

**Journée**  
**GT-CPNL & GT-MEA**

# Model Predictive Control and its Applications to Some Mechatronic Systems



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- Context of Mechatronic systems
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  - ✓ Basic definitions in Mechatronics
  - ✓ Main technical areas (applications)
- **Example 1 : One-leg hopping robot**
  - ✓ System description and dynamics
  - ✓ Control problem formulation
  - ✓ Proposed predictive controller
  - ✓ Simulation results
- **Example 2 : Hard disc drive**
  - ✓ System description and dynamics
  - ✓ Control problem formulation
  - ✓ Proposed predictive controller
  - ✓ Simulation results
- **Example 3 : Inertia wheel inverted pendulum**
  - ✓ System description and dynamics
  - ✓ Control problem formulation
  - ✓ Proposed predictive controller
  - ✓ Real-time experimental results
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# Context of Mechatronic systems



## Introductory example : Adaptive cruise control (ACC)

In the cars of today



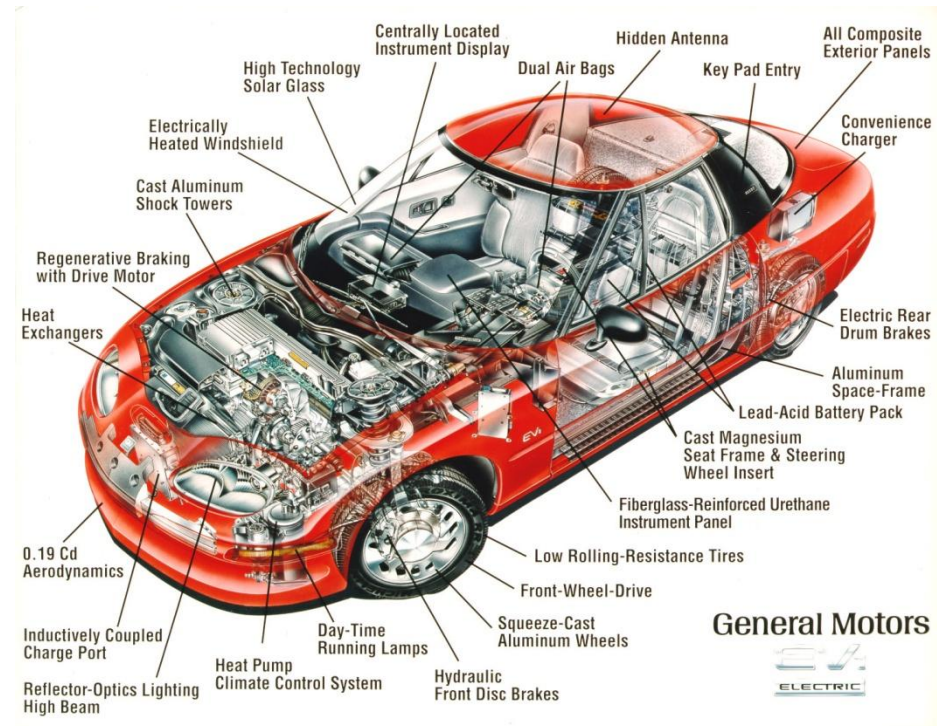
There are more than 100 microprocessors controlling various systems



The more complicated systems are not just on/off control

### Examples :

- ✓ Speed control
- ✓ Active vibration control
- ✓ Trajectory control
- ✓ Transmission control
- ✓ Adaptive cruise control
- ✓ Autonomous Emergency Braking (AEB)
- ✓ ... etc



A very complex multi-integrated system !

## Introductory example : Adaptive cruise control (ACC)

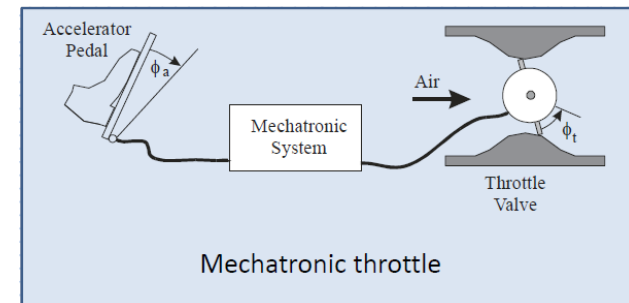
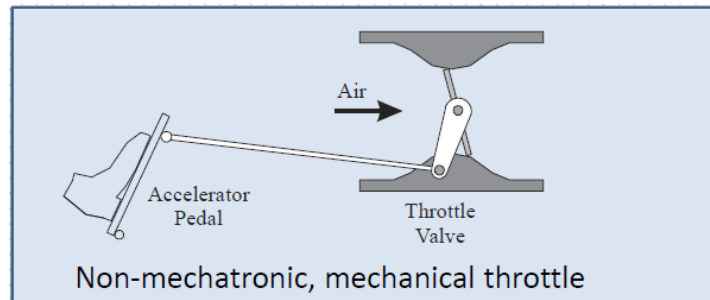
Cruise control is not a more convenience option than a safety feature !

Conventional cruise control systems simply maintain a preset speed :



### How does it work ?

- 1- The driver presses a button to set the speed,
- 2- Cruise control system controls the speed by adjusting the throttle position,
- 3- The control is deactivated if the driver steps on the brake, changes the speed setting, or press the button.



### However

- 1- It is not aware of other vehicles' movement,
- 2- The driver must be always aware → Possibility of mistakes !
- 3- Possibility of collision with the leading car if not manually slowed down.

## Introductory example : Adaptive cruise control (ACC)

**Adaptive cruise control (ACC)**, is an enhancement to a conventional cruise control system which allows the vehicle to follow a forward vehicle at an appropriate distance.

It actively maintains a preset distance between vehicles rather than a preset speed.

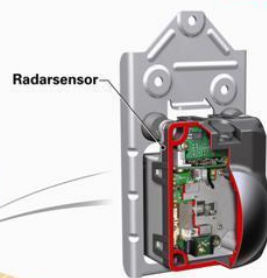
### How does it work ?

- 1- A laser or radar sensor in the front of the vehicle measures the distance to the vehicle ahead.
- 2- The driver then selects a distance that suits the driving conditions.
- 3- The system automatically maintains that distance as traffic speeds up and slows down.

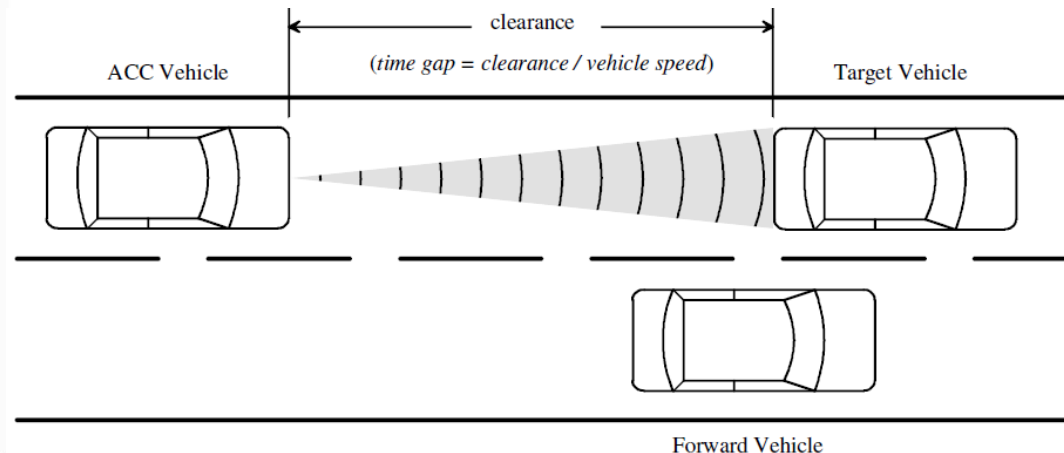
- ✓ Adaptive cruise control is much better than conventional cruise control in heavy traffic.
- ✓ It reduces the risk of rear ending another vehicle if the driver isn't paying attention.

adaptive cruise control

05/03



audipassion.com

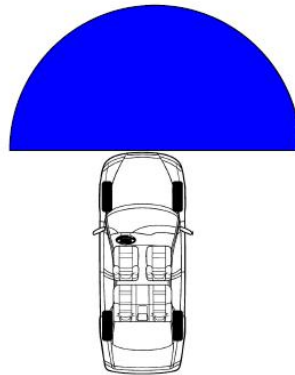




## Introductory example : Adaptive cruise control (ACC)



Radar sensor  
Detection field



Laser sensor  
detection field



Example of Laser sensor  
From IBEO

Autoliv-CelsiusTech Electronics	
Modulation characteristics	Modulation type FMCW
Radar scanning principle	Mechanical scanning
Frequency	76-77 GHz
Transmitted power	10mW
Minimum tracking distance	2 m
Maximum tracking distance	200 m
Update rate of radar	10 Hz
Field of view	24°
Angle resolution	0.1°
Distance resolution	1 m

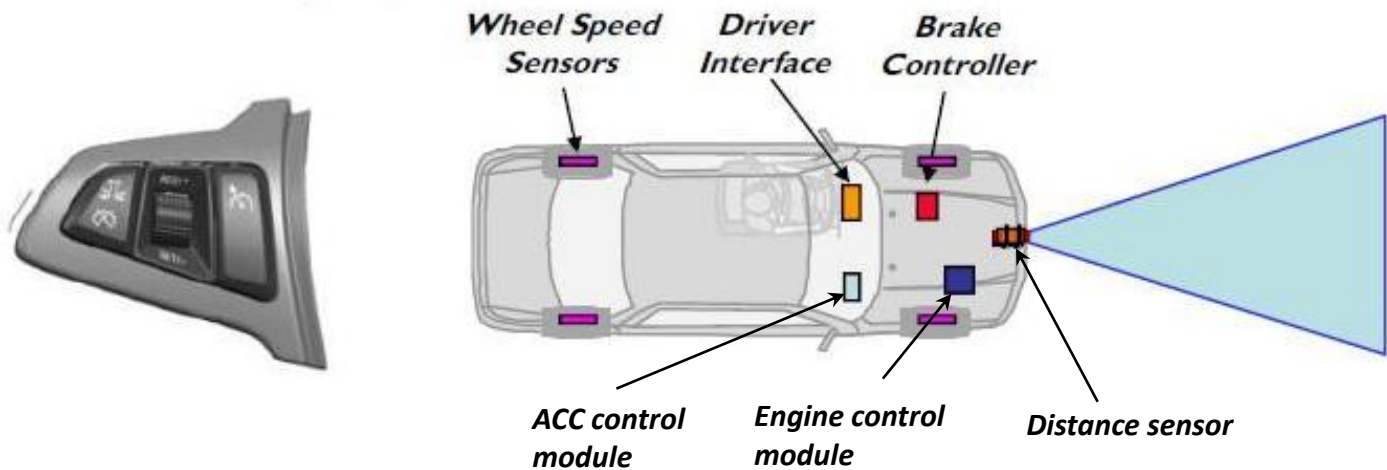
Technical specifications

IBEO Laser scanner LD Automotive	
Minimum tracking distance	0.4 m
Maximum tracking distance	100 m
Update rate of laser	10 Hz
Field of view	up to 270°
Angle resolution	0.25°
Distance resolution	0.004 m

Technical specifications

## Introductory example : Adaptive cruise control (ACC)

- The ACC module interacts with the throttle, the brake system to speed up or slow down



### Sensors:

- Four wheel sensors
- Brake pedal sensor
- Throttle pedal sensor
- Laser/radar sensor
- ...

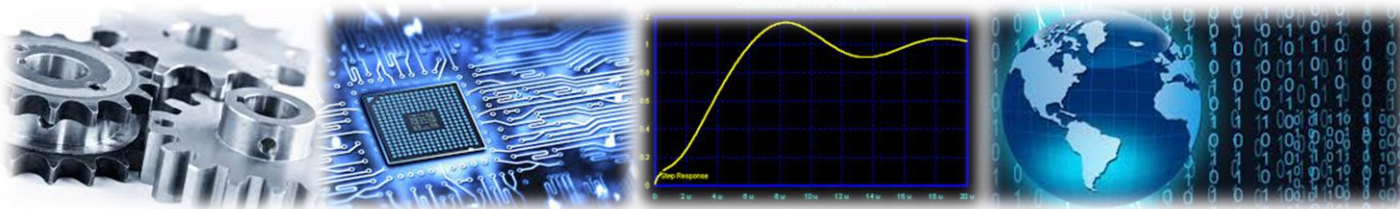
### Actuators :

- Brake actuator
- Throttle actuator



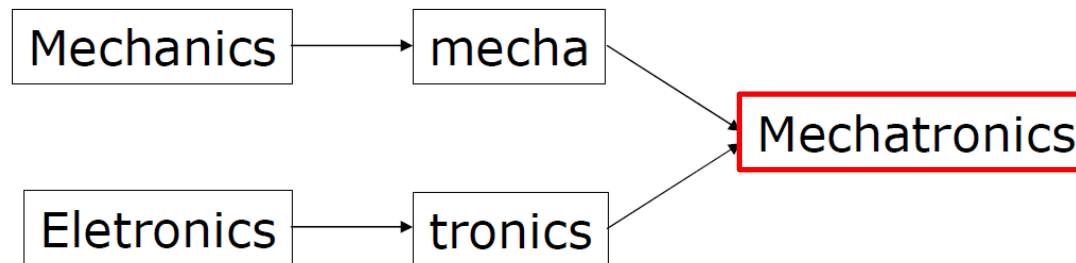


# Basic definitions in Mechatronics



## What is Mechatronics ?

- The word **Mechatronics** was first coined by a senior engineer of the Japanese company Yaskawa, in 1969, and the company was granted trademark rights on the word in 1971
- Initially, the word **Mechtronics** was defined as the combination of "mecha" of **mechanics** and "tronics" of **electronics**



- The word soon received broad acceptance in industry and has taken a wider meaning and is now widely being used as a technical jargon to describe a philosophy in engineering technology
- In order to allow its free use, Yaskawa decided to abandon his rights on the word in 1982

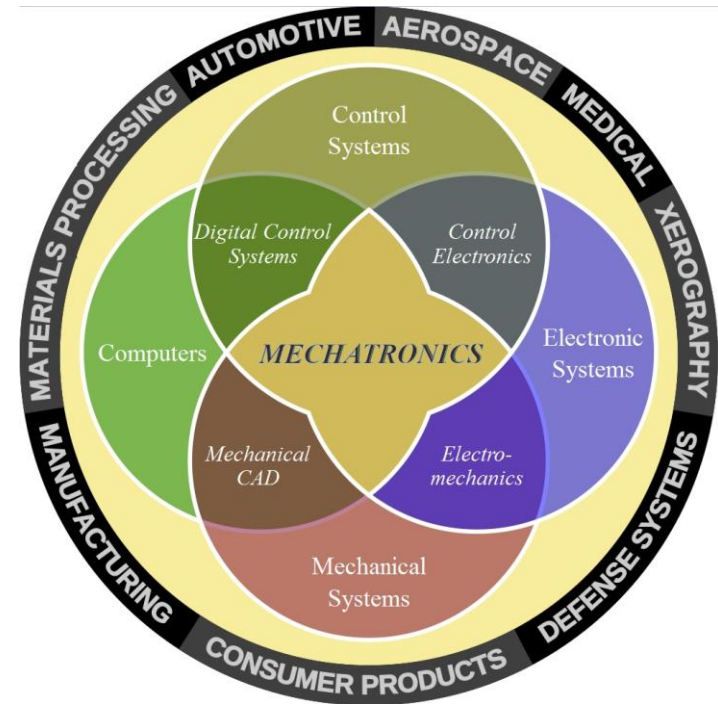
## What is Mechatronics ?

- For this wider concept of Mechatronics, different definitions has been proposed within the scientific community
- The most commonly used definition is the following :

Mechatronics is the synergistic integration of :

- ✓ **Mechanical engineering,**
- ✓ **Electronics,**
- ✓ **Control engineering, and**
- ✓ **Computer technology**

in the design and manufacture of products and processes



Rensselaer Polytechnic Institute

Synergistic integration means that the Mechatronic engineers have to study the aspects of engineering that are vital for the design and manufacture of products.

## Brief history of Mechatronics

### ● The first stage [late 60s] :

- ✓ This period corresponds to the years when this term was introduced.
- ✓ During this stage, **technologies** used in Mechatronic systems **developed** rather **independently** and **individually**.

### ● The second stage [early 80s] :

- ✓ A **synergistic integration** of different **technologies** started taking place.
- ✓ The notable example is **optoelectronics**.
- ✓ The concept of **hardware/software co-design** also started in this period.

### ● The third stage [Early 90s] :

- ✓ The most notable aspect of this stage is the increased use of **computational intelligence** in Mechatronic products and systems.
- ✓ Another important achievement is the possibility of **miniaturization** of components; in the form of **micro actuators** and **micro sensors** (i.e. Micro Mechatronics).

## Different levels of Mechatronics

● Mechatronics has evolved through the following stages [Gera, 2006]:

✓ **Primary Level Mechatronics:** Integrates electrical signaling with mechanical action at the basic control level for e.g conveyors, and fluid valves.



✓ **Secondary Level Mechatronics:** Integrates microelectronics into electrically controlled devices for e.g. DC motors.



✓ **Tertiary Level Mechatronics:** Incorporates advanced control strategy using microelectronics, microprocessors and other application specific integrated circuits for e.g. automotive systems.



