A new Interval/Graph approach for nonlinear control

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Introduction

Système dynamique non-linéaire :

$$\begin{cases} \dot{x}(t) & = & f\left(x(t), u(t)\right) \qquad (t > 0) \\ x(0) & = & x_0 \end{cases}$$

avec

$$x(t) \in K \subset \mathbb{R}^n,\ x(0) \in K$$

$$u : [0, \infty) \longrightarrow \mathbb{U}$$

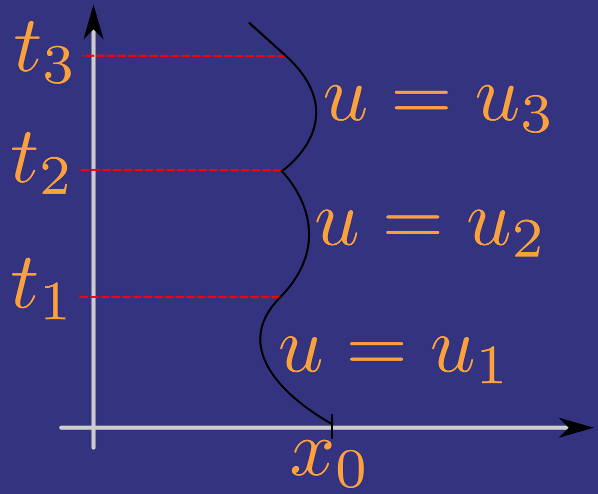
$$f : \ K \times \mathbb{U} \longrightarrow \mathbb{R}^n$$

Introduction

On note

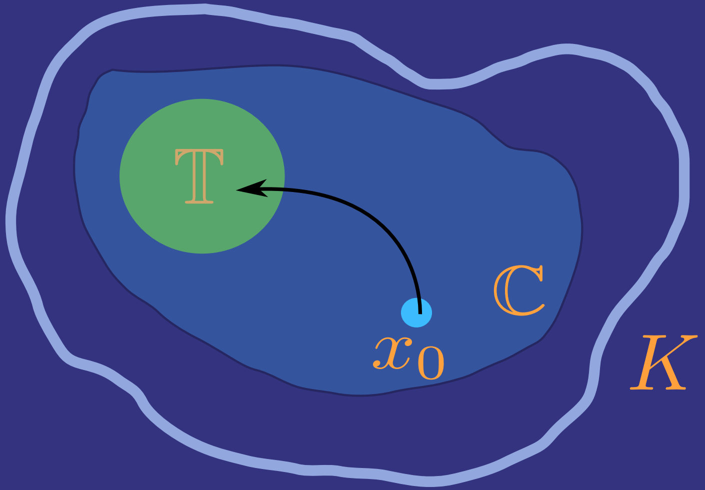
$$x\left(.,u\left(.\right),x_0\right):\mathbb{R}_{\geq 0}\times\mathbb{U}\rightarrow\mathbb{K}\subset\mathbb{R}^n$$

une trajectoire du sys tème :



