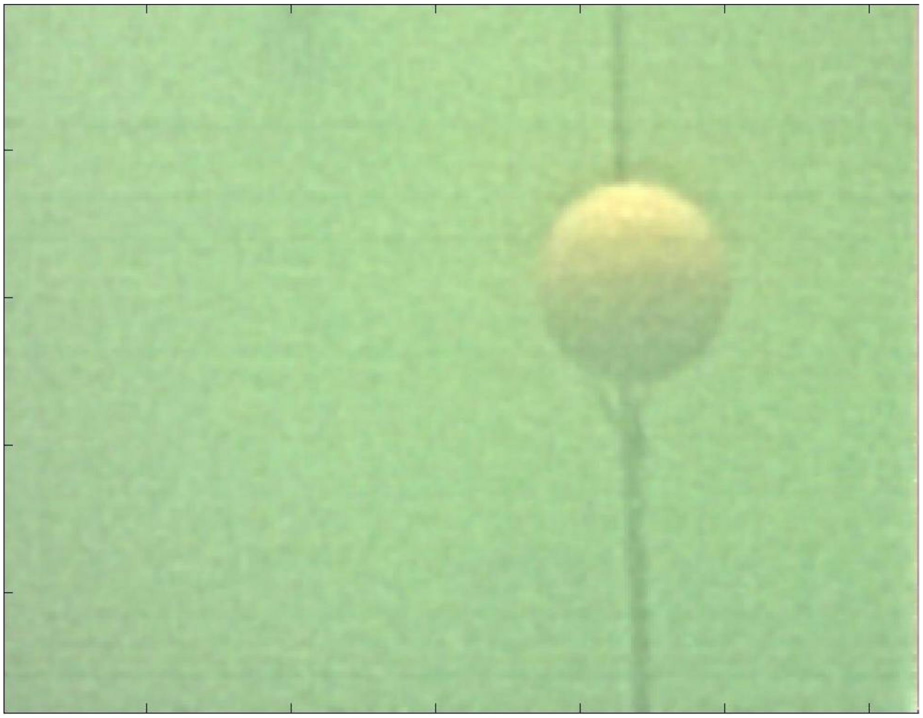
du GDR Macs
Jeudi 13 novembre 2008 de 10h-17h