✓ **Quaternary Level Mechatronics:** This level attempts to improve smartness a step ahead by introducing intelligence ( artificial neural network and fuzzy logic ) and fault detection and isolation ( F.D.I.) capability into the system.



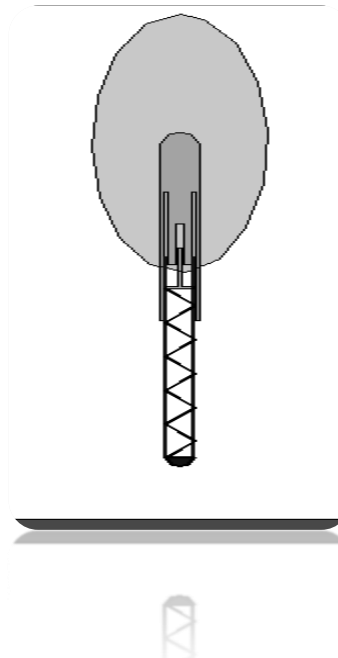


## Main technical areas (applications)

- ✓ Robotics (industrial and special)
- ✓ Automotive systems (ACC, AEB, vibrations, ...)
- ✓ Transportation and Vehicle Systems
- ✓ Actuators and sensors
- ✓ Computer facilities (printers, plotters, HDD, ...)
- ✓ Simulators for training of pilots and operators
- ✓ Manufacturing (machine tool, laser cutting, ...)
- ✓ Micro devices and optoelectronics
- ✓ Vibrations and noise control
- ✓ Power and energy devices
- ✓ Medical mechatronic systems
- ✓ Consumer Products
- ✓ Photo and video equipment (cameras, DVD players)
- ✓ Show industry
- ✓ ... etc





**Example 1****One-Leg Hopping Robot**

## From natural to artificial hopping

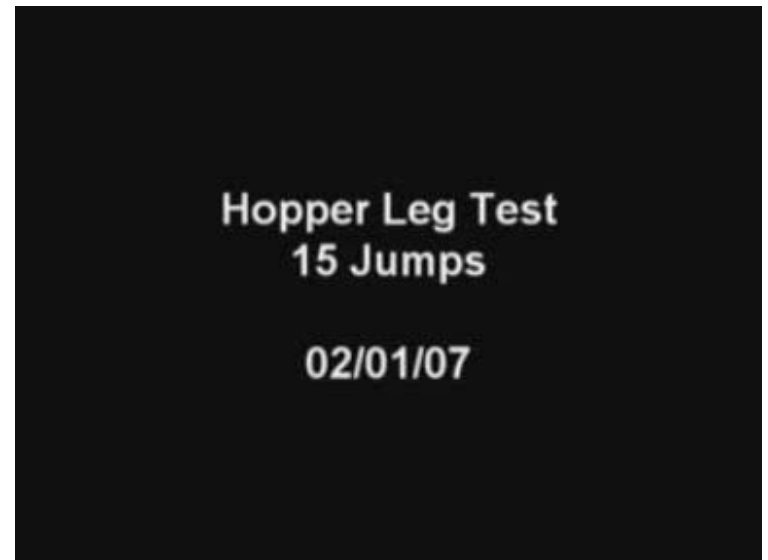
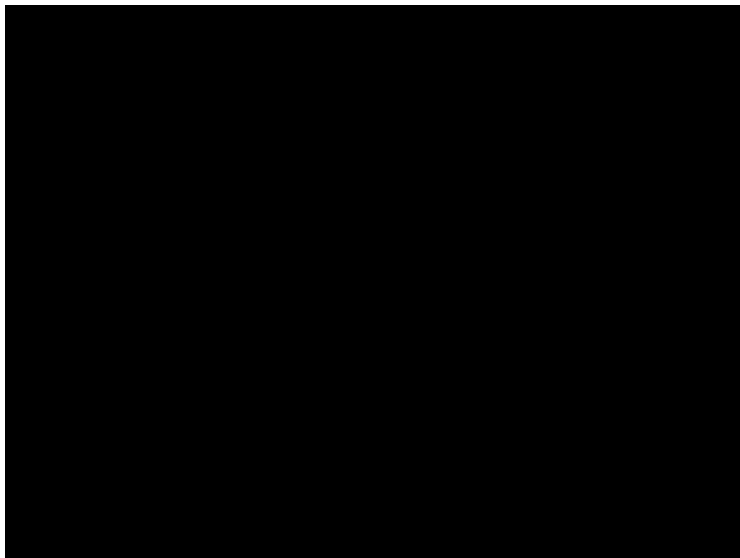


Kangaroo (Australia)

Kangaroos have a very performant locomotion

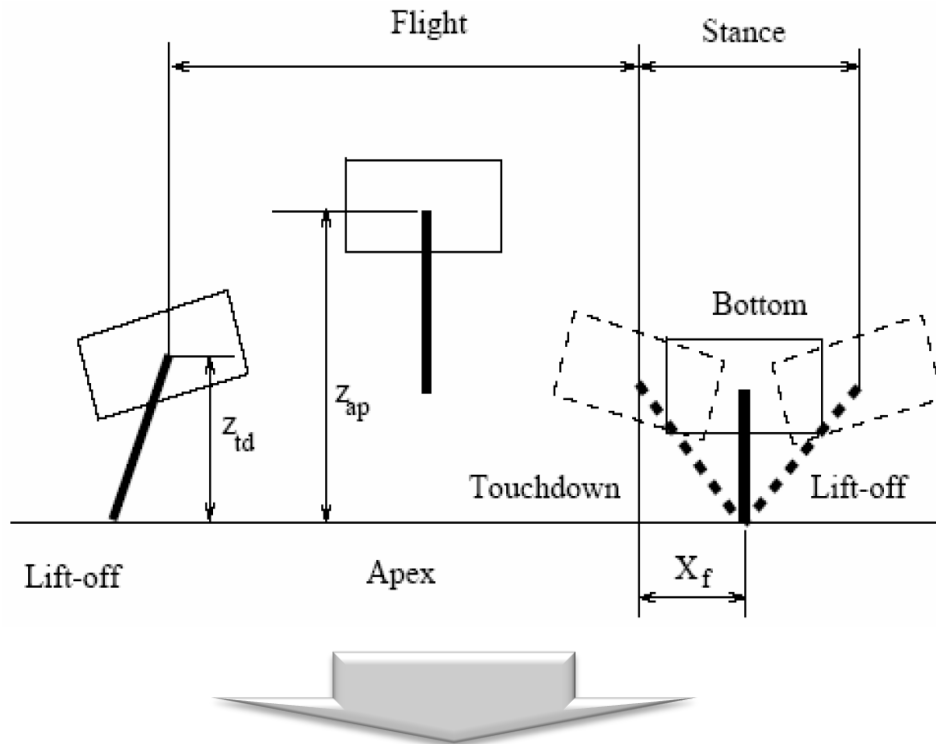
- ✓ Speed : Up to 90 km/h
- ✓ 70% of energy stored in Achilles' tendon
- ✓ Jumps : Up to 3.5m height and 13m length

One-leg hopper (Ohio University)



## Problem formulation

Hopping cycle decomposition



- ✓ Two main phases : stance / Flight
- ✓ Two transition phases : take off / landing

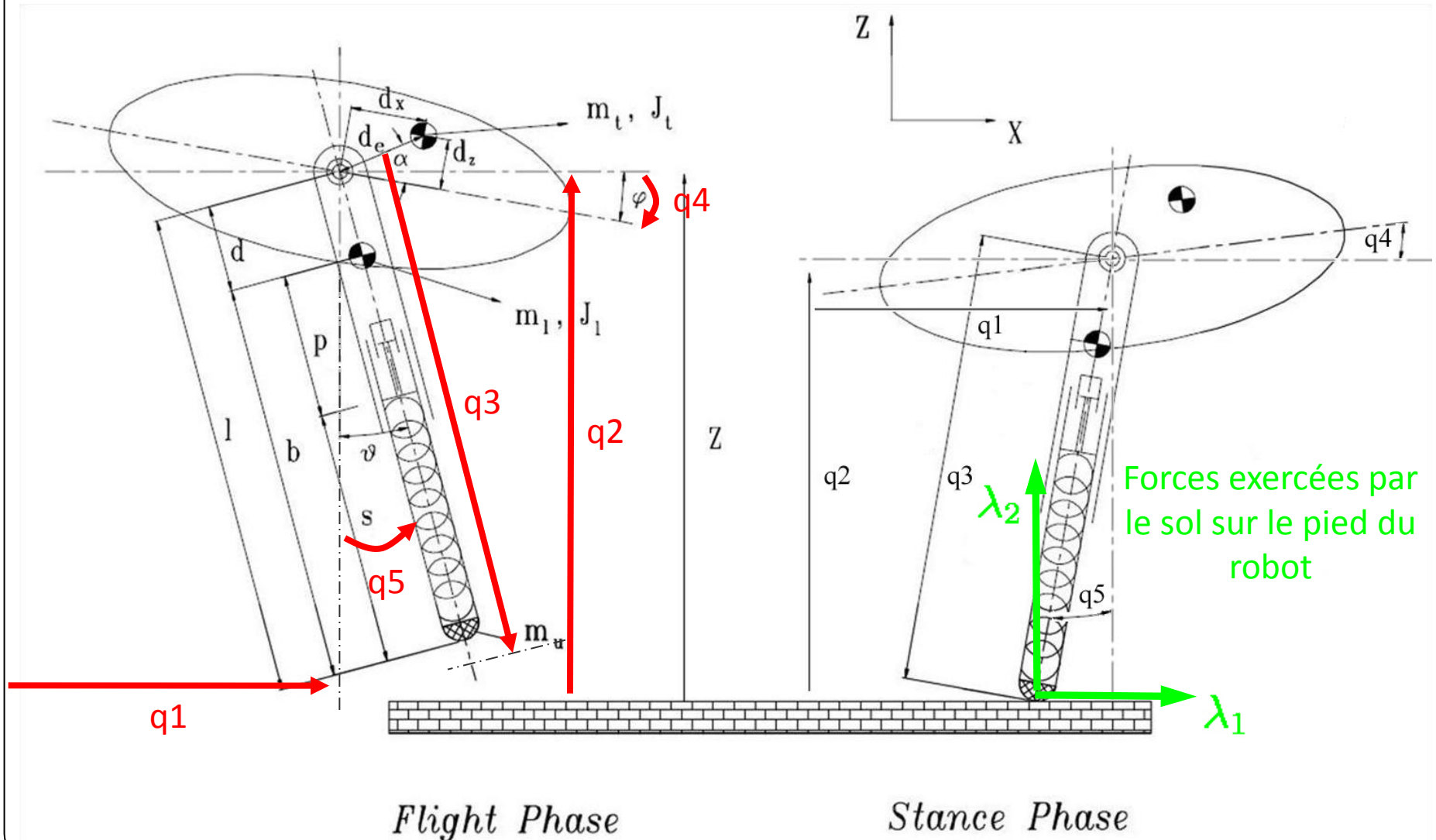
Objective:

Design controllers to achieve stable periodic jumps for a one-leg hopping robot

Difficulties:

- ✗ High nonlinear dynamics
- ✗ Hybrid dynamics
- ✗ Underactuated system
- ✗ Very instable

## Description of the robot



## Dynamic modelling of the robot

### Flight phase :

Lagrange formulation : 
$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} + \frac{\partial F}{\partial q_i} = Q_i$$

Dynamics in a matrix form: 
$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} + F(q) + G(q) = Su$$

$q, \dot{q}, \ddot{q} \in \mathbb{R}^5 \quad u \in \mathbb{R}^2 \quad \longrightarrow \quad \text{Underactuated system}$

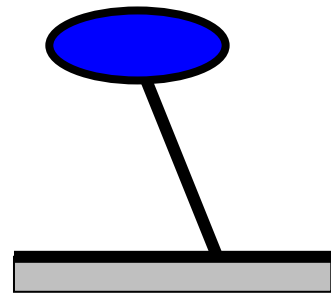
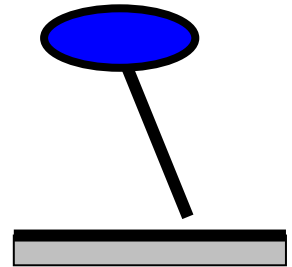
**Stance phase :** Can be deduced from the flight phase model

$$\Phi(q) = 0 \Rightarrow J(q)\dot{q} = 0 \Rightarrow J(q)\ddot{q} + P(q, \dot{q}) = 0$$

$$\begin{cases} M(q).\ddot{q} + N(q, \dot{q}).\dot{q} + F(q) + G(q) = U + J(q)^T \lambda \\ J(q)\ddot{q} + P(q, \dot{q}) = 0 \end{cases}$$

$J \in \mathbb{R}^{2 \times 5}$  : Jacobian matrix of constraints

$\lambda \in \mathbb{R}^2$  : Lagrange multipliers



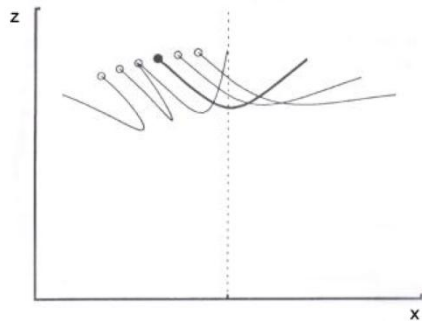
## Proposed control scheme

Raibert's controller includes three parts

Control of the length of the leg :

$$F_l = cte$$

Control of the foot placement :

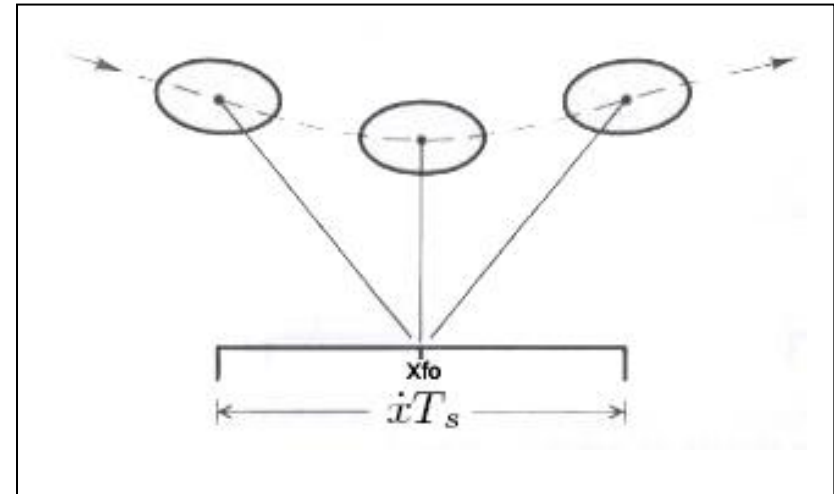


$$x_f = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d)$$

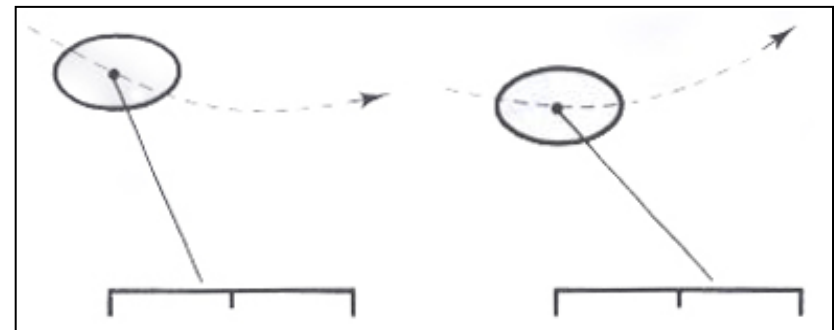
Control of the behavior of the robot :

$$\Gamma = k_p(\phi_d - \phi) - k_v(\dot{\phi})$$

Symmetrical trajectory of the robot



Foot placement of the robot :  
(asymmetrical trajectory)