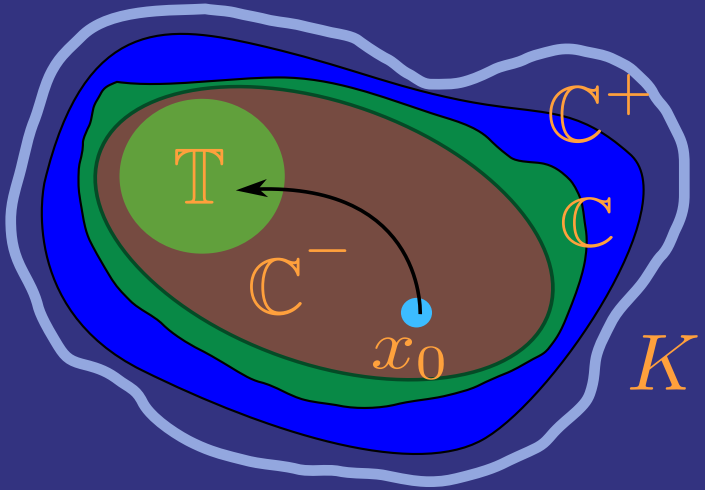
Introduction

Bassin de capture



$$\mathbb{C} = \{x_0 \mid \exists T \geq 0, \exists u \in [0,T] \to \mathbb{U},\\ x(T, u(T), x_0) \in \mathbb{T}\\ \text{et } \forall t \in [0,T], x(t, u(t), x_0) \in K\}$$

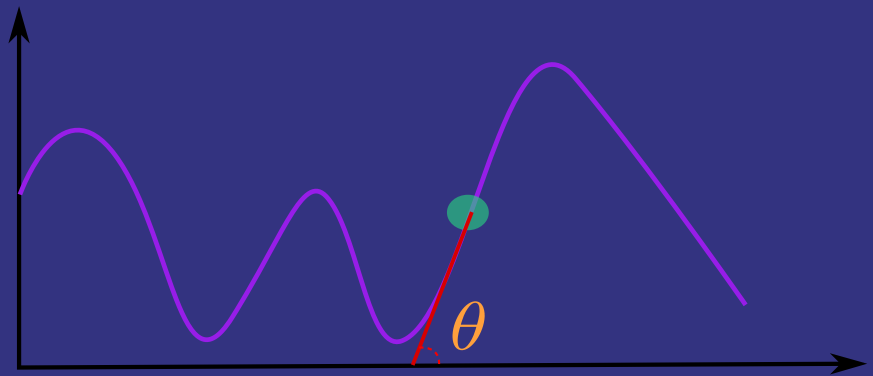
Bassin de capture



Objectif

$$\mathbb{C}^- \subset \mathbb{C} \subset \mathbb{C}^+$$

Grand huit



Etat du système

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} s \\ \dot{s} \end{pmatrix}$$

Librairie : VNODE (Ned Nedialkov)

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$x(., u(.), x_0)$ **est une trajectoire de**

$$\dot{x}(t) = f\left(x(t), u(t)\right)$$

Intégration Numérique Garantie

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$x(., u(.), x_0)$ **est une trajectoire de**

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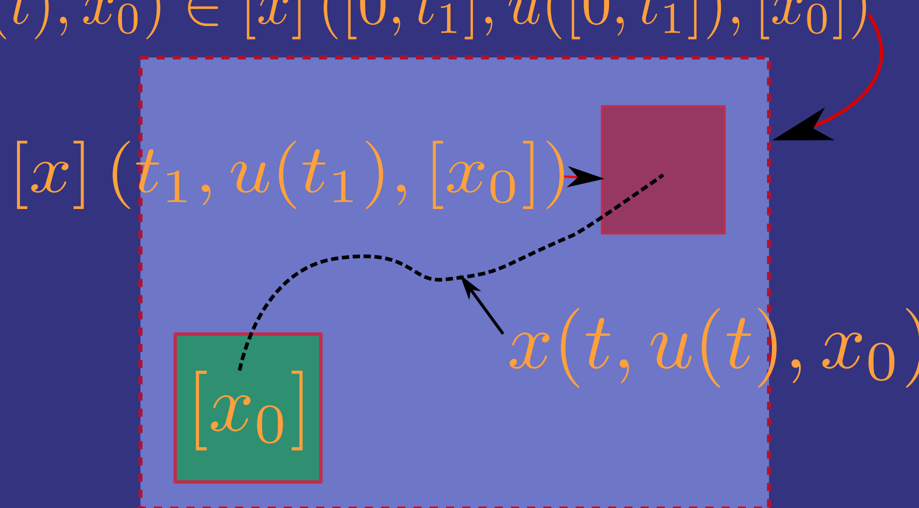
Alors

$$\forall t \in [0, t_1], x(t, u(t), x_0) \in [x]\left([0, t_1], u([0, t_1]), [x_0]\right)$$