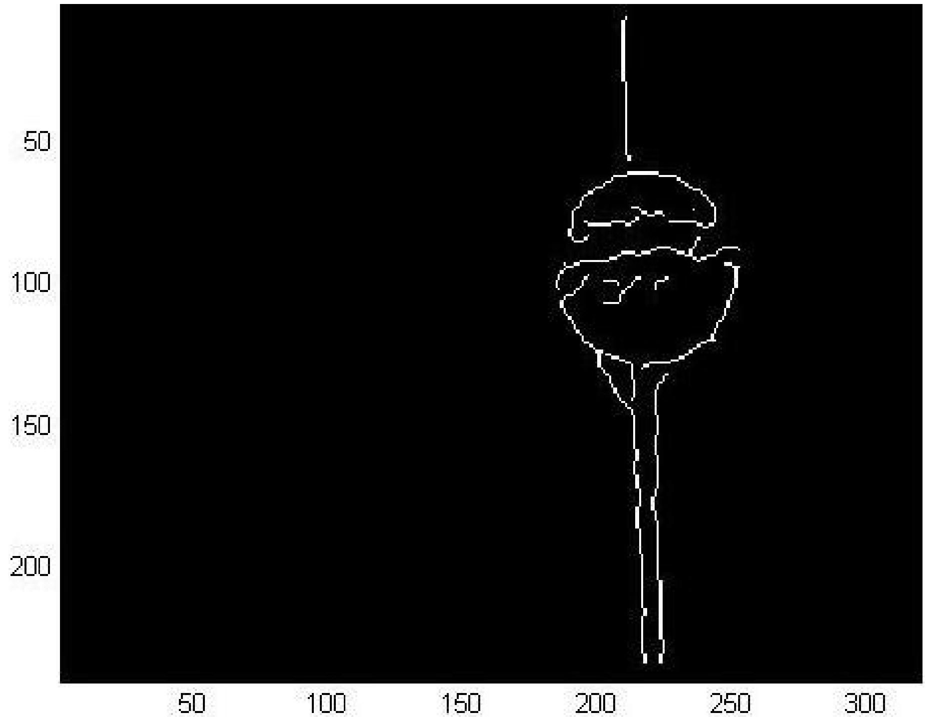
1 Shape detection problem



Sauc'isse robot swimming inside a pool



A spheric buoy seen by Sauc'isse



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300

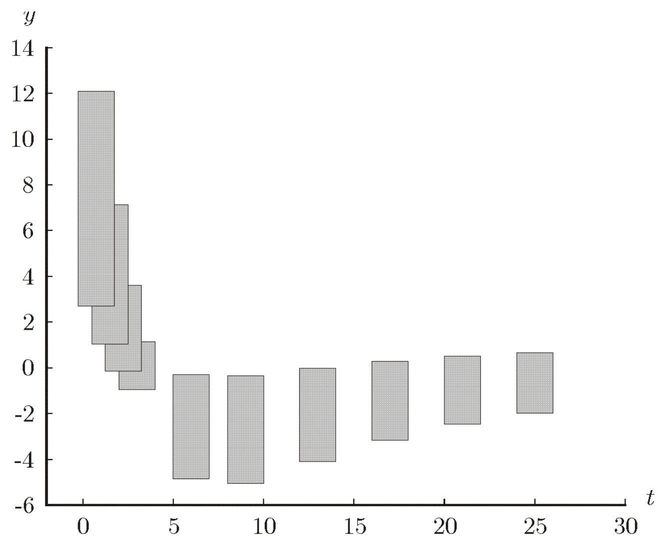
2 Set estimation

An *implicit parameter set estimation problem* amounts to characterizing

$$\mathbb{P} = \bigcap_{i \in \{\mathbf{1},\ldots,m\}} \underbrace{\{\mathbf{p} \in \mathbb{R}^n, \exists \mathbf{y} \in [\mathbf{y}](i), \mathbf{f}(\mathbf{p}, \mathbf{y}) = \mathbf{0}\}}_{\mathbb{P}_i}$$

where $\mathbf{p}$ is the parameter vector, $[\mathbf{y}](i)$ is the ith measurement box and $\mathbf{f}$ is the model function.

Example: Find the set of all $\mathbf{p} = (p_1, p_2)^\mathsf{T}$ such that $20\exp(-p_1 t) - 8\exp(-p_2 t)$ goes through all ten boxes



For this problem, the model function is

$$f(\mathbf{p}, \mathbf{y}) = 20\,\mathrm{exp}(-p_1 y_1) - 8\,\mathrm{exp}(-p_2 y_1) - y_2,$$

and the boxes $[\mathbf{y}](1), \ldots, [\mathbf{y}](10)$ are those represented on the figure.

3 Shape extraction as a set estimation problem

Consider the *shape function* $\mathbf{f}(\mathbf{p}, \mathbf{y})$, where $\mathbf{y} \in \mathbb{R}^2$ corresponds to a pixel and $\mathbf{p}$ is the shape vector.

The *shape* associated with $\mathbf{p}$ is

$$\mathcal{S}(\mathbf{p}) \stackrel{\text{def}}{=} \left\{\mathbf{y} \in \mathbb{R}^2, \mathbf{f}(\mathbf{p}, \mathbf{y}) = \mathbf{0}\right\}.$$

Consider a set of (small) boxes in the image

$$\mathcal{Y} = \{[\mathbf{y}](1), \ldots, [\mathbf{y}](m)\}.$$

Each of this box is assumed to intersect the edge of the shape we want to extract.

In our buoy example,

- $\mathcal{Y}$ corresponds to edge pixel boxes.

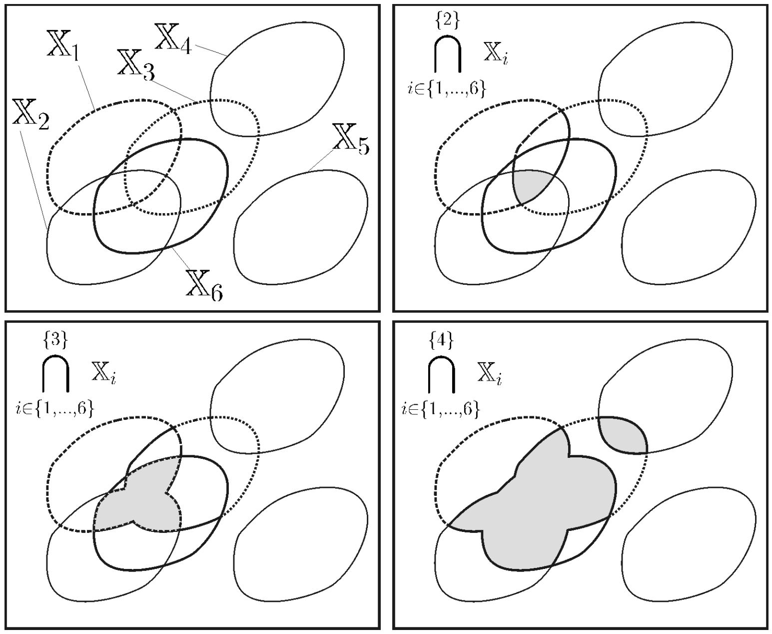
- $f(\mathbf{p}, \mathbf{y}) = (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2$.