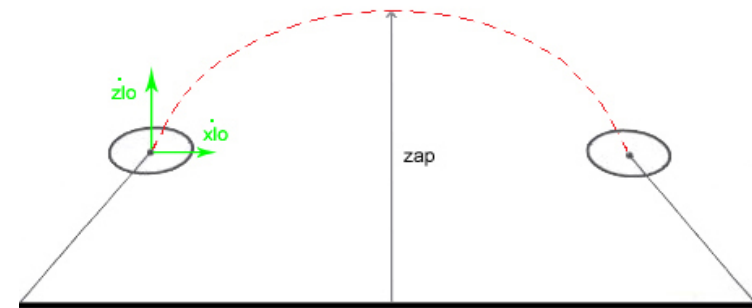
## Proposed control scheme

### Basic idea :

Replace the controller of the length of the leg by a NMPC

### Ballistic trajectory

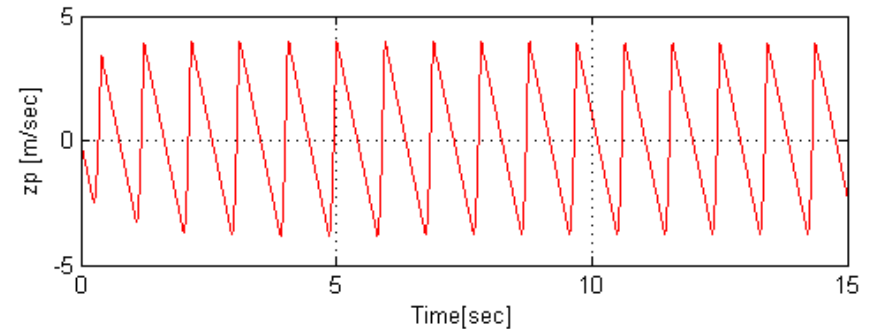
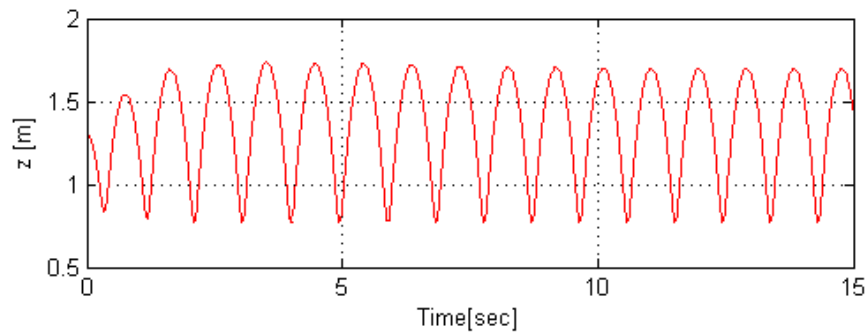
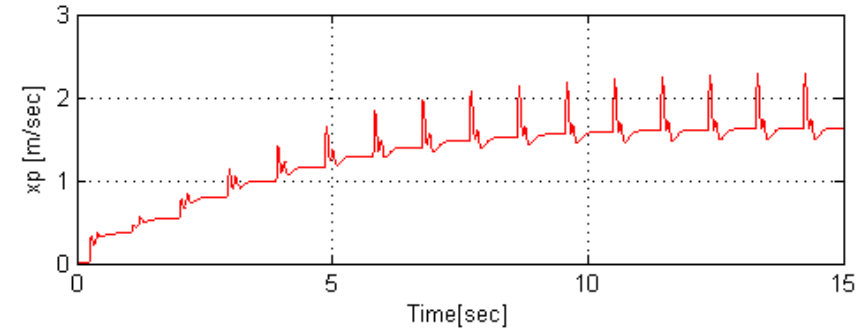
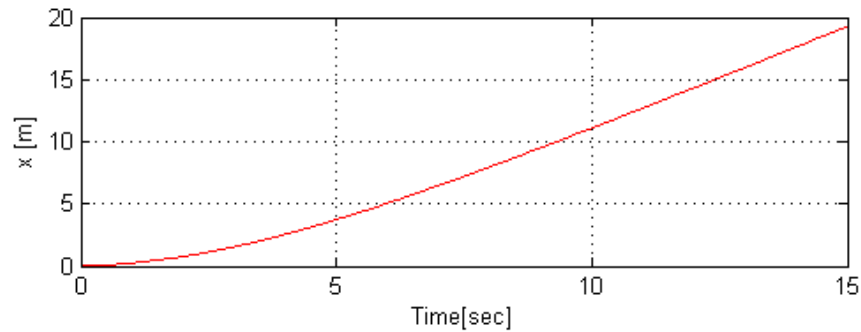
$$\begin{cases} \dot{x}_{lo} = \dot{x}_d \\ \dot{z}_{lo} = \sqrt{2g(z_{ap} - z_{lo})} \end{cases}$$



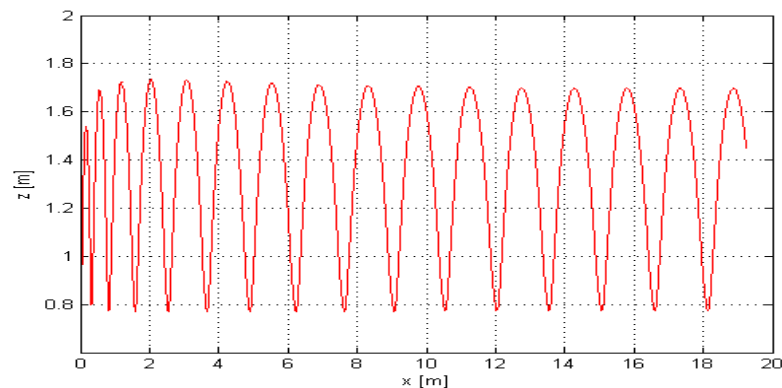
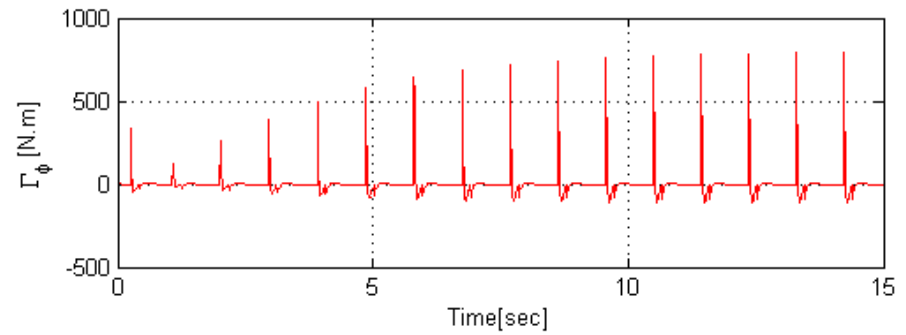
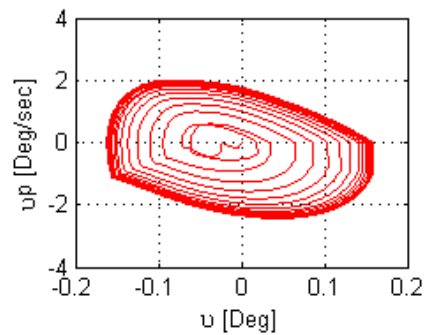
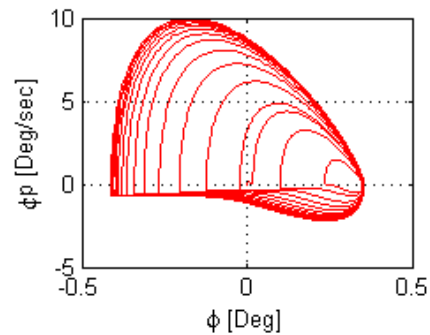
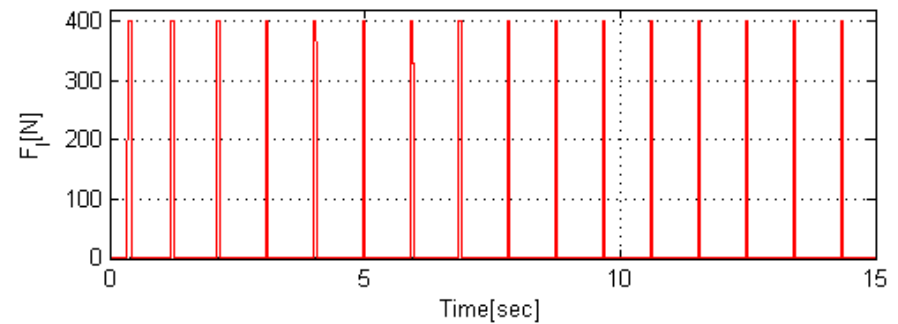
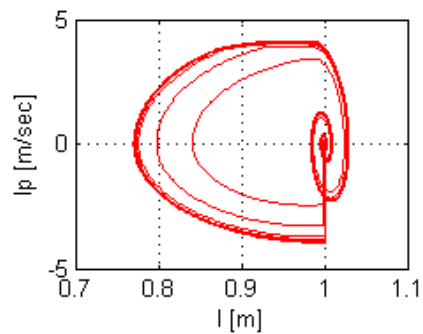
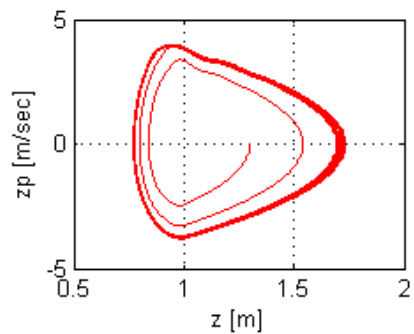
$$\min_u J = \sum_{k=1}^{N_p} Q_1(l_0(k) - l(k))^2 + Q_2(\dot{x}_{lo}(k) - \dot{x}(k))^2 + Q_3(\dot{z}_{lo}(k) - \dot{z}(k))^2$$

sous  $|u| < u_{max}$

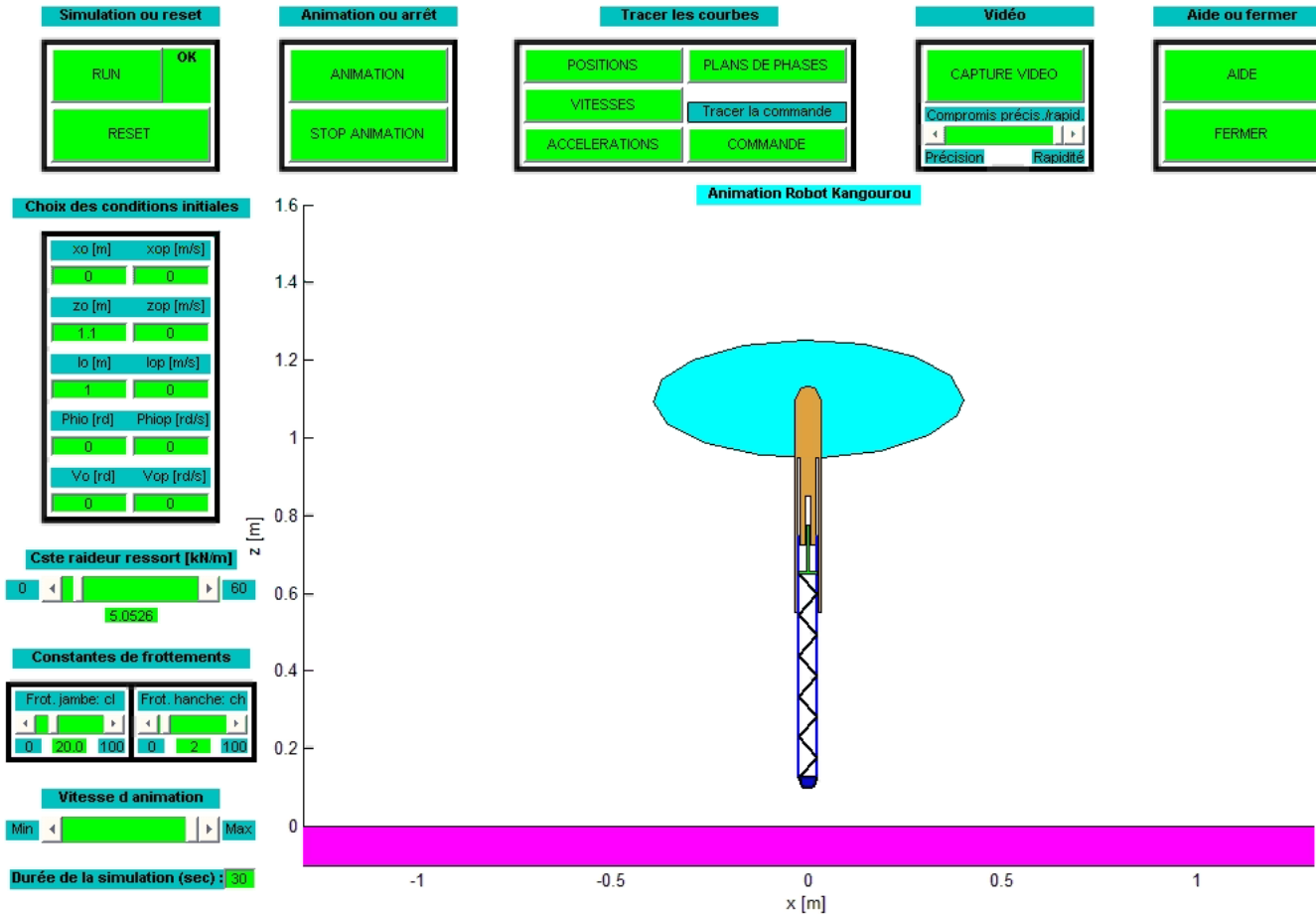
## Simulation results



## Simulation results



## Illustrative movie in the simulator



## Example 2

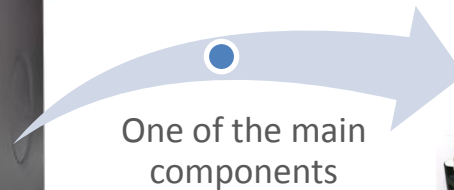
# Hard Disc Drive



## What is a Hard Disc Drive (HDD)?



Computer, laptop, or server



Hard Disc Drive

- It's a data **storage device** (in binary form 0/1)
- It consists of **one or more rigid rotating magnetic disks** (platters)
- A HDD **retains its data** even when **powered off**
- Data is read in a **random-access** manner
- Individual **blocks** of data can be **stored** or **retrieved** in **any order** rather than **sequentially**
- With a **magnetic head** arranged on a moving **actuator arm**
- To **read** and **write** data **to the surface**



## A brief history of HDD

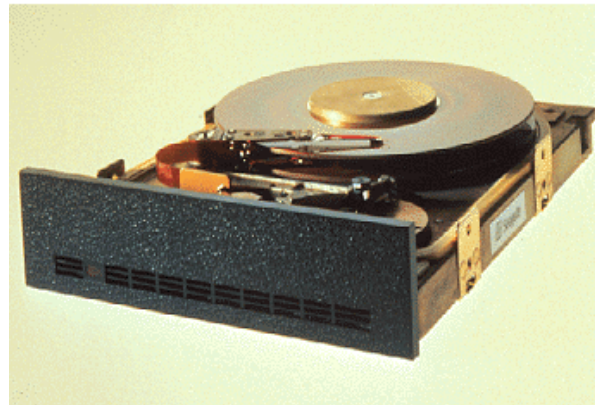
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### In 1956

- ✓ IBM's RAMAC
- ✓ First machine with HD
- ✓ 50 platters
- ✓ 2 feet diameter
- ✓ 100,000 characters
- ✓ Equivalent to 5 MB

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### In 1979

- ✓ By Seagate Technology
- ✓ First 5.25" hard disk
- ✓ The ST506's is 5 MB
- ✓ 10 times as much as the RAMAC at a fraction of its size

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### In 1998

- ✓ 4 Decades after RAMAC
- ✓ Seagate Technology
- ✓ 47 GB was impressive
- ✓ Store 100,000 times
- ✓ On the same surface

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### In 2005

- ✓ World smallest HD
- ✓ Toshiba
- ✓ 0.85" hard drive
- ✓ Size of a postage stamp
- ✓ 4 GB (2005), 8GB (2007)

## Inside a Hard Disc Drive



[www.letheonline.net](http://www.letheonline.net)



- Initialization
- Deleting a folder
- Copy past task

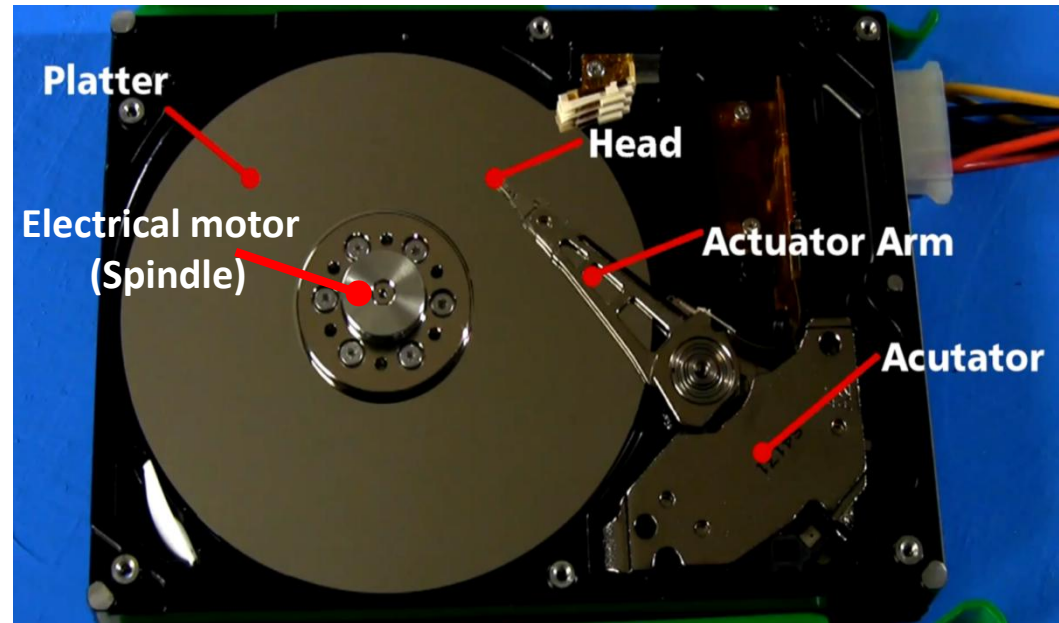
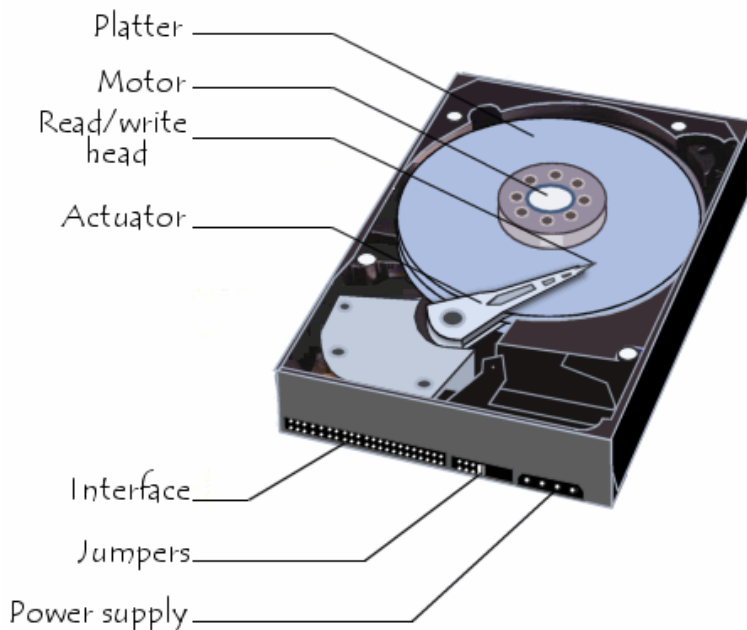
- ✓ In order to **read/write** data the head needs to be **physically moved** to the **correct position** on the disc