Algorithme – Bassin de Capture

Soient

$\mathbb{C}^- \subset \mathbb{C} \subset \mathbb{C}^+, [x_0] \in \mathbb{R}^n$ **et** $t_1 \geq 0$

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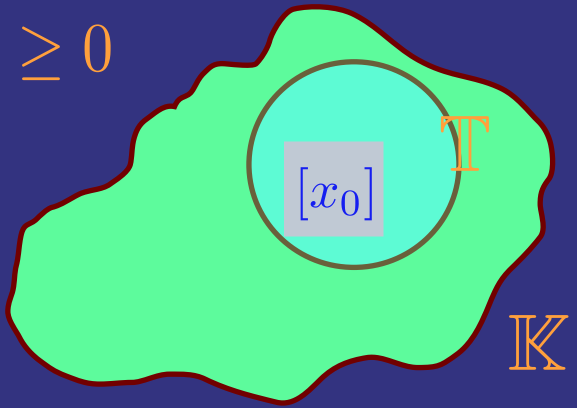

Propriétés

1. $[x_0] \subset \mathbb{T} \Rightarrow [x_0] \subset \mathbb{C}$

Algorithme - Bassin de Capture

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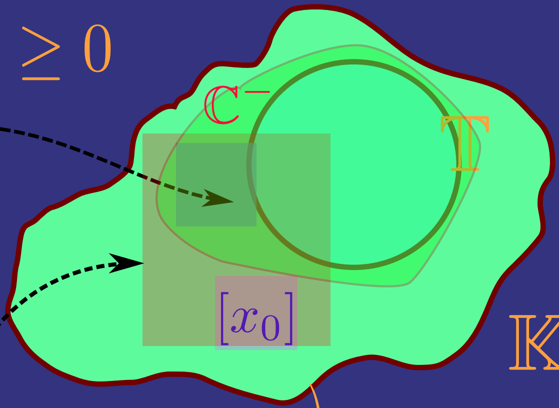

Propriétés

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2. $\left([x]\,(t_1, u(t_1), [x_0]) \subset \mathbb{C}^{-} \textbf{ et } [x]\,([0, t_1], u([0, t_1]), [x_0]) \subset \mathbb{K} \right) \Rightarrow [x_0] \subset \mathbb{C}$

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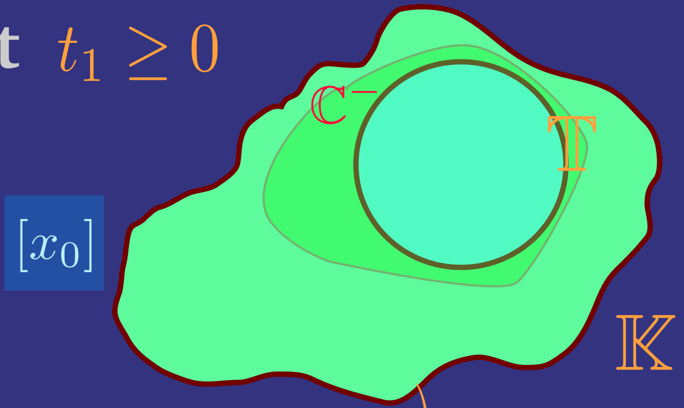

Propriétés

1. $[x_0] \subset \mathbb{T} \Rightarrow [x_0] \subset \mathbb{C}$

2. $\left([x](t_1, u(t_1), [x_0]) \subset \mathbb{C}^- \textbf{ et } [x]([0, t_1], u([0, t_1]), [x_0]) \subset \mathbb{K}\right) \Rightarrow [x_0] \subset \mathbb{C}$

3. $[x_0] \cap \mathbb{K} = \emptyset \Rightarrow [x_0] \cap \mathbb{C} = \emptyset$