- $\mathbf{p} = (p_1, p_2, p_3)^\mathsf{T}$ where p_1, p_2 are the coordinates of the center of the circle and p_3 its radius.

Now, in our shape extraction problem, a lot of $[\mathbf{y}](i)$ are outlier.

4 Robust set estimation

The q-relaxed intersection denoted by $\bigcap^{\{q\}} \mathbb{X}_i$ is the set of all $\mathbf{x}$ which belong to all $\mathbb{X}_i$'s, except q at most.



$\mathbb{X}_1$
$\mathbb{X}_2$
$\mathbb{X}_3$
$\mathbb{X}_4$
$\mathbb{X}_5$
$\mathbb{X}_6$
$\bigcap_{i\in\{1,\ldots,6\}}^{\{2\}} \mathbb{X}_i$
$\bigcap_{i\in\{1,\ldots,6\}}^{\{3\}} \mathbb{X}_i$
$\bigcap_{i\in\{1,\ldots,6\}}^{\{4\}} \mathbb{X}_i$

The q relaxed feasible set is

$$\mathbb{P}^{\{q\}} \overset{\mathsf{def}}{=} \bigcap_{i \in \{\mathbf{1},\ldots,m\}}^{\{q\}} \{\mathbf{p} \in \mathbb{R}^n, \exists \mathbf{y} \in [\mathbf{y}](i), \mathbf{f}(\mathbf{p},\mathbf{y}) = \mathbf{0}\}.$$

5 Interval propagation

An optimal contractor for the set

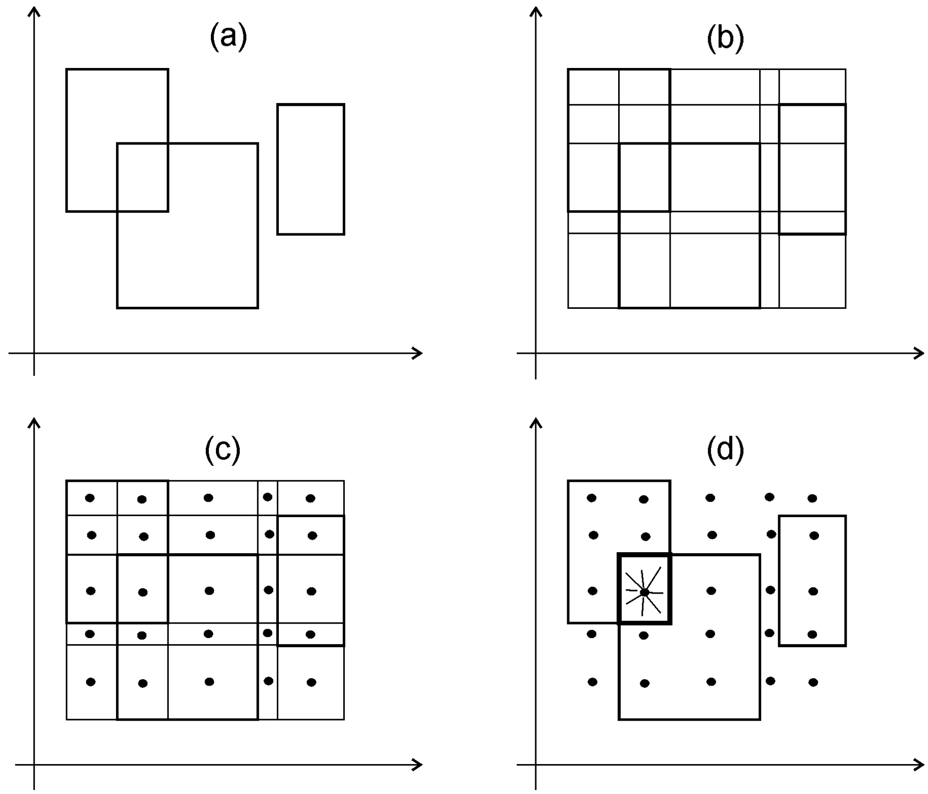
$$\left\{\mathbf{p} \in [\mathbf{p}], \exists \mathbf{y} \in [\mathbf{y}], (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2 = 0\right\}.$$