- ✓ That is a **key reason** why HDD suffer from **slow access time** w.r.t **RAM** for instance

- ✓ The triangular-shaped head arm holds the R/W head
- ✓ Able to move the head from the hub to the edge of the drive

## Main components of a Hard Disk Drive

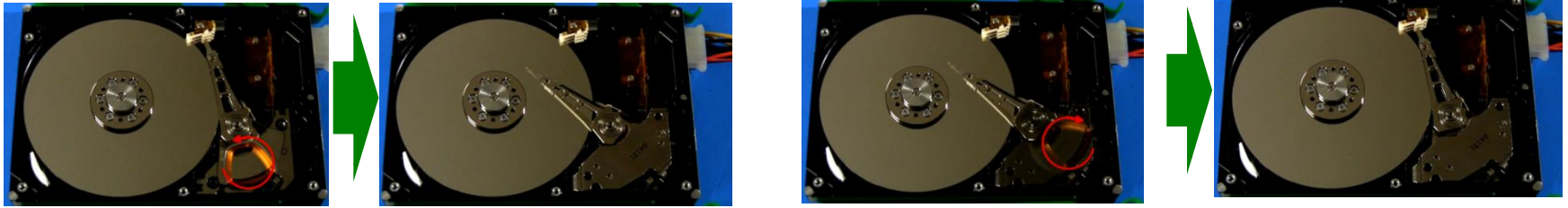


A typical HDD consists of 5 main parts :

- **Electric motor** : Spins the rotating discs (at 7200 rpm)
- **Disc platters** : Made of magnetic material used to store data
- **Actuator** : Called Voice Coil Motor (VCM)
- **Actuator arm** : Holds and moves the head to write/access data up to 60 times/s
- **Head** : is the most crucial part → R/W data on surface

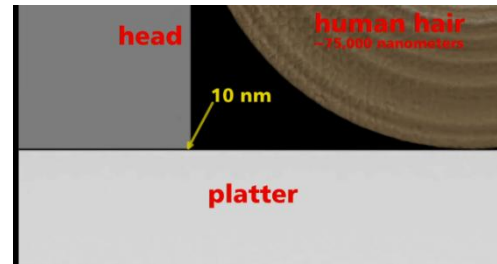
## How does it work ?

1. **Moving the head** : *Arm moves according to the Lorentz force*



*The sense of the force depends on the sense of the current (force ~ current)*

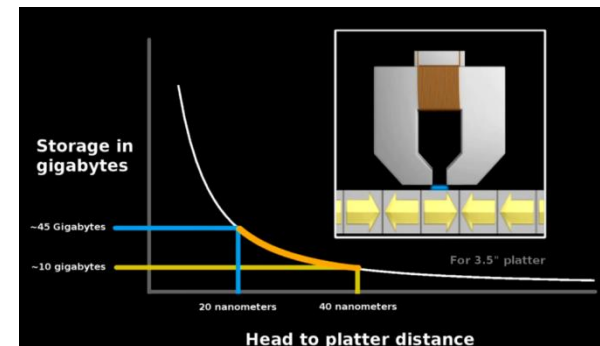
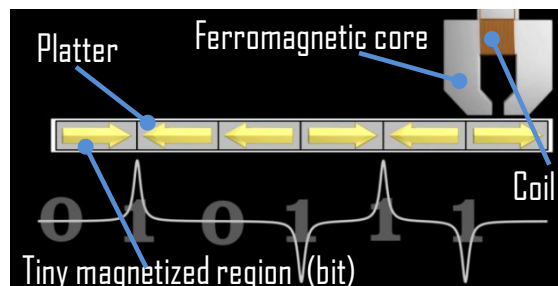
2. **Recording data** : *(All the data is stored using a basic form as series of 1 and 0)*



*Distance : Head / platter  
10 – 100 nm*

*Based on Faraday's Law :*

*A change in magnetization produces a voltage in the coil*





## Dynamic modeling of a HDD

- A **VCM** can be modeled including **hysteresis friction nonlinearity** from **pivot bearing**
- At **low frequencies** the dynamics is given by [San P. P. et al. 2011] :

$$M\ddot{q} + F(q, \dot{q}) = u$$

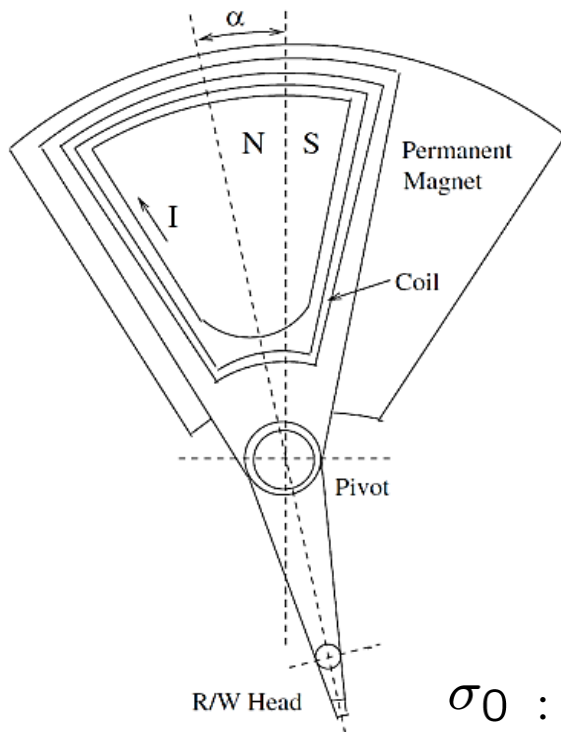
$$y = q + w_{out}$$



- $M$  is the inertia of the system,
- $q$  ,  $\dot{q}$  , and  $\ddot{q}$  are position, velocity and acceleration of the VCM actuator head tip,
- $u$  is the control input,
- $y$  is the output displacement,
- $F(q, \dot{q})$  is the nonlinear function representing the pivot bearing hysteresis friction,
- $w_{out}$  is an output disturbance.

## Dynamic modeling of a HDD

- The nonlinear function  $F(q, \dot{q})$  is presented by the LuGre function [Liu. X, et al. 1999]
- The static & dynamic characteristics of the hysteresis function are expressed as:



$$F(q, \dot{q}) = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{q}$$

$$\dot{z} = \dot{q} - \alpha(\dot{q}) |\dot{q}| z$$

$$\alpha(\dot{q}) = \frac{\sigma_0}{f_c + (f_s - f_c) e^{-\left(\frac{\dot{q}}{\dot{q}_s}\right)^2}}$$

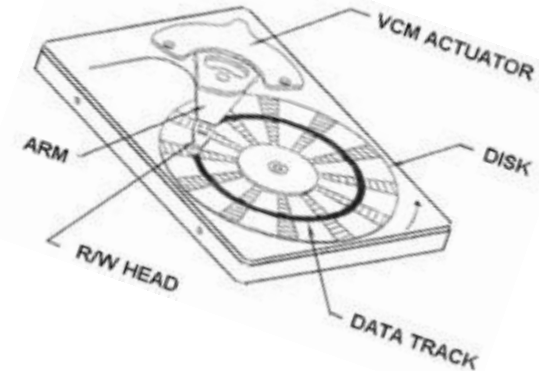
$\sigma_0$  : Stiffness coefficient,  
 $\sigma_1$  : Micro damping ratio, and  
 $\sigma_2$  : Viscous coefficient

$f_c$  : Coulomb friction force,  
 $f_s$  : Stiction force, and  
 $\dot{q}_s$  : Stribeck velocity

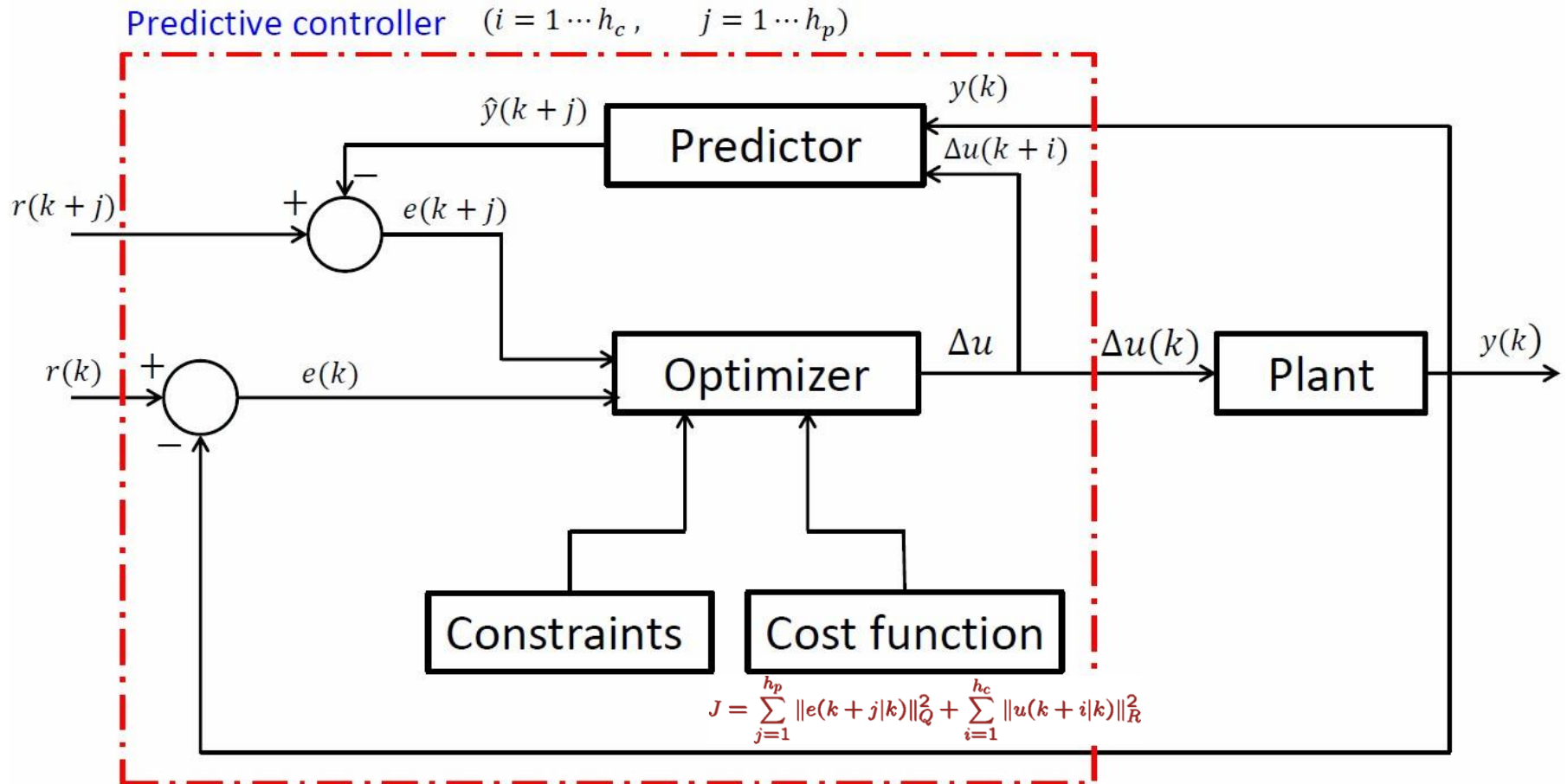


## Control problem

- ✓ **Two problems: Track seeking / Track following**
- ✓ **Second one** : keep the **head** closed to the **desired track**
- ✓ Both **precision** and **rapidity** are required (**robustness**)
- ✓ **Bounded** control input
- ✓ **Uncertain** nonlinear system
- ✓ **Implementation** of the proposed control architecture in **simulation**
- ✓ Tests in various operating conditions
  - *Nominal conditions with noise*
  - *External disturbance rejection*
  - *Robustness towards parameters' uncertainties*
- ✓ **Proposed solution** : Nonlinear Model Predictive Control (**NMPC**)



## Proposed control solution

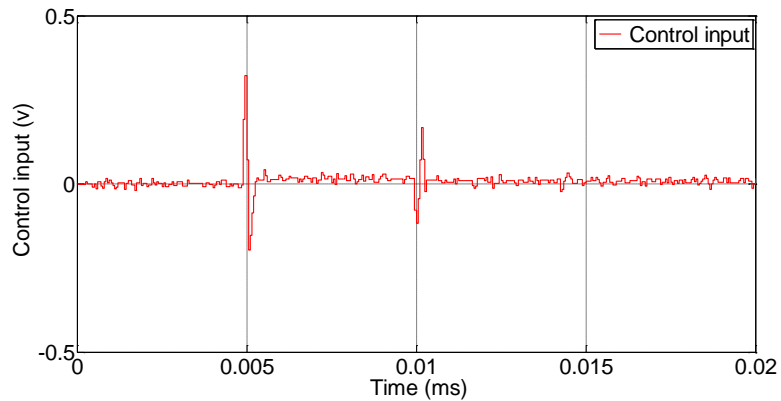
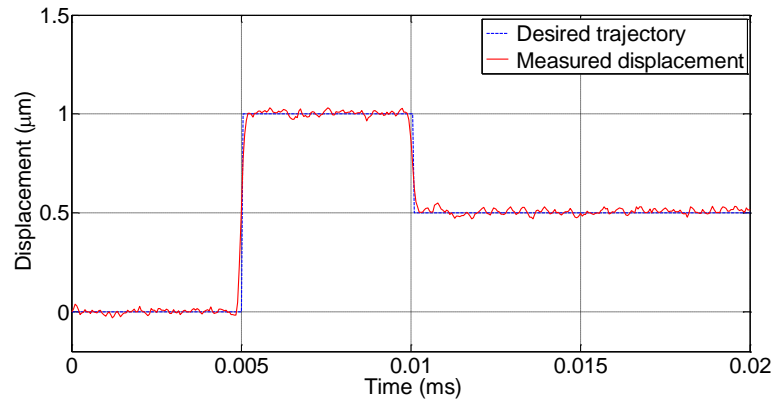


# Simulation results

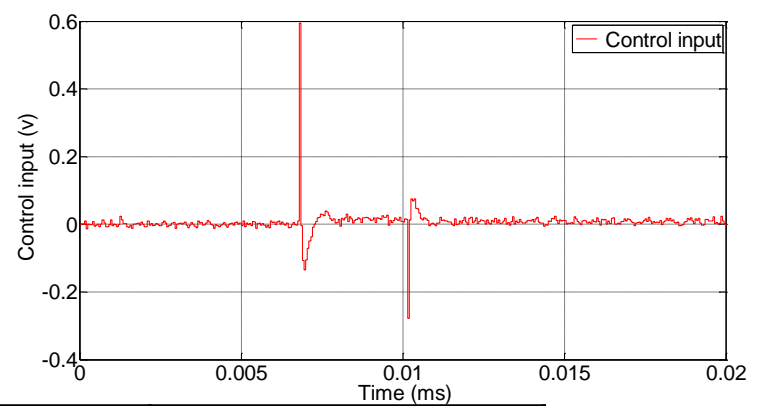
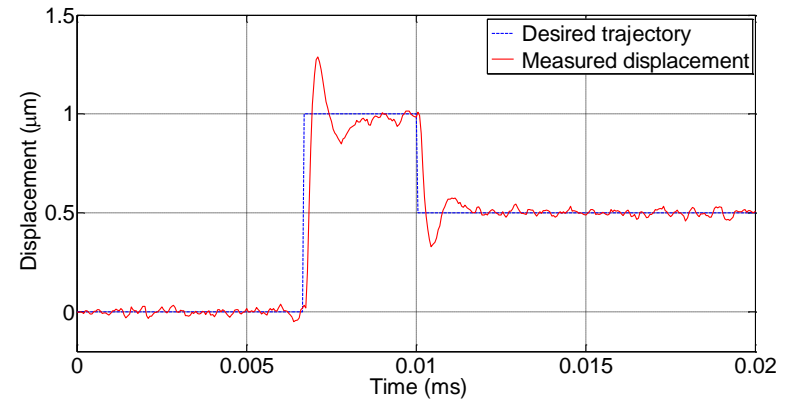
- ✓ **Scenario 1** : Nominal case
- ✓ **Scenario 2** : External disturbance rejection
- ✓ **Scenario 3** : Robustness towards parameters' uncertainties