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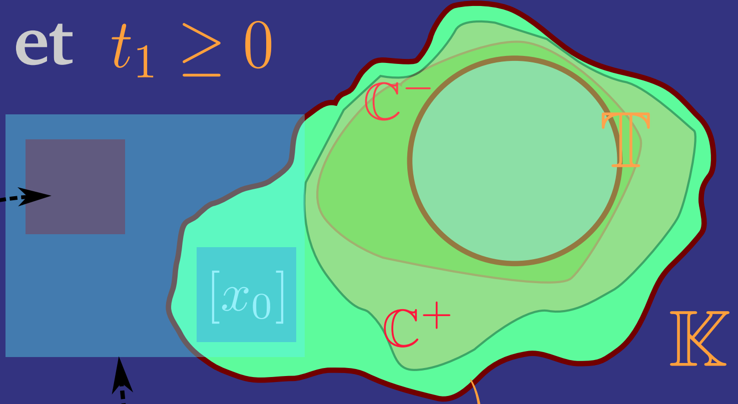

Propriétés

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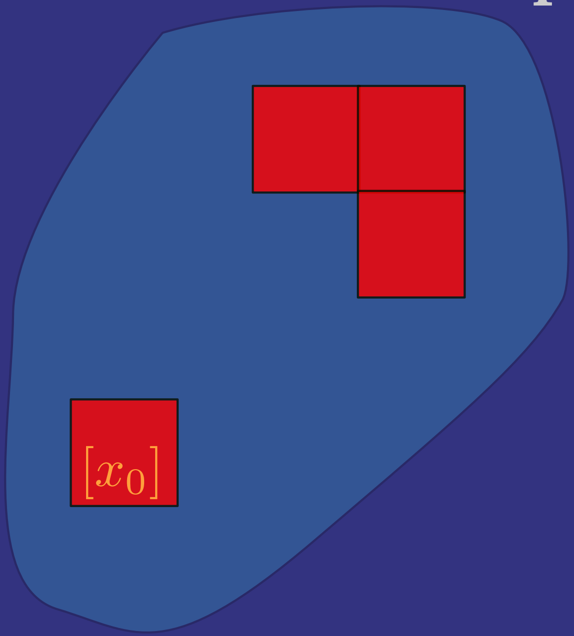
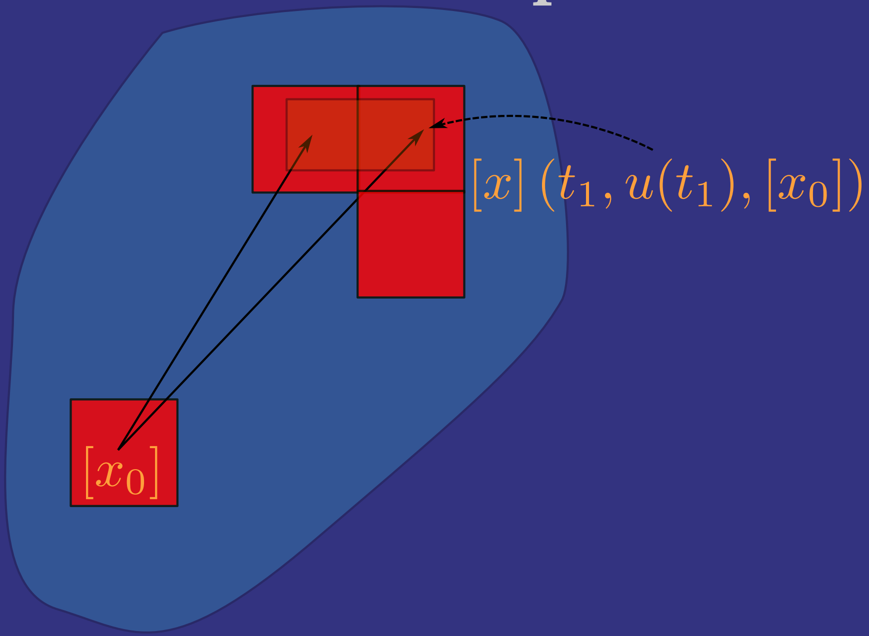
3. $[x_0] \cap \mathbb{K} = \emptyset \Rightarrow [x_0] \cap \mathbb{C} = \emptyset$