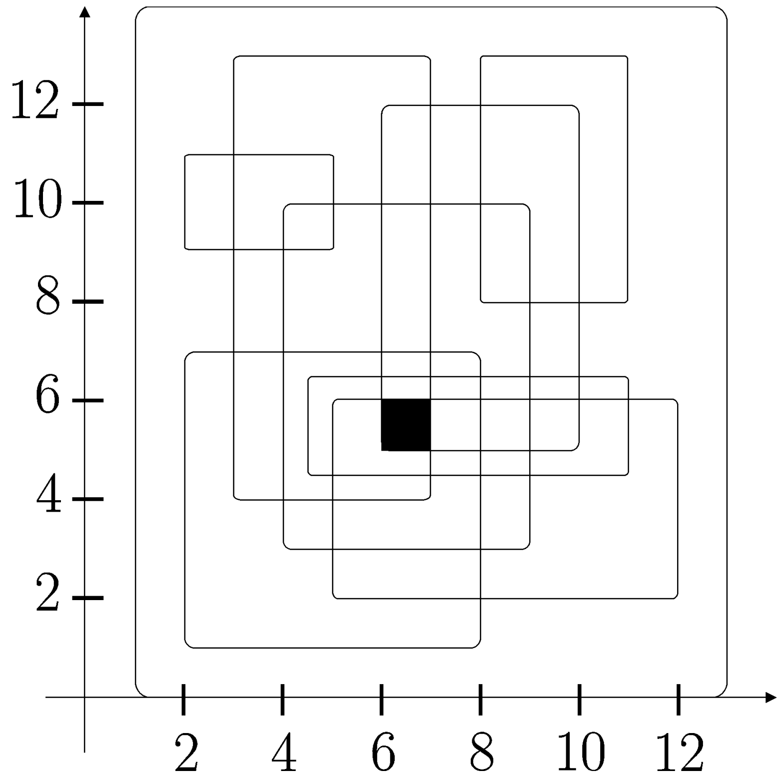
FB(in: $[\mathbf{y}], [\mathbf{p}]$, out: $[\mathbf{p}]$)

$[d_1] := [y_1] - [p_1]\,;$
$[d_2] := [y_2] - [p_2]\,;$
$[c_1] := [d_1]^2\,;$
$[c_2] := [d_2]^2\,;$
$[c_3] := [p_3]^2\,;$
$[e] := [0,0] \cap ([c_1] + [c_2] - [c_3])\,;$
$[c_1] := [c_1] \cap ([e] - [c_2] + [c_3])\,;$
$[c_2] := [c_2] \cap ([e] - [c_1] + [c_3])\,;$
$[c_3] := [c_3] \cap ([c_1] + [c_2] - [e])\,;$
$[\bar{p}_3] := [p_3] \cap \sqrt{[c_3]};$
$[d_2] := [d_2] \cap \sqrt{[c_2]};$
$[d_1] := [d_1] \cap \sqrt{[c_1]};$
$[p_2] := [p_2] \cap ([y_2] - [d_2])\,;$
$[p_1] := [p_1] \cap ([y_1] - [d_1])\,;$