## Scenario 1 : Nominal case with noise

### NMPC



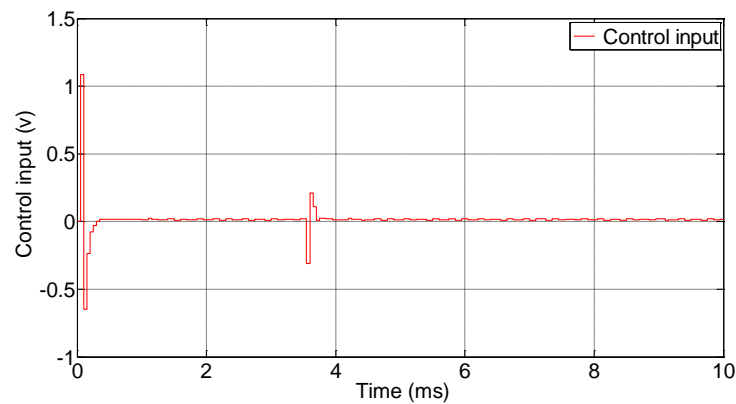
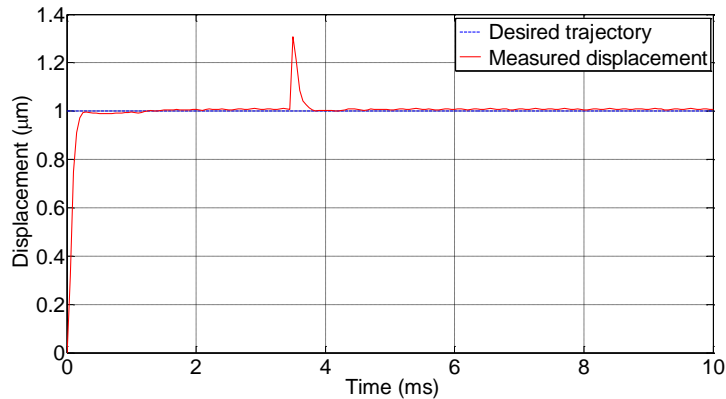
### PID



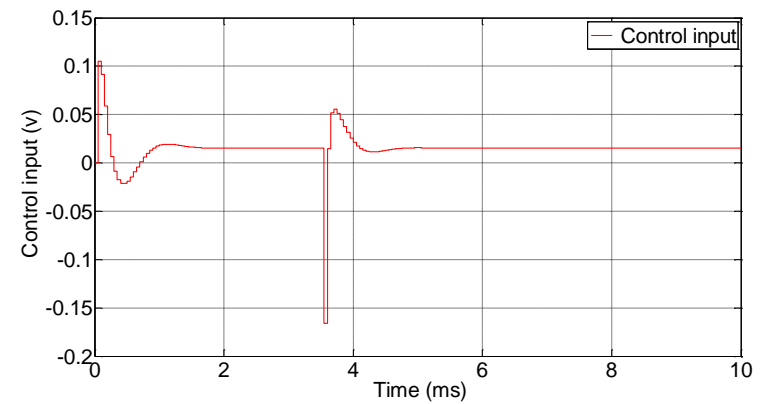
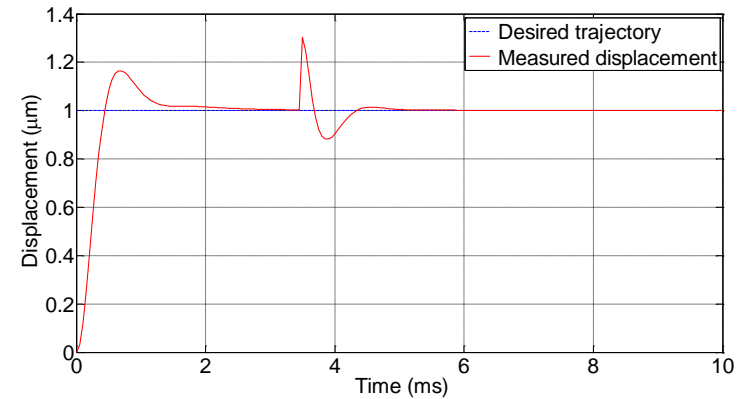
	Settling time (ms)	Maximum overshoot
<b>PID controller</b>	2,07	31%
<b>NMPC controller</b>	0,33	4%

## Scenario 2 : External disturbance rejection

### NMPC

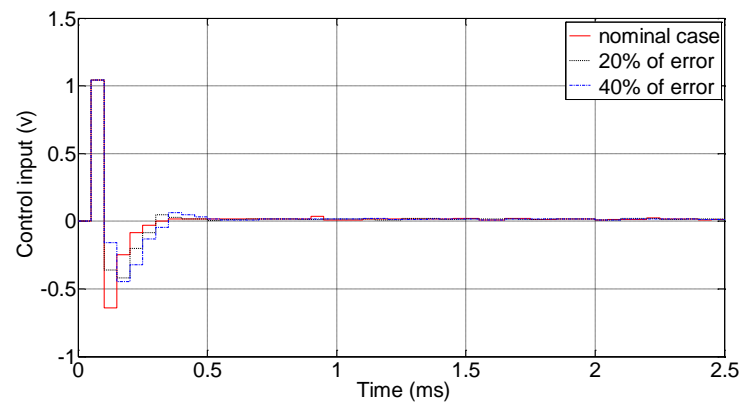
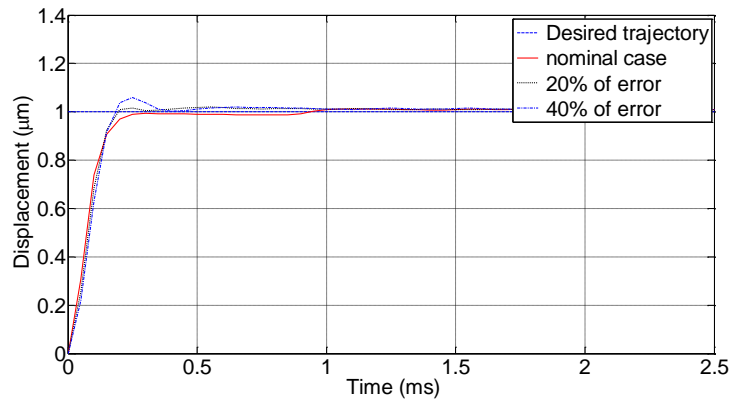


### PID

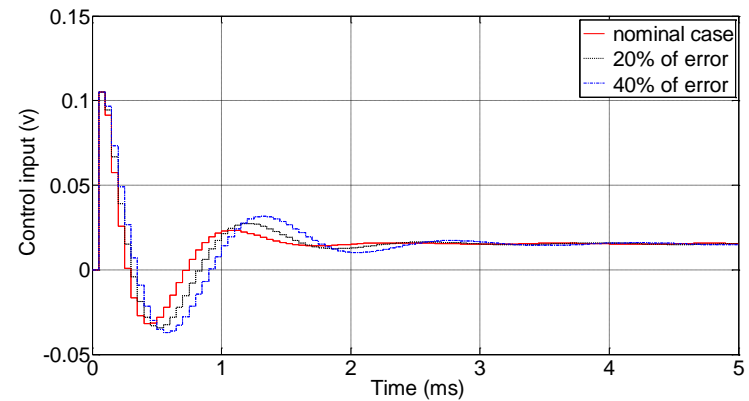
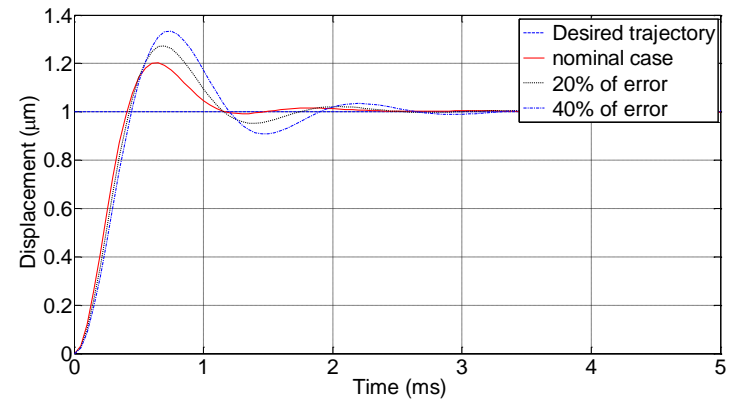


## Scenario 3 : Robustness towards uncertainties

### NMPC

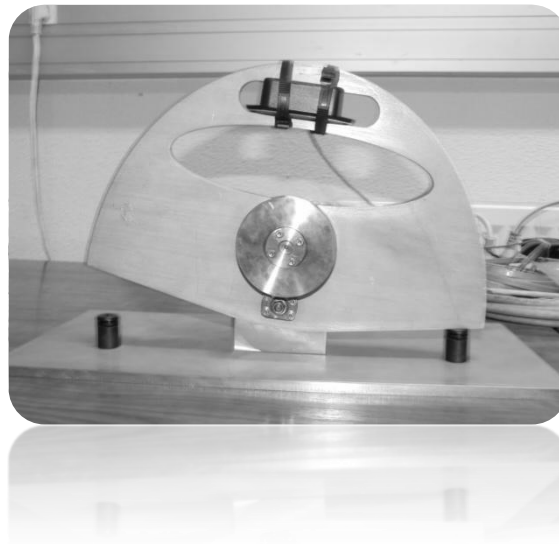


### PID



**Example 3**

# Inertia Wheel Inverted Pendulum





## Principle of the gyrostabilizer

- The effect of a torque (e.g. gravity / excitation moments)



Causes a variation in the spin axis

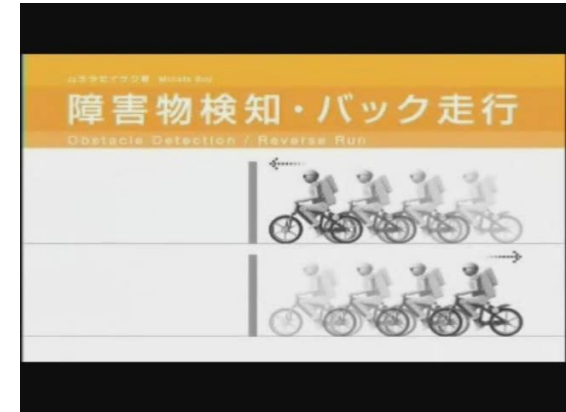


Reaction of a spinning wheel

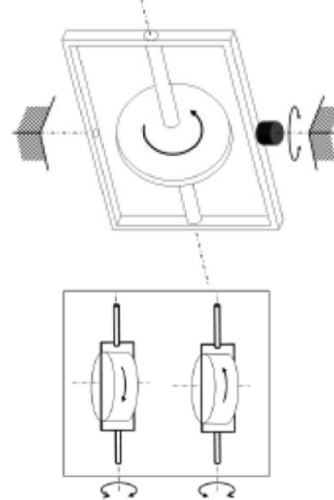


Output torque orthogonal to the input torque and spin axes

- The phenomenon provides an effective means of motion control and balance
- Gyro's flywheel must be in motion to resist gravity
- **Two early examples of application :**

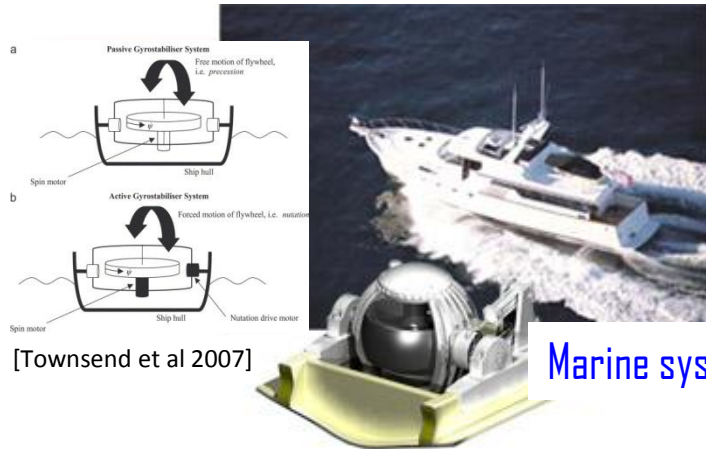


The Schilovski Gyrocar (1914)



- ✓ Single gimbal active stabilizer unit
- ✓ With 40 inch diameter & 4.5 inch thick
- ✓ Flywheel operated at 2000-3000 rpm
- ✓ Twin type active stabilizer system (3000 rpm)
- ✓ 40 feet long and weighted 22 tons
- ✓ Developed primary for military applications