4. $\left([x](t_1, \mathbb{U}, [x_0]) \cap \mathbb{C}^+ = \emptyset\right.$ **et** $\left.[x]([0, t_1], \mathbb{U}, [x_0]) \cap \mathbb{T} = \emptyset\right) \Rightarrow$ $[x_0] \cap \mathbb{C} = \emptyset$

Intervalles et Graphe



Intervalles et Graphe



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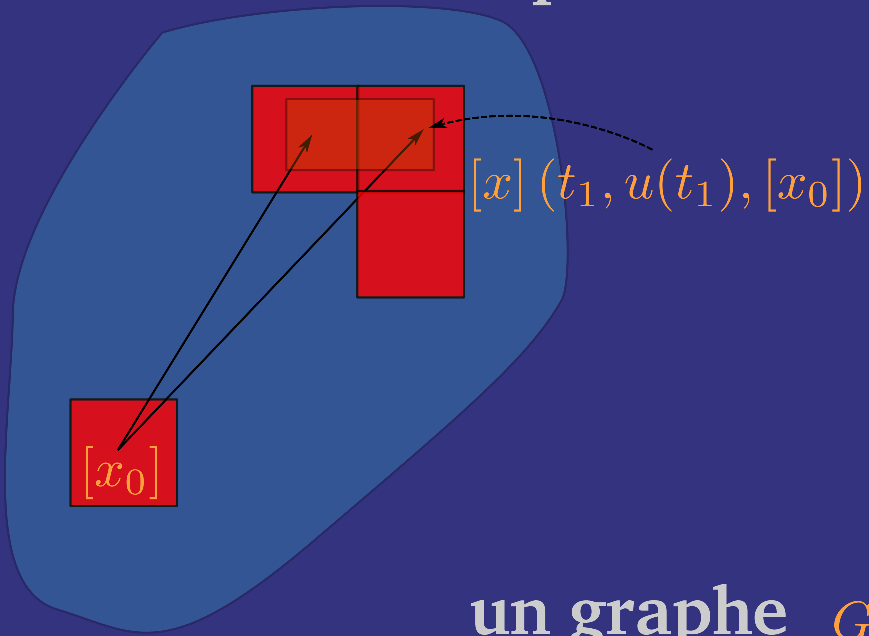
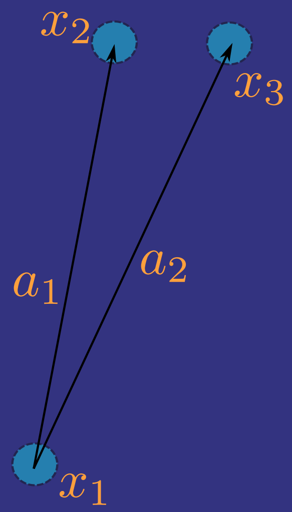