5.1 Relaxed intersection

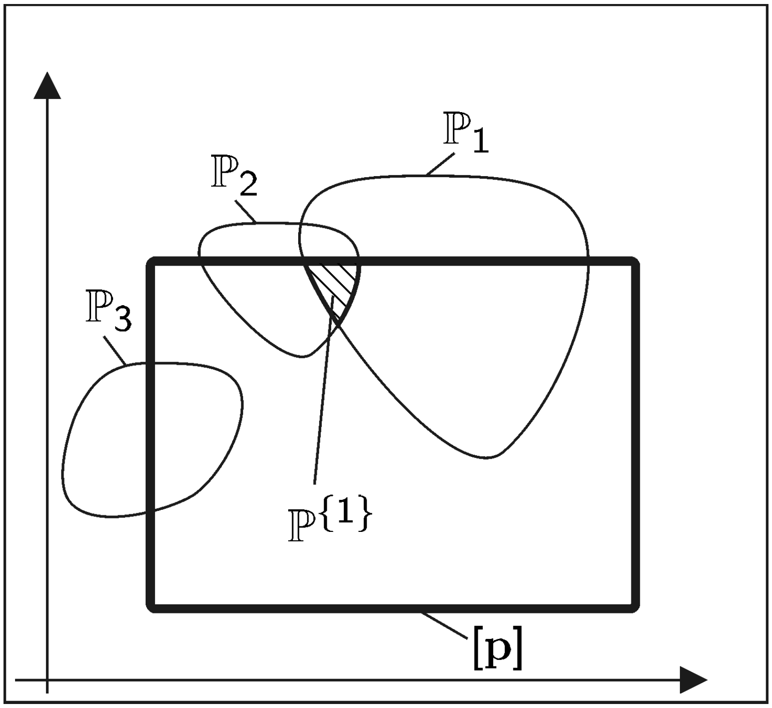
Computing the q relaxed intersection of m boxes is tractable.


(a)
(b)
(c)
(d)

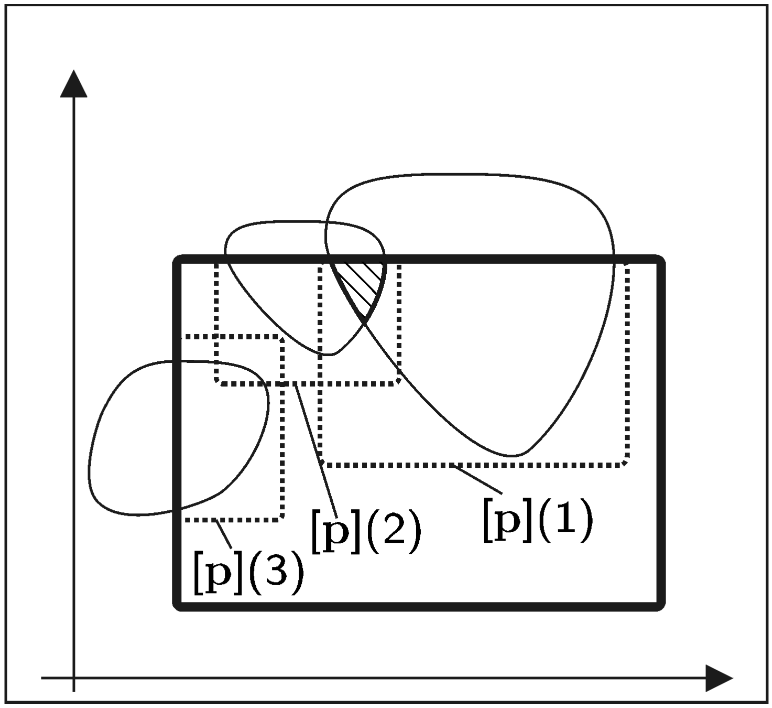


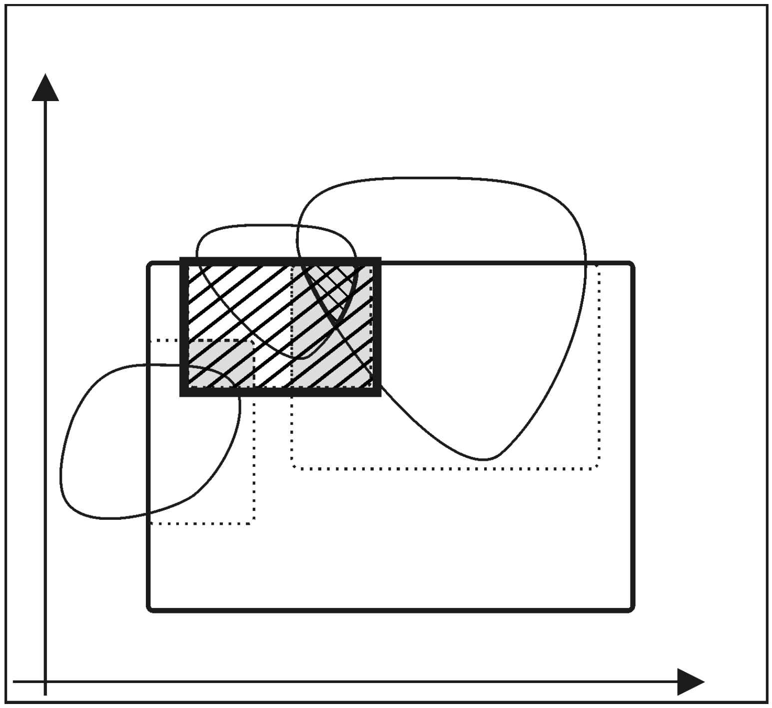
The black box is the 2-intersection of 9 boxes