## Examples of some applications



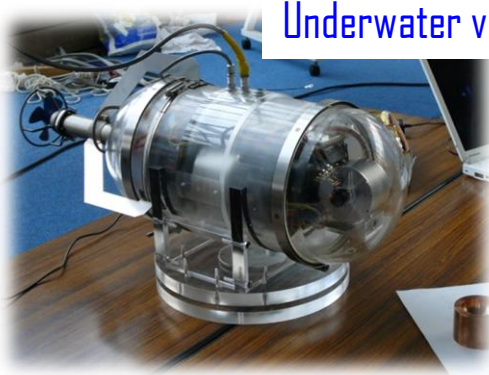
Marine systems



Robotics



Aerospace



Underwater vehicles

IKURA AUV

## Gyrostabilizer Applications

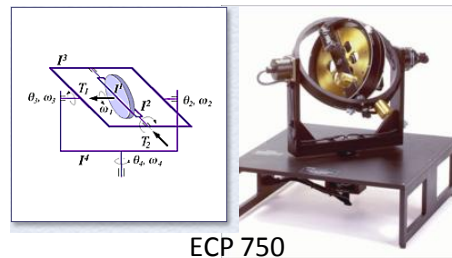
Monorails



Cars

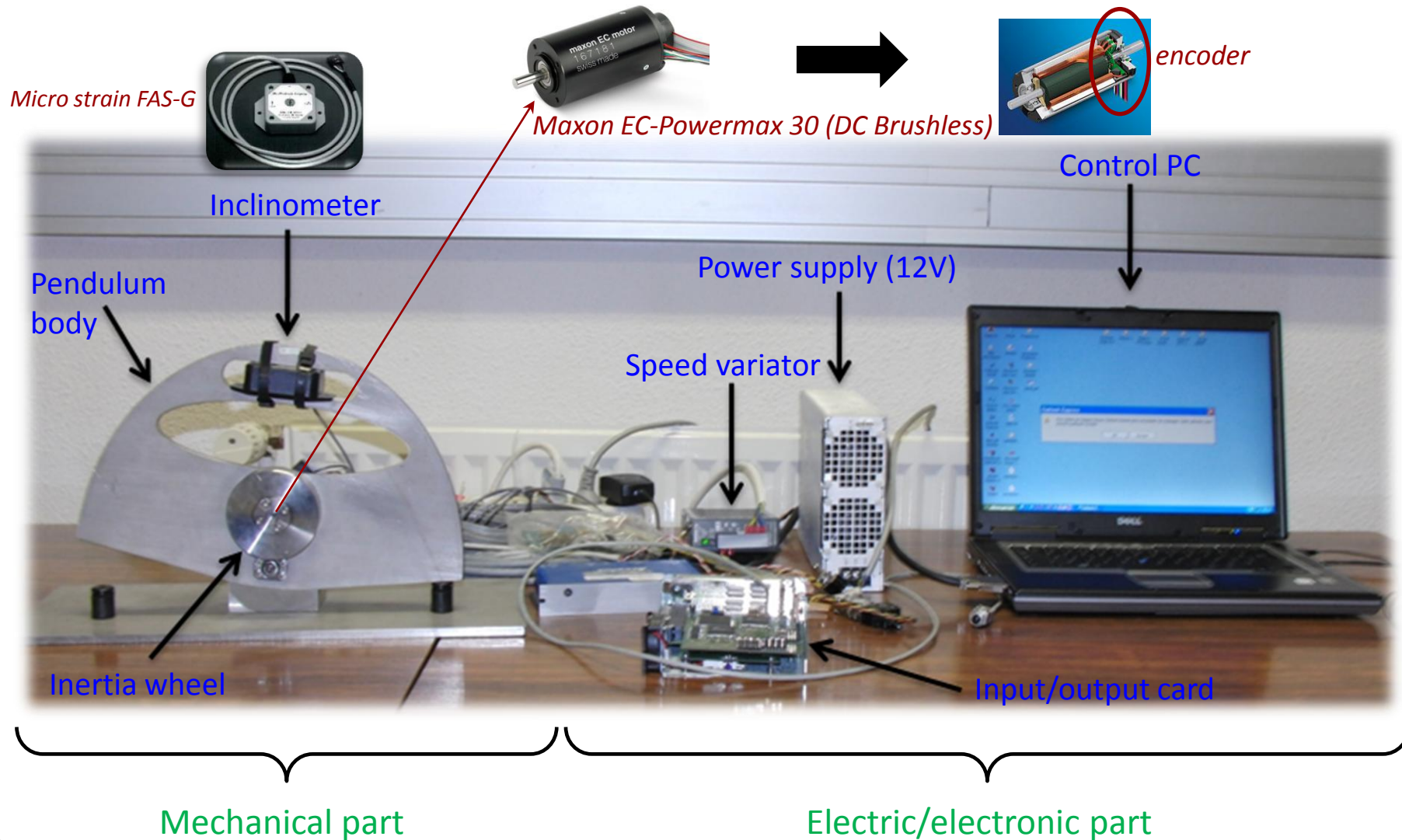


Two-wheel gyro car

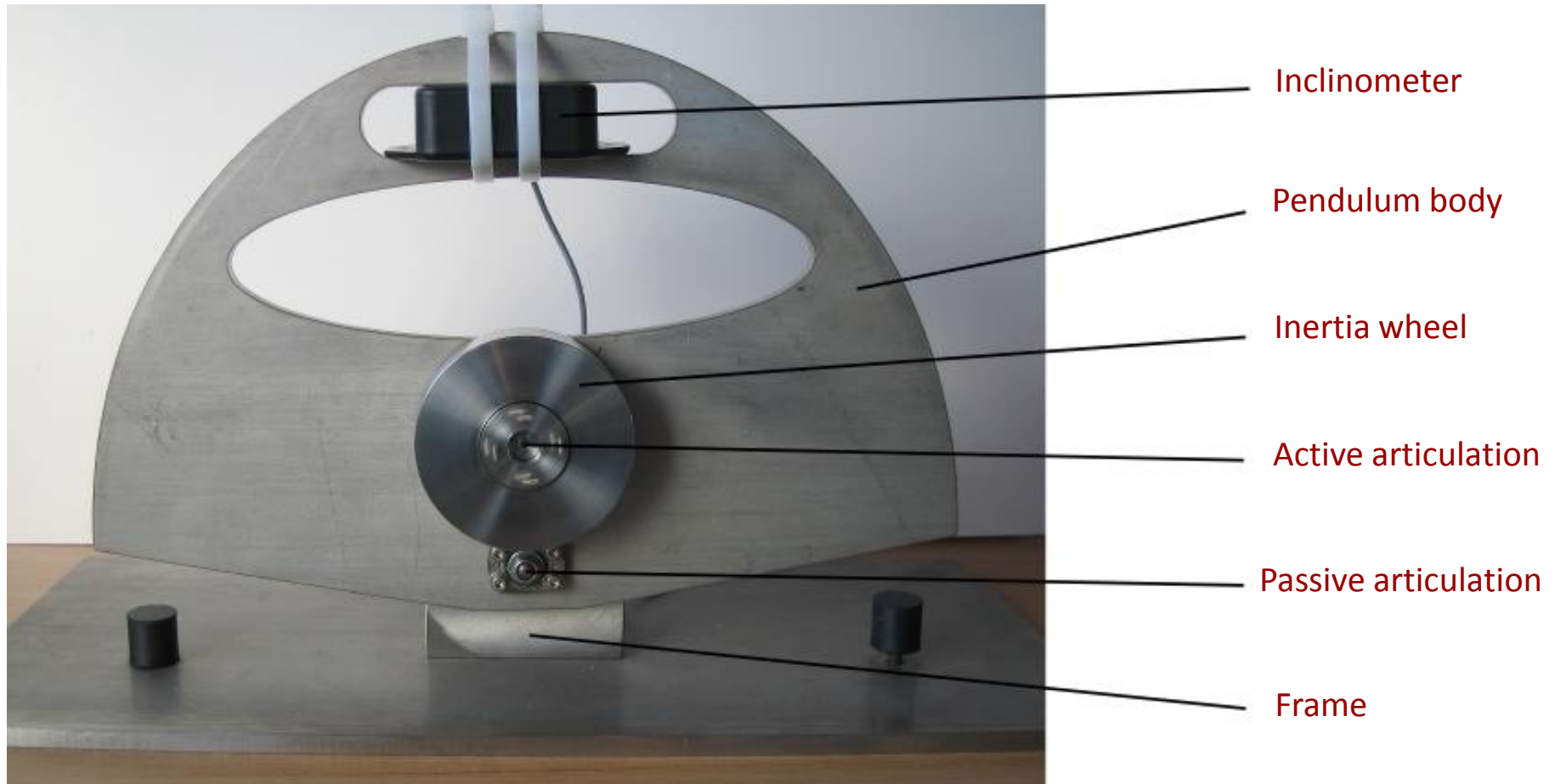


ECP 750

## Description of the experimental testbed



## Description of the mechanical part

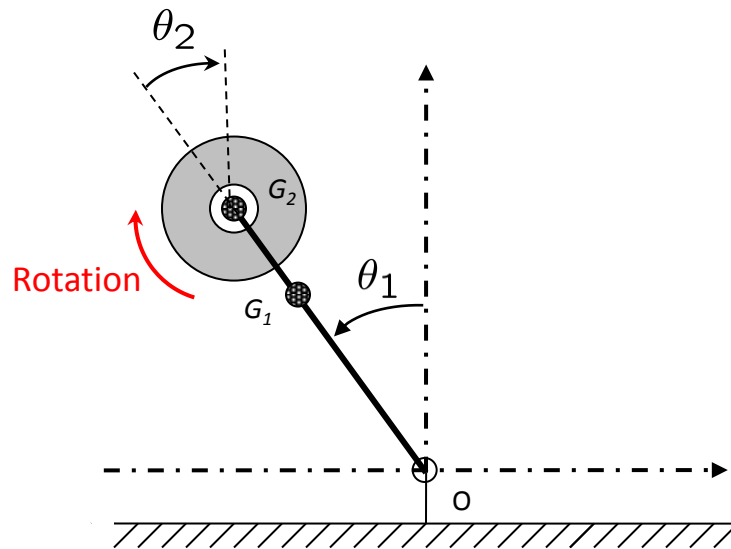


Mechanical part of the system : **inertia wheel inverted pendulum**

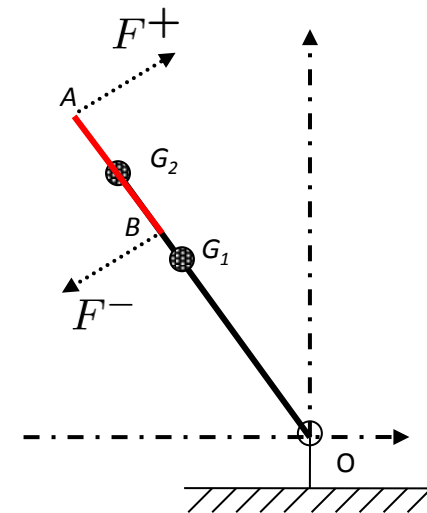


## Functioning principle

- The **actuator** is controlled to produce a **torque** on the **inertia wheel**
- **Torque** can produce an **acceleration** of the rotating wheel
- Thanks to the **dynamic coupling**, a torque acting on the **passive joint** is generated
- This **passive joint** can be controlled through the **acceleration** of the **inertia wheel**

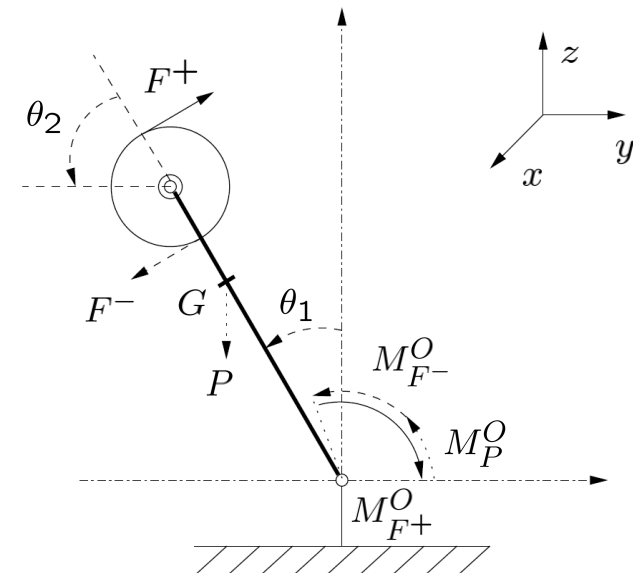
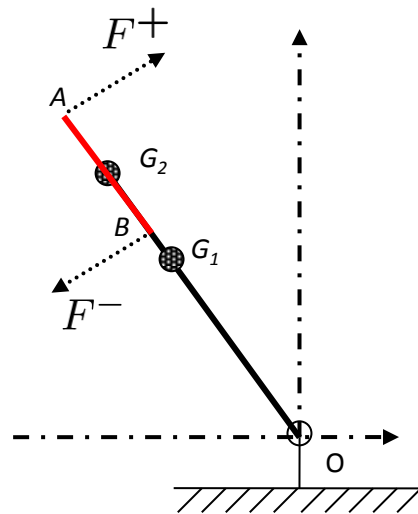


Initial mechanical model



Equivalent mechanical model

## Functioning principle



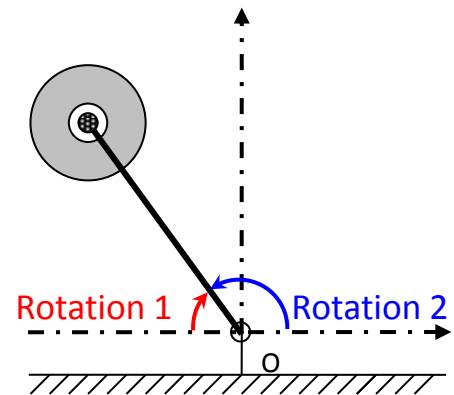
Three moments are acting on the passive joint :

- ✓ Moment relative to the force :  $F^+$
- ✓ Moment relative to the force :  $F^-$
- ✓ Moment due to the gravity force :  $P$



$$M_{F+}^O > M_{F-}^O + M_P^O$$

$$M_{F+}^O < M_{F-}^O + M_P^O$$



One moment (torque of actuator) is acting on the active joint (inertia wheel)

## Dynamic modeling

- Generalized coordinates :  $q_1 = \theta_1$  ,  $q_2 = \theta_2$
- Propose to use the formalism of Lagrange :  $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$  ,  $i = 1..2$
- The application of Lagrange principle gives :

$$T = \frac{1}{2}(m_1 l_1^2 + m_2 l_2^2 + i_1)\dot{\theta}_1^2 + i_2(\dot{\theta}_1 + \dot{\theta}_2)^2$$

$$V = (m_1 l_1 + m_2 l_2)g \cos(\theta_1)$$

$$L = T - V = \frac{1}{2}I\dot{\theta}_1^2 + i_2(\dot{\theta}_1 + \dot{\theta}_2)^2 - \overline{m}l g \cos(\theta_1)$$

$$\overline{m}l = m_1 l_1 + m_2 l_2$$

$$I = m_1 l_1^2 + m_2 l_2^2 + i_1$$

$$\begin{cases} \ddot{\theta}_1 &= \frac{1}{I}[-\tau_2 + \overline{m}l g \sin \theta_1] \\ \ddot{\theta}_2 &= \frac{1}{I i_2}[(i_2 + I)\tau_2 - i_2 \overline{m}l g \sin \theta_1] \end{cases}$$

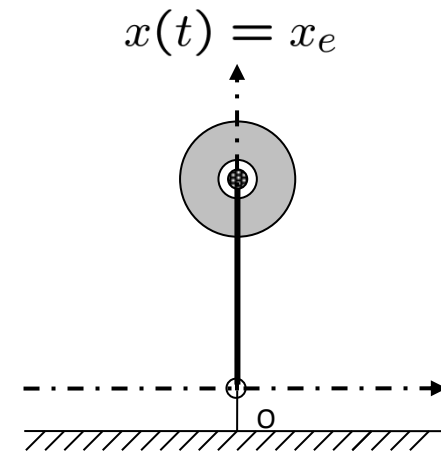
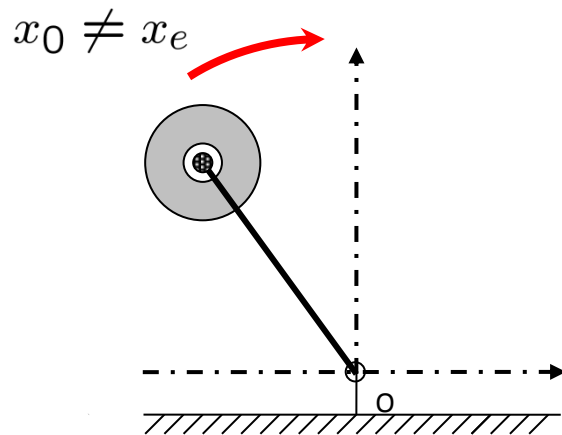
$$\begin{bmatrix} I + i_2 & i_2 \\ i_2 & i_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} \overline{m}l g \sin \theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \tau_2 \end{bmatrix}$$

$$M(q)\ddot{q} + H(q, \dot{q}) + G(q) = Ru$$



## Control problem formulation

- Assume the system in some initial condition
- Find a control input  $u$  to bring  $x(t)$  to  $x_e$  and maintain it around this point



- Proposed solutions :

- ✓ State feedback control
- ✓ Optimal control
- ✓ Sliding mode control
- ✓ **Generalized Predictive Control (GPC)** → Linear discrete dynamics

## Linearization of the dynamics

- Recall the nonlinear dynamics : 
$$\begin{cases} \ddot{\theta}_1 &= \frac{1}{I}[\tau_1 - \tau_2 + \overline{m}l g \sin \theta_1] \\ \ddot{\theta}_2 &= \frac{1}{I i_2}[-i_2 \tau_1 + (i_2 + I)\tau_2 - i_2 \overline{m}l g \sin \theta_1] \end{cases}$$

- Consider the state vector :  $x = [\theta_1 \quad \dot{\theta}_1 \quad \dot{\theta}_2]^T$  and  $\tau_1 = 0$  ,  $\tau_2 = u$

- The nonlinear dynamics can be written in nonlinear state space as :

$$\dot{x} = f(x) + g(x)u$$

- The unstable equilibrium of the system :  $x_e = [0 \quad 0 \quad 0]^T$

- The linearization of the dynamics around the unstable equilibrium gives :

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad \text{With :}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \overline{m}l g / I & 0 & 0 \\ -\overline{m}l g / I & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1/I \\ (I + i_2)/i_2 I \end{bmatrix}, \quad C = [1 \quad 0 \quad 0], \quad D = [0]$$

## Discretization of the dynamics

- Recall the linearized dynamics : 
$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$
- The discretization of the dynamics gives : 
$$\begin{cases} x_{k+1} = A_d x_k + B_d u_k \\ y_k = C_d x_k + D_d u_k \end{cases}$$

### Summary of geometric and dynamic parameters of the system

Paramtre	Description	Valeur	unit
$m_1$	Masse du pendule	3.30810	$Kg$
$m_2$	Masse du volant	0.33081	$Kg$
$l_1$	Distance pivot / centre de gravit du pendule	0.06	$m$
$l_2$	Distance pivot / centre de gravit du pendule	0.044	$m$
$i_1$	Moment d'inertie du pendule	0.0314683	$Kgm^2$
$i_2$	Moment d'inertie du volant d'inertie	0.0004176	$Kgm^2$
$g$	Acclration de la pesanteur	9.81	$ms^{-2}$

## Proposed control approach : GPC

- Consider the case of the GPC approach with penalty on the end-state
- Recall the discrete dynamics of the system : 
$$\begin{cases} x(k+1) = A_d x(k) + B_d u(k) \\ y(k) = C_d x(k) + D_d u(k) \end{cases}$$
- Consider the extended state vector :

$$x_e(k+1) = \begin{pmatrix} x(k+1) \\ u(k) \end{pmatrix}$$

- The variation on the control input : 
$$\Delta u(k) = u(k) - u(k-1)$$

- The dynamics can be written as : 
$$\begin{cases} x_e(k+1) = A_e x(k) + B_e \Delta u(k) \\ y_e(k) = C_e x_e(k) + D_e \Delta u(k) \end{cases}$$

with :

$$A_e = \begin{pmatrix} A_d & B_d \\ 0 & 1 \end{pmatrix} \quad B_e = \begin{pmatrix} B_d \\ 1 \end{pmatrix} \quad C_e = (C \quad 0) \quad D_e = (0)$$

This last dynamics will be used in the controller design

## Proposed control approach : GPC

- Consider the following optimization function :

$$\begin{aligned}
 J(N_1, N_p, N_u, Q, \lambda) = & \sum_{j=N_1}^{N_p} [y(k+j) - w(k+j)]^T [y(k+j) - w(k+j)] + \\
 & \sum_{j=N_1}^{N_u} \Delta u(k+j-1)^T \Delta u(k+j-1) + \\
 & [x(k+N_p) - w_x(k+N_p)]^T Q [x(k+N_p) - w_x(k+N_p)]
 \end{aligned}$$

- From the system model one can have the state predictions :

$$\begin{aligned}
 x_e(k+1) &= A_e x_e(k) + B_e \Delta u(k) \\
 x_e(k+2) &= A_e^2 x_e(k) + B_e \Delta u(k+1) \\
 &\vdots \\
 x_e(k+j) &= A_e^j x_e(k) + \sum_{i=0}^{j-1} A_e^{j-i-1} B_e \Delta u(k+i)
 \end{aligned}$$

- The prediction of future outputs be :

$$y_e(k+j) = C_e A_e^j x_e(k) + \sum_{i=0}^{j-1} C_e A_e^{j-i-1} B_e \Delta u(k+i)$$