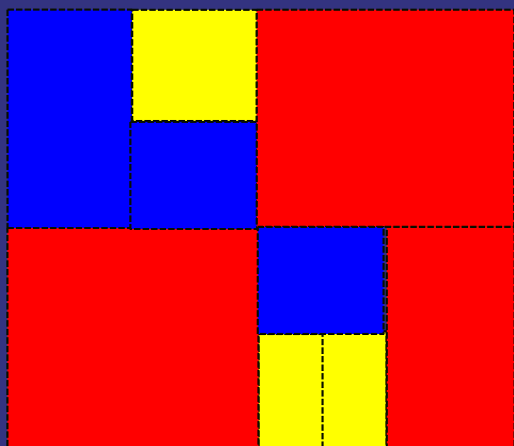
un graphe $G = (X, A)$ **avec**

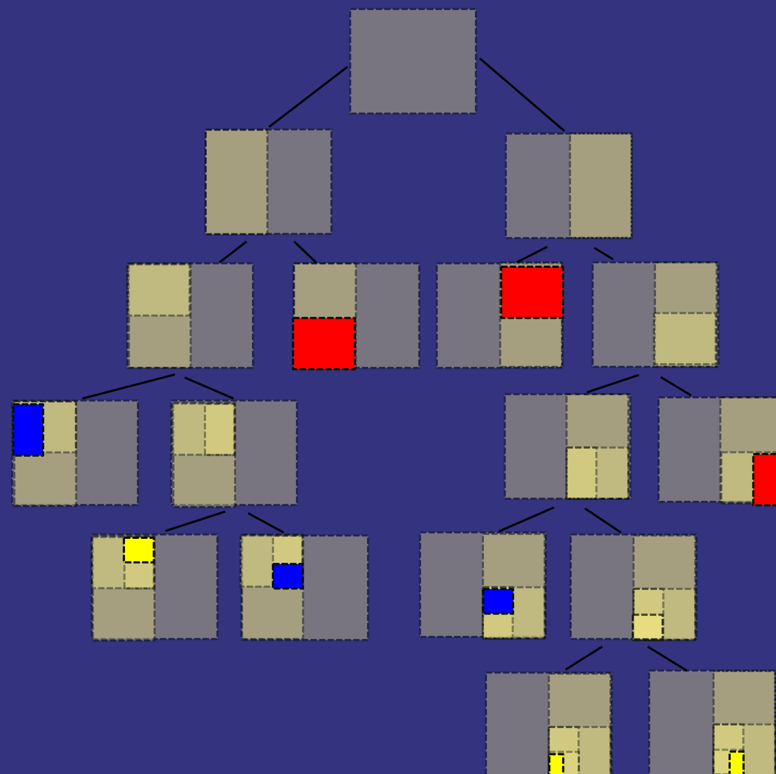
$$X = \{x_1, x_2, x_3, \ldots, x_n\}$$

$$A = \{a_1, a_2, a_3, \ldots, a_n\}$$

Algorithme - Implémentation

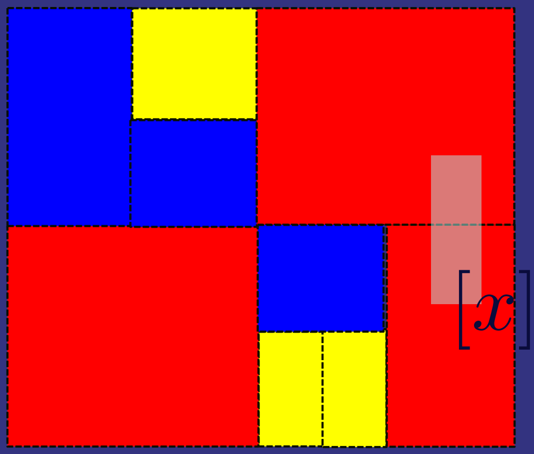
Représentation du sous-pavage

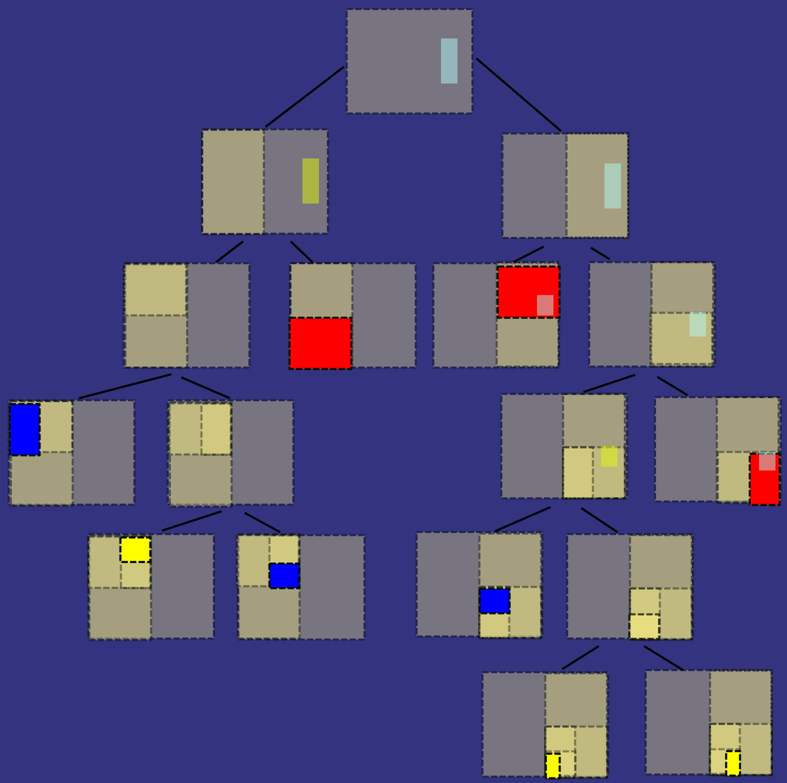




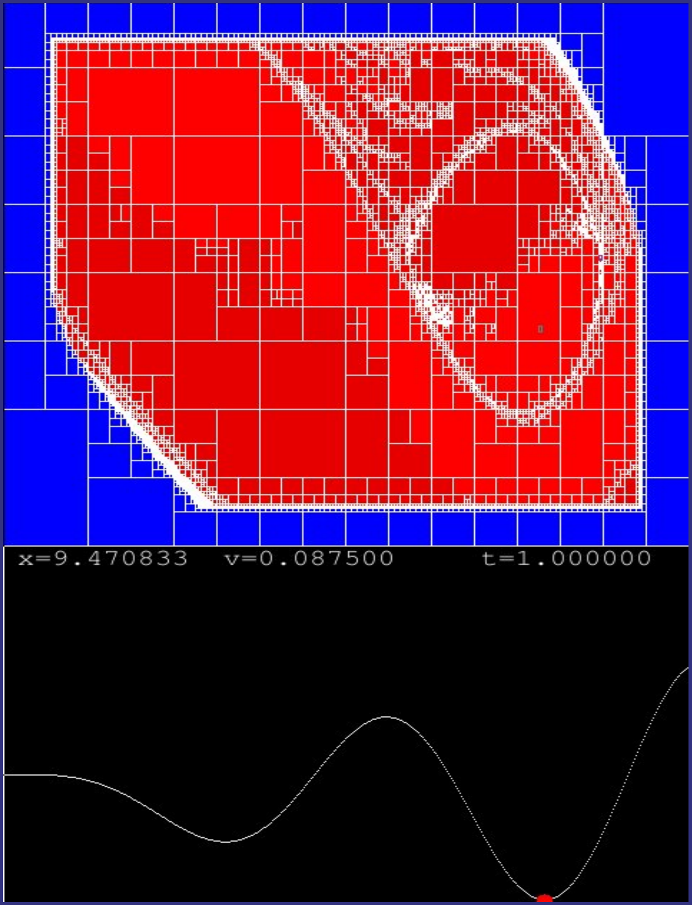
Algorithme - Implémentation

le pavé $[x] \subset \mathbb{C}^-$?





Algorithme – Démonstration



Conclusion

- Approximation garantie du bassin de capture
  - diminuer le temps de calcul
  - extension aux s ystèmes hybrides

- Caractérisation du n oyau de viabilité

$\mathrm{viab}(\mathbb{K}) := \{x_0 \in \mathbb{K} \mid \exists x(.,u(.),x_0), t.q. \forall t \geq 0, x(t,u(t),x_0) \in \mathbb{K}\}$