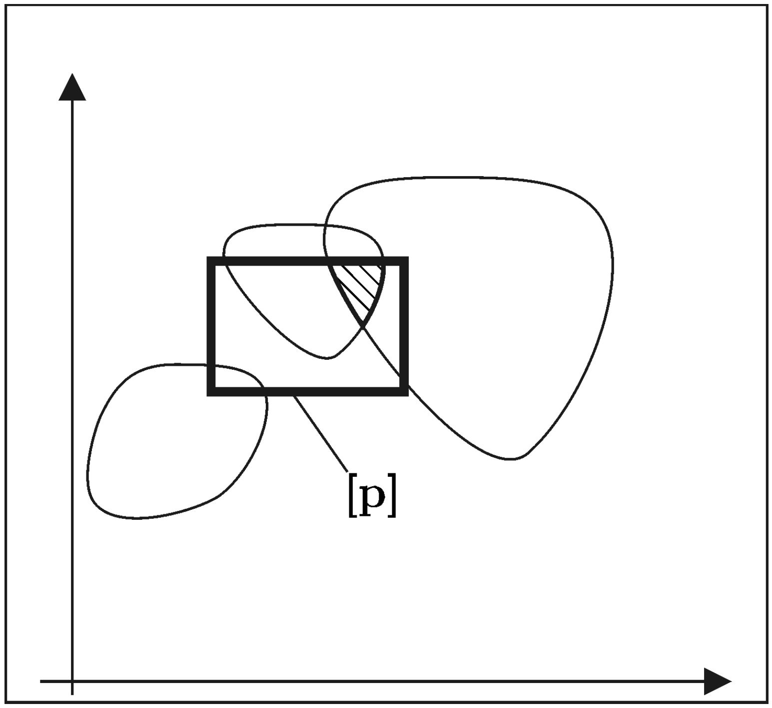
5.2 Algorithm

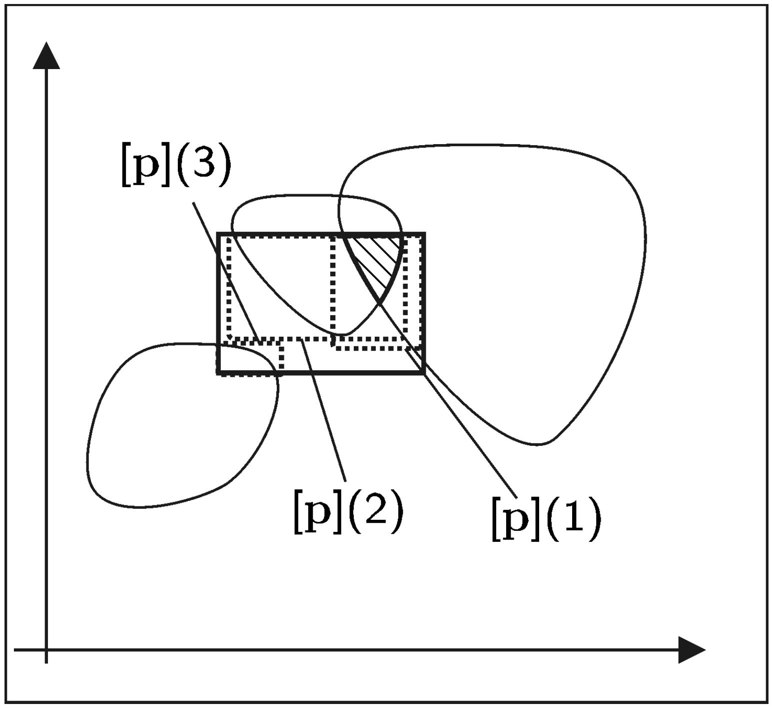


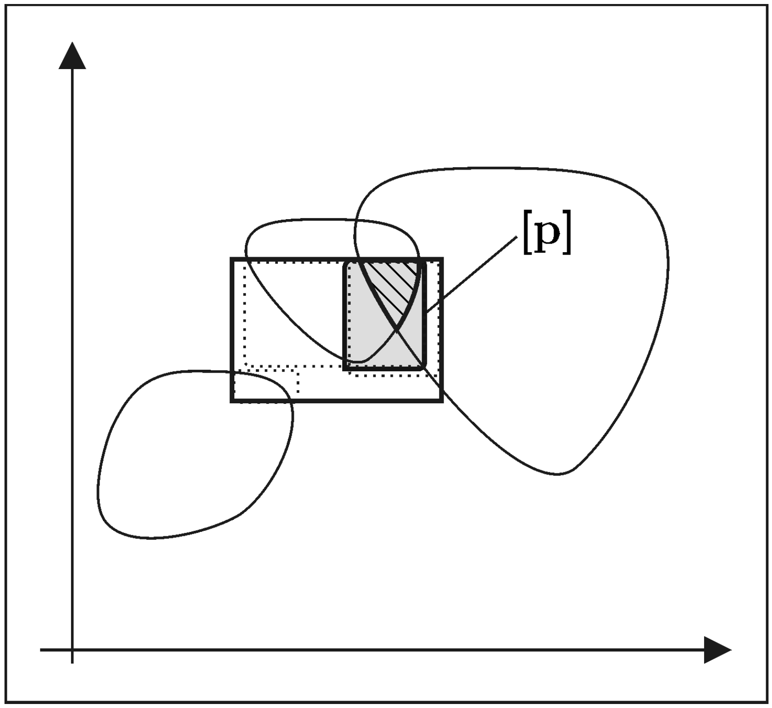
$\mathbb{P}_1$
$\mathbb{P}_2$
$\mathbb{P}_3$
$\mathbb{P}^{\{1\}}$
[p]


[p](1)
[p](2)
[p](3)




[p]

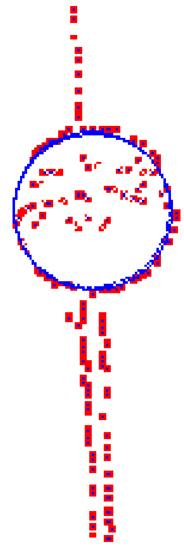

[p](3)
[p](2)
[p](1)


[p]

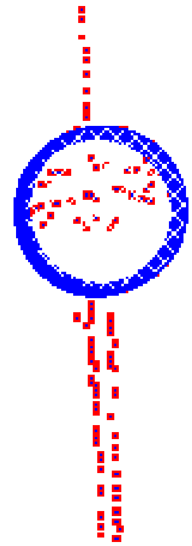
Algorithm Enclose(in: $[\mathbf{p}], [\mathbf{y}](1), \ldots, [\mathbf{y}](m), q$, out: $\mathcal{L}$)

$\mathcal{L} := \{[\mathbf{p}]\}$;
repeat