## Proposed control approach : GPC

- In a matrix form we have :

$$Y = \begin{bmatrix} y_e(k+1) \\ \vdots \\ y_e(k+j) \\ \vdots \\ y_e(k+N_p) \end{bmatrix} ; \quad \Delta U = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u) \end{bmatrix}$$

$$Y = G\Delta U + f$$

$$G = \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ CA^{N_1-1}B & CA^{N_1-2}B & CA^{N_1-3}B & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \dots & CA^{N_p-N_u}B \end{bmatrix} \quad f = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_1} \\ \vdots \\ CA^{N_p} \end{bmatrix} x(k) = Lx(k)$$

- The control input is computed as the minimum of :

$$J_N = \frac{1}{2} \Delta u^T H \Delta u + b^T \Delta u + f_0$$

with

$$\begin{aligned} H &= 2(G^T G + \lambda I + \bar{C}^T Q \bar{C}) ; \\ b &= 2x(k)^T (L^T G + (A^N)^T Q \bar{C}) ; \\ f_0 &= x(k)^T (L^T L + (A^N)^T Q A^N) x(k). \end{aligned}$$

- The optimal solution is :  $\Delta u^* = -K x(k)$

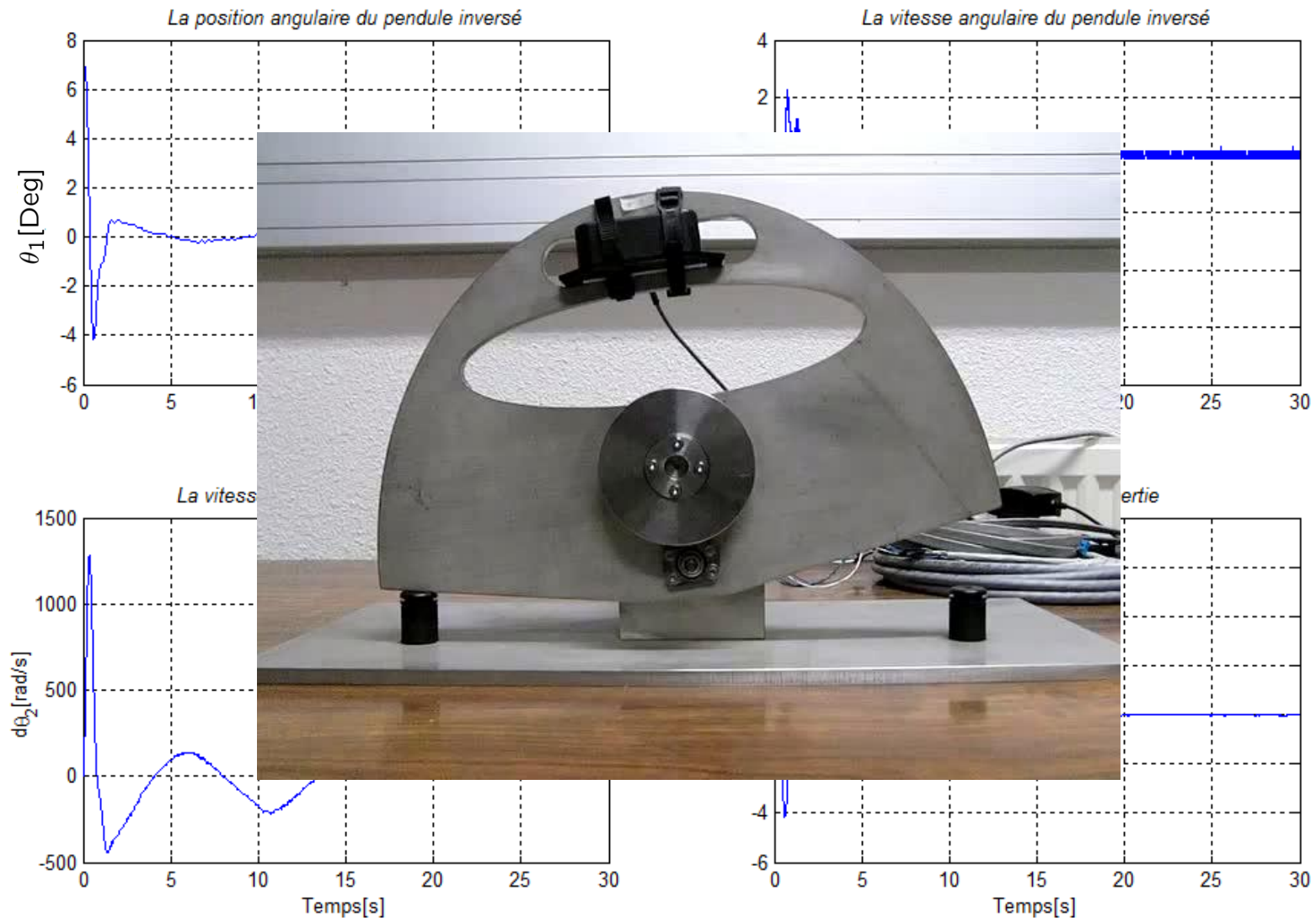
$$K = -(G^T G + \lambda I + \bar{C}^T Q \bar{C})^{-1} (G^T + \bar{C}^T Q A^N) x(k).$$

# Real-time experiments

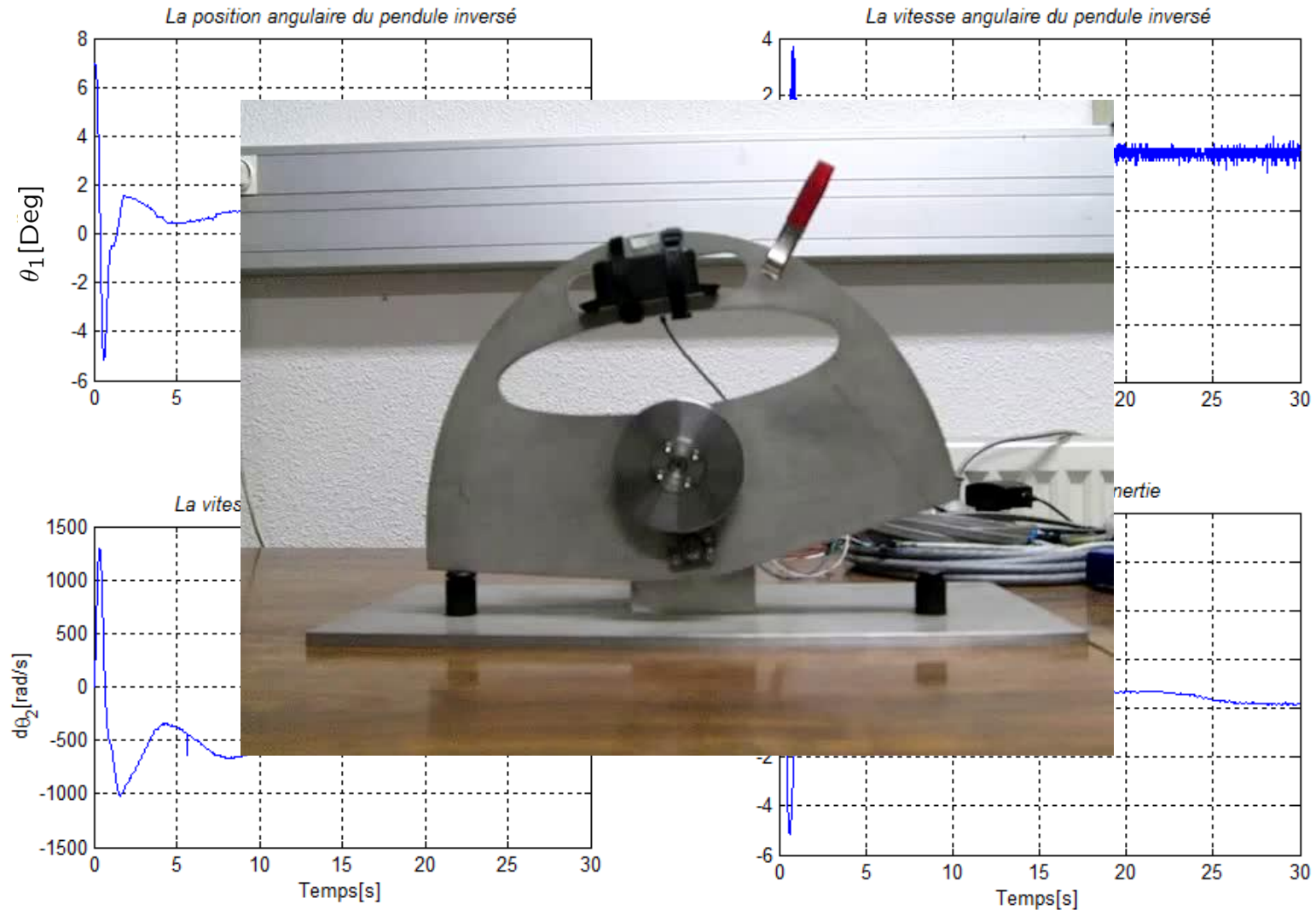
- ✓ **Scenario 1** : Stabilization in the nominal case
- ✓ **Scenario 2** : Case with persistent disturbance
- ✓ **Scenario 3** : Case with punctual disturbance
- ✓ **Scenario 4** : Combination of the two disturbances



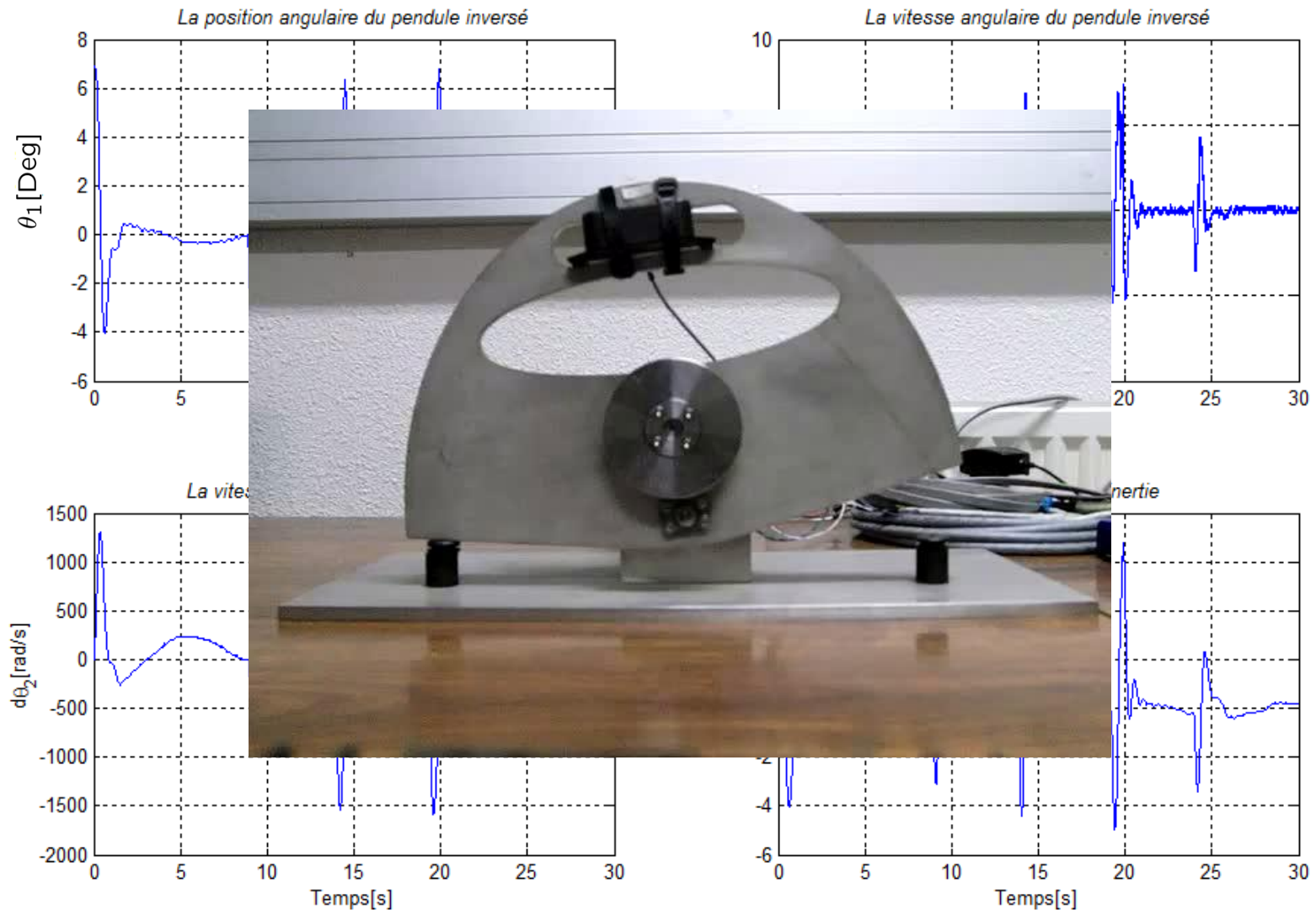
## Scenario 1 : Stabilization in the nominal case



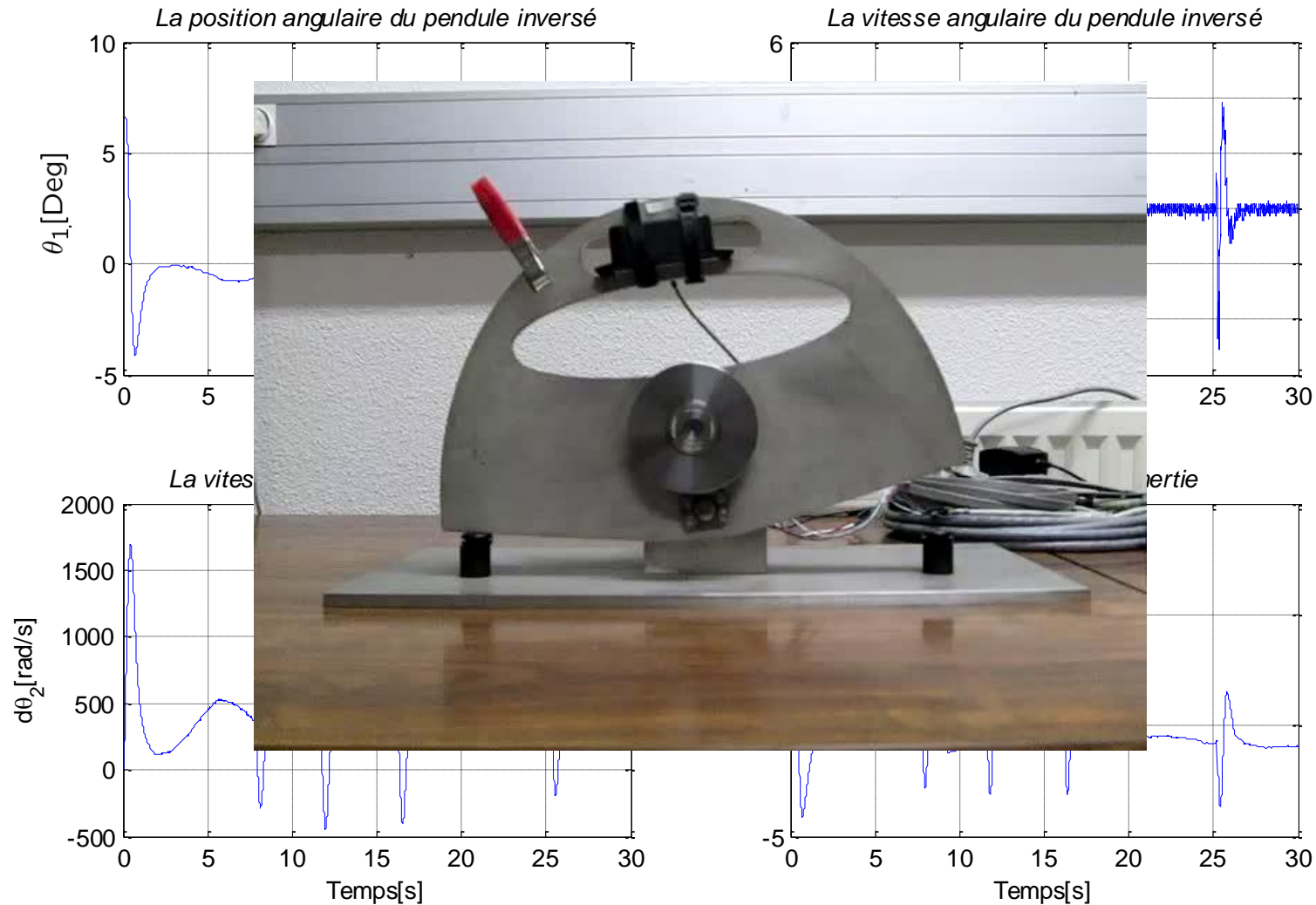
## Scenario 2 : Case with persistent disturbance