    pull $([\mathbf{p}],\mathcal{L})$;
    while the contraction are significant
        for $i = 1$ to m, compute $[\mathbf{p}](i)$ enclosing $[\mathbf{p}] \cap \mathbb{P}_i$
        $[\mathbf{p}] := \left[\bigcap_{i \in \{1,\ldots,m\}}^{\{q\}} [\mathbf{p}](i) \right]$
    end repeat
    bisect $[\mathbf{p}]$ and push the resulting boxes into $\mathcal{L}$
until all boxes of $\mathcal{L}$ have a width smaller than ε.

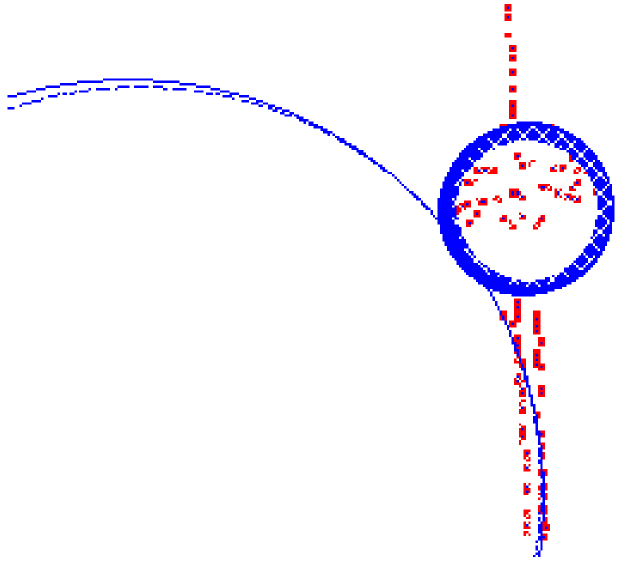
6 Results



$q = 0.70\ m$ (i.e. 70% of the data can be outlier)