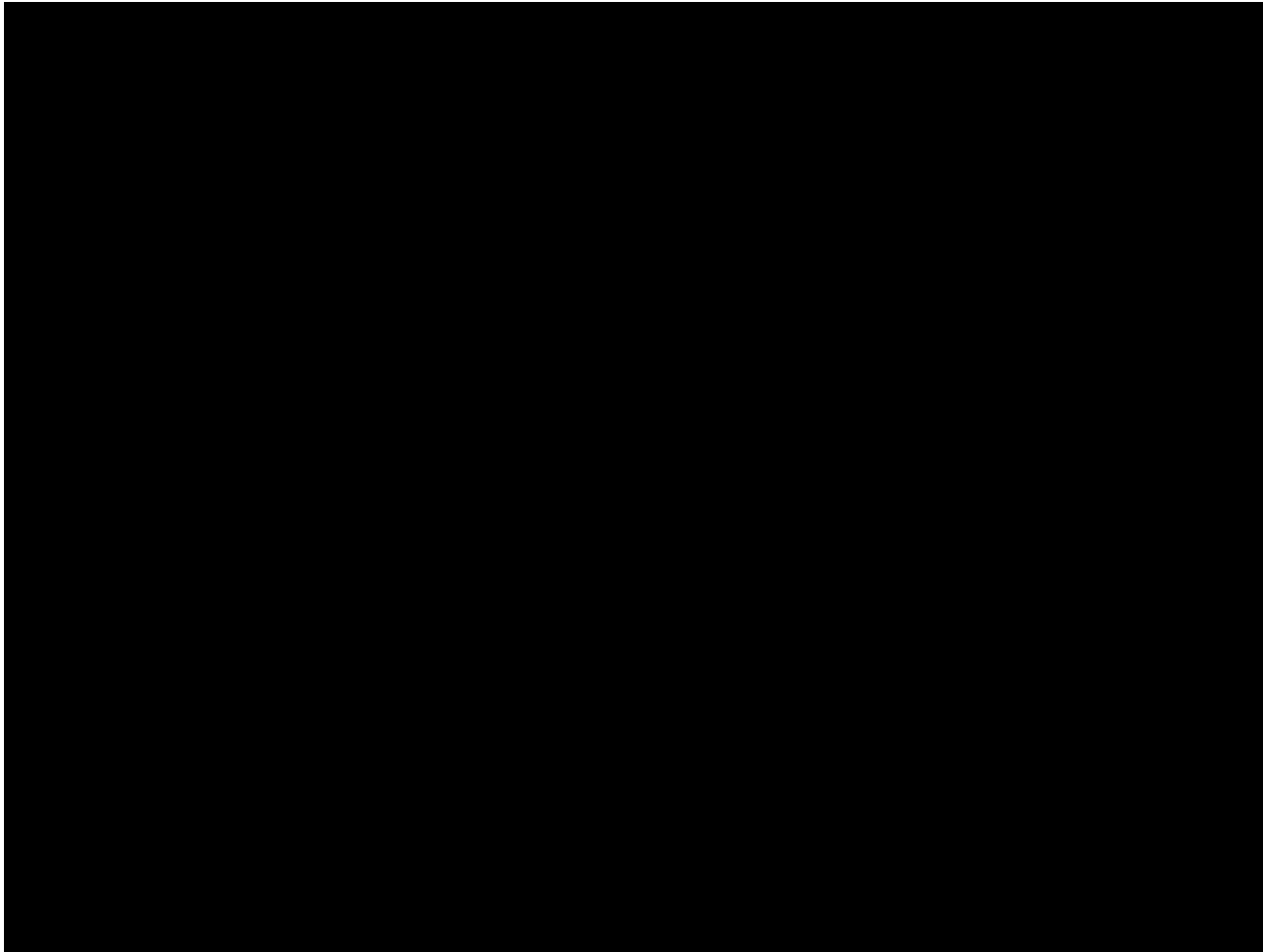
### Scenario 3 : Case with punctual disturbance



## Scenario 4 : Combination of the two disturbances



**Stable limit cycle generation with**  
**A prediction based optimal trajectories tracking**

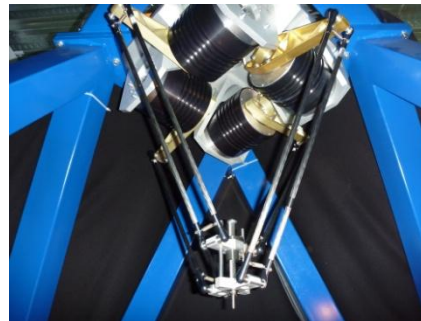


## Conclusion & future work

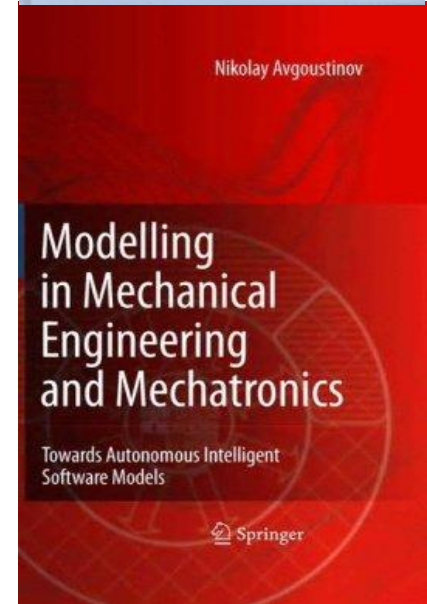
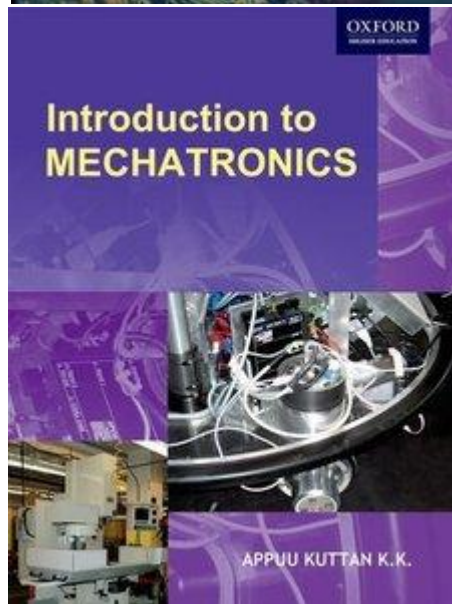
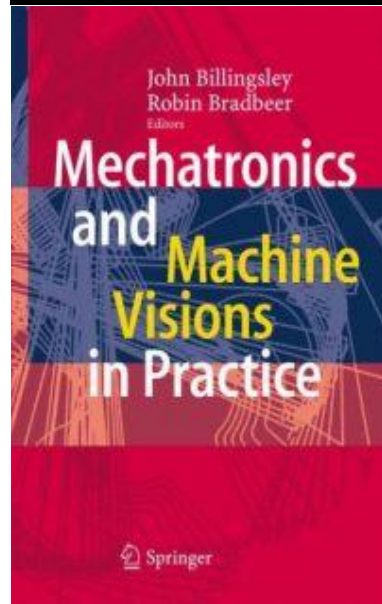
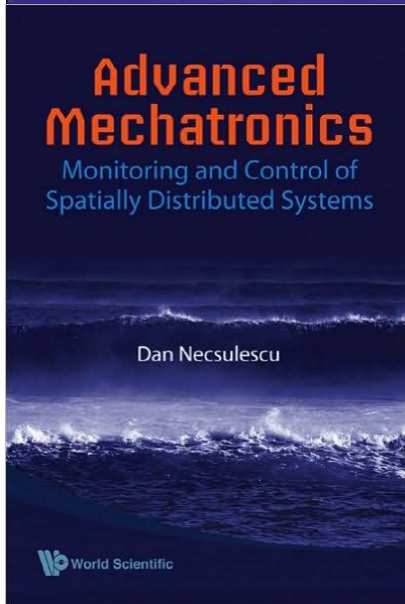
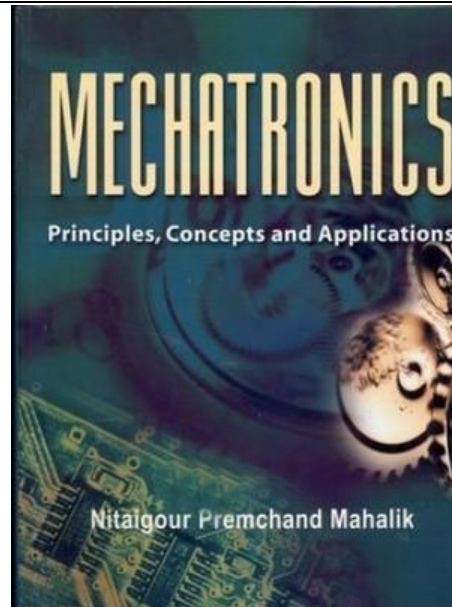
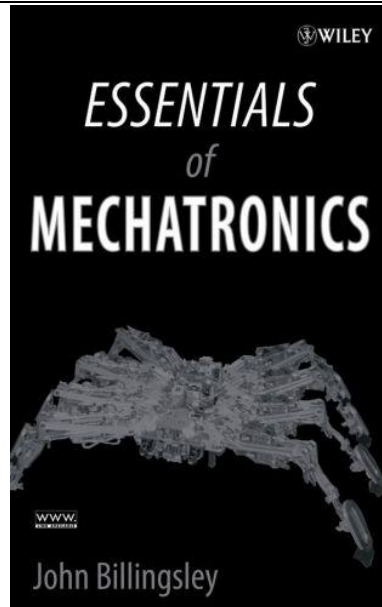


## Conclusion

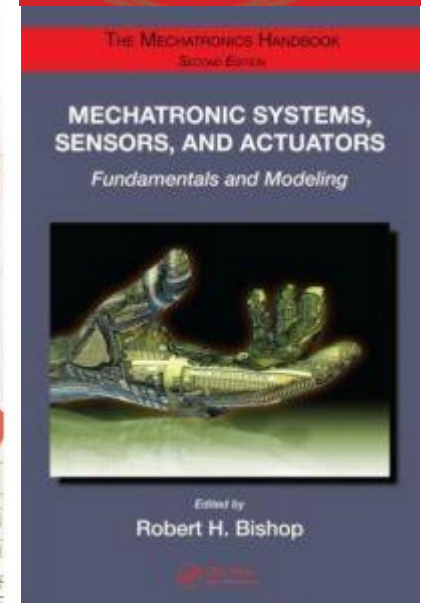
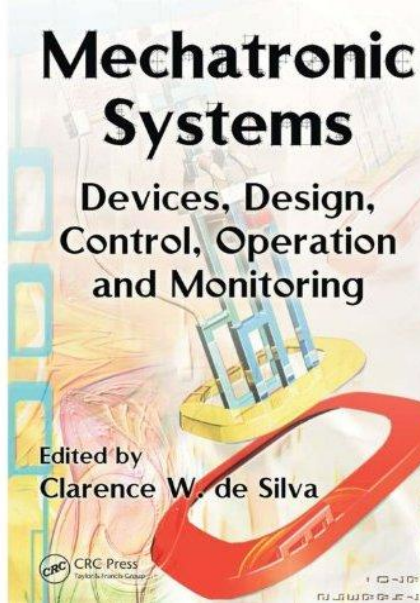
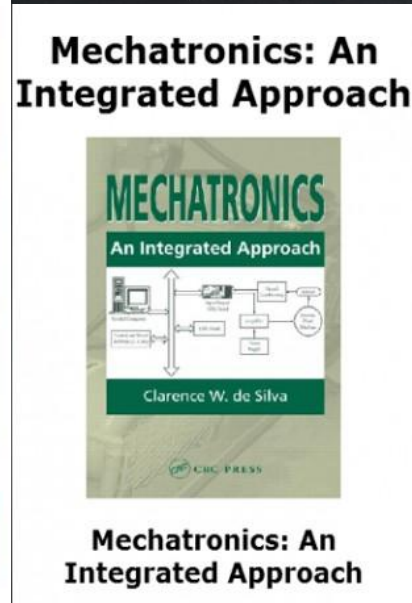
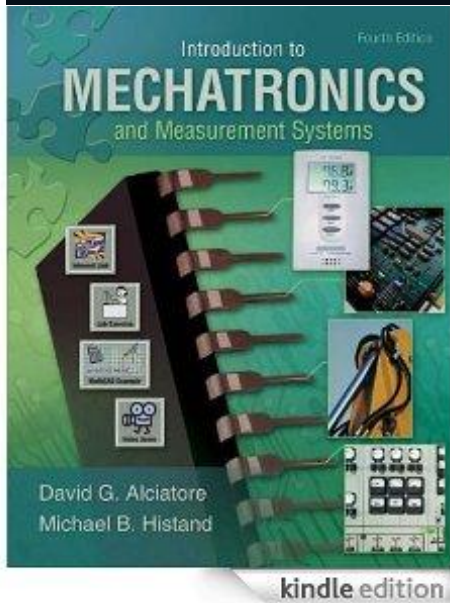
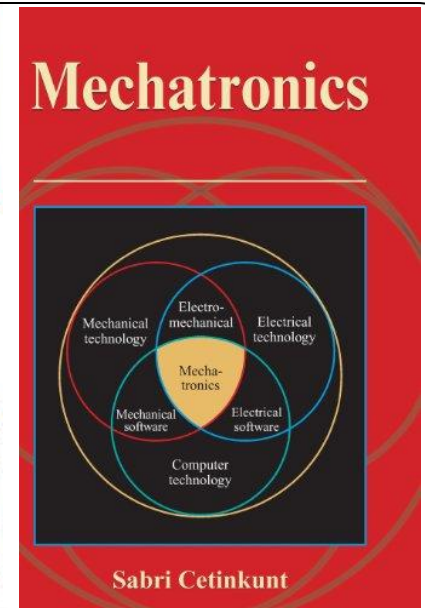
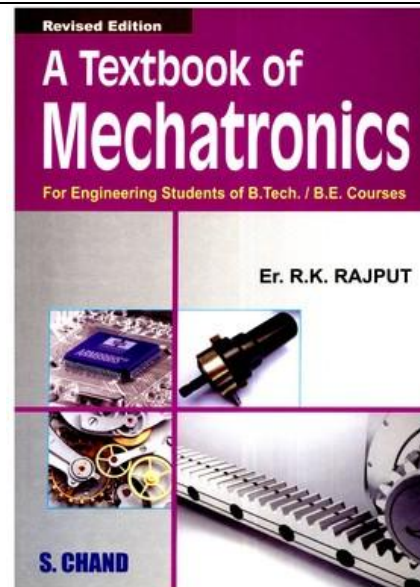
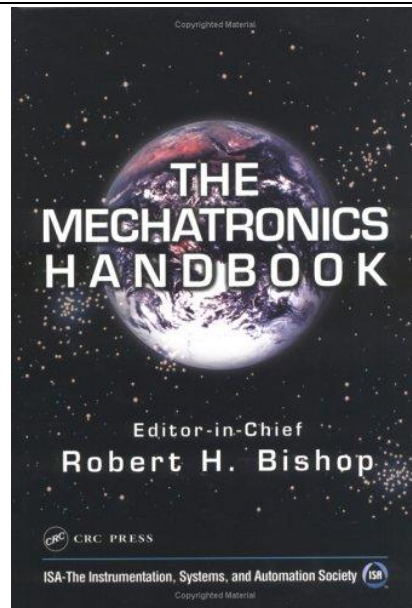
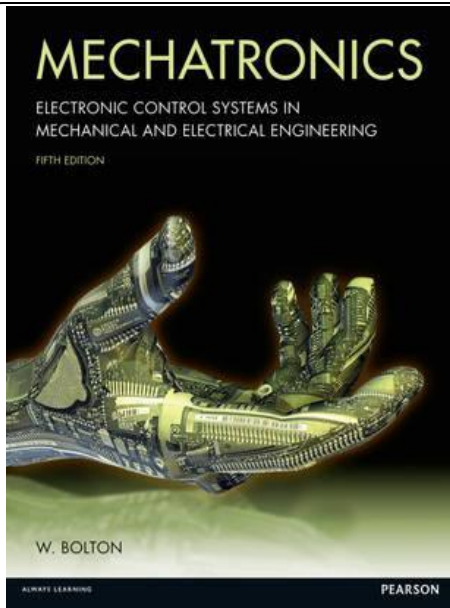
- Control of Mechatronic systems
- Proposed controllers : Model predictive control
- **Applications** : 3 different examples of Mechatronic systems
  - ✓ One-leg hopping robot : NMPC
  - ✓ Hard disk drive : NMPC
  - ✓ Inertia wheel pendulum : GPC
- **Validation** : in simulation and real-time experiments
- **Future work** : control of more complex systems



For further reading ...







[www.lirmm.fr/~chemori/](http://www.lirmm.fr/~chemori/)

## Ahmed Chemori

**Main menu**


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**Research activities**


- Research topics
- Current projects
- Past projects
- Publications
- Students
- Seminars and plenaries
- Collaborations

**Scientific animation**

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*CNRS Researcher in Automatic Control and Robotics*



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**Ahmed CHEMORI.** was born in Constantine. He received his B.Sc. and M.Sc. degrees in electronics from the university of Constantine, Algeria in 1998 and 2000. He received his M.Sc. in Automatic Control from Institut National Polytechnique de Grenoble (INPG), France. In 2005 he received his Ph.D. in automatic control from Institut National Polytechnique de Grenoble (Department of Automatic Control), France. In 2004/2005 he has been a Temporary Teaching and Research Attached to the University Paris 11 and Laboratoire des Signaux et Systèmes (LSS). In 2005/2006 he has been at Gipsa-Lab (Former LAG) as a postdoctoral researcher. Since 2006 he has been working at Laboratoire d'Informatique, de Robotique et de Microélectronique de Montpellier (LIRMM - UMR 5506) as "Chargé de recherche CNRS", where he conducts research in the area of nonlinear control of mechanical and robotic systems. His research topics includes: Humanoid robot control, underactuated mechanical systems control and parallel robots control.

**New experiments**

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