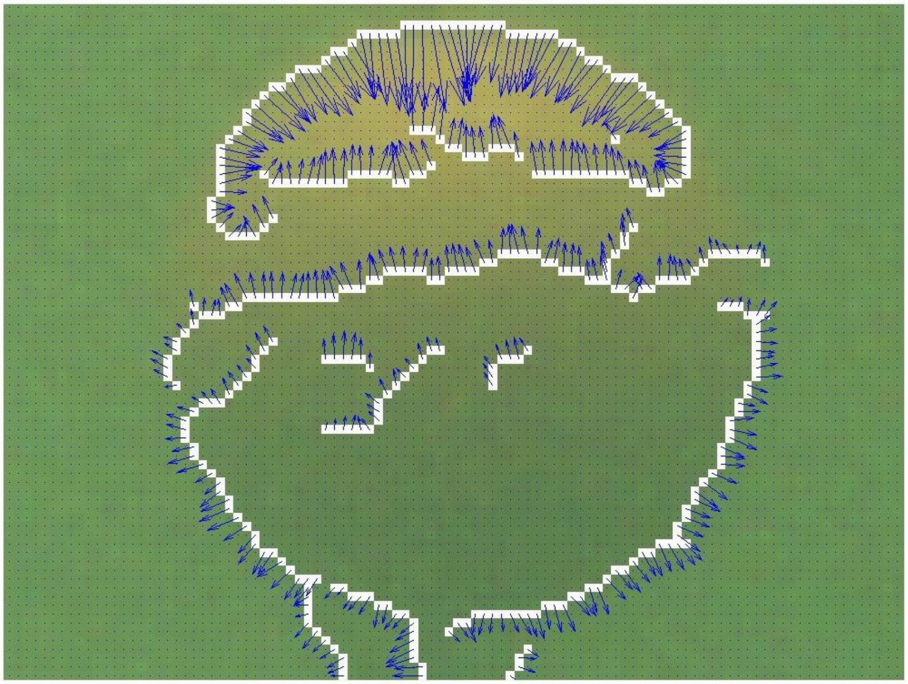
$q = 0.80\ m$ (i.e. 80% of the data can be outlier)



$q = 0.81\ m$ (i.e. 81% of the data can be outlier)

O'Gorman and Clowes (1976), in the context of the Hough transform (1972):

the local gradient of the image intensity is orthogonal to the edge.



Now, $\mathbf{y} = (y_1, y_2, y_3)^\top$ where y_3 is the direction of the gradient.

The gradient condition is

$$\det \begin{pmatrix} \frac{\partial f(\mathbf{p},\mathbf{y})}{\partial y_1} & \cos(y_3) \\ \frac{\partial f(\mathbf{p},\mathbf{y})}{\partial y_2} & \sin(y_3) \end{pmatrix} = 0.$$

For $f(\mathbf{p},\mathbf{y}) = (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2$, we get

$$\mathbf{f}(\mathbf{p},\mathbf{y}) = \begin{pmatrix} (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2 \\ (y_1 - p_1)\sin(y_3) - (y_2 - p_2)\cos(y_3) \end{pmatrix}.$$

New outliers: the edge points that are on the shape, but that do not satisfy the gradient condition.

The computing time is now 2 seconds instead of 15 seconds.

7 Hough transform

The Hough transform is defined by

$$\eta\left(\mathbf{p}\right) = \text{card}\left\{i \in \{\mathbf{1},\ldots,m\}, \exists \mathbf{y} \in [\mathbf{y}](i), \mathbf{f}\left(\mathbf{p},\mathbf{y}\right) = \mathbf{0}\right\},$$

Hough method keeps all $\mathbf{p}$ such that $\eta\left(\mathbf{p}\right) \geq m - q$.

Instead, our approach solves $\eta\left(\mathbf{p}\right) \geq m - q$.

8 Perspective

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0.9
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0.7
0.6
0.5
0.4
0.3
0.2
0.1