# Set-membership state estimation for uncertain systems based on zonotopes and ellipsoids

S. Ben Chabane [1], C. Stoica Maniu [1], D. Dumur [1], T. Alamo [2], E. F. Camacho [2]

[1]SUPELEC Systems Sciences (E3S) – Automatic Control Department, Gif sur Yvette, France

[2]Department of Ingeniería de Sistemas y Automática, Universidad de Sevilla, Sevilla, Spain

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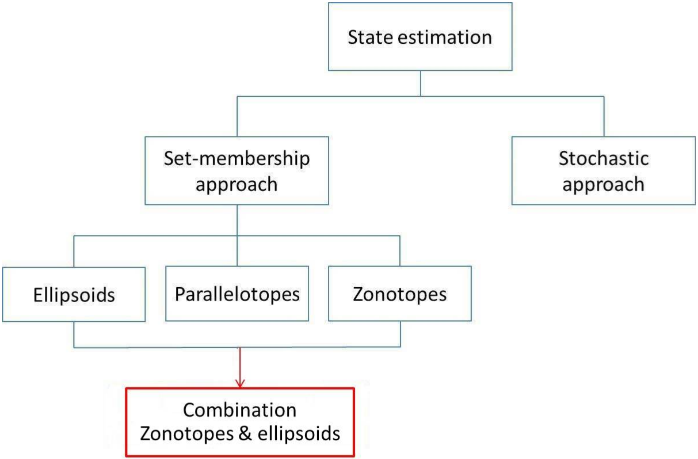
## Outline

## Two approaches for estimation

Knowing the state of a system is crucial for solving many control problems.



## System

Consider the linear discrete-time invariant system:

$$\begin{cases} x_{k+1} = Ax_k + F\omega_k \\ y_k = c^\top x_k + \sigma v_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}$, $\omega_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}$. The pair $(c^\top, A)$ is detectable.

## Objective

Assume that $x_0 \in \mathcal{X}_0$, $\omega_k \in \mathcal{W}$ and $v_k \in \mathcal{V}$ for all $k > 0$.

Set-membership approach:

$$x_k \in \hat{\mathcal{X}}_k \Rightarrow x_{k+1} \in \hat{\mathcal{X}}_{k+1}.$$

$\rightarrow$ Find the optimal set $\hat{\mathcal{X}}_{k+1}$ that contains $x_{k+1}$.

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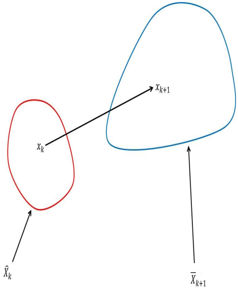
$$\bar{\mathcal{X}}_{k+1} = A\hat{\mathcal{X}}_k \cup F\mathcal{W}$$

where $\bar{\mathcal{X}}_{k+1}$ is a predicted state estimation set.



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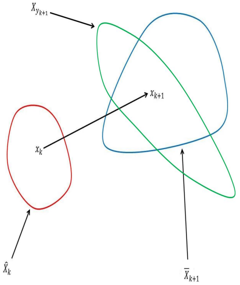
- Measurement

$$\mathcal{X}_{y_{k+1}} = \{x_{k+1} \in \mathbb{R}^n : |c^\top x_{k+1} - y_{k+1}| \leq \sigma\}$$

where $\mathcal{X}_{y_{k+1}}$ is the consistent state set by using the measurement $y_{k+1}$.



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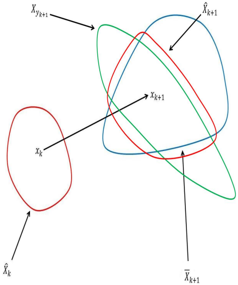
- Correction step

$$\hat{\mathcal{X}}_{k+1} \supseteq \bar{\mathcal{X}}_{k+1} \cap \mathcal{X}_{y_{k+1}}$$

where $\hat{\mathcal{X}}_{k+1}$ is the guaranteed state estimation set.



## Mathematical notations

- **Interval**: $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$
  Unitary interval: $\mathbf{B} = [-1, 1]$
  Box (interval vector): $([a_1, b_1], \ldots, [a_n, b_n])^\top \subset \mathbb{R}^n$
  Unitary box: $\mathbf{B}^n = \{x \in \underbrace{([-1, 1], \ldots, [-1, 1])}_{n \text{ times}}{}^\top\} \subset \mathbb{R}^n$.
- **Strip**: $\mathcal{S}(y, d) = \{x \in \mathbb{R}^n : |y - d^\top x| \leq 1\}$.
- **Minkovsky sum**: $\mathcal{A} \oplus \mathcal{B} = \{a + b : a \in \mathcal{A}, b \in \mathcal{B}\}$.
- **Ellipsoid**: $\mathcal{E}(c, P) = \{x : (x - c)^\top P^{-1}(x - c) \leq 1\}$, where $c \in \mathbb{R}^n$ is the center and a matrix $P = P^\top \succ 0$ characterizes its shape and size.
- **Zonotope**: a convex symmetric polytope.
  $m$-zonotope: the set $\mathcal{Z} = p \oplus H\mathbf{B}^m = \{x \in \mathbb{R}^n : x = p + Hz, z \in \mathbf{B}^m\}$, with a vector $p \in \mathbb{R}^n$ and a matrix $H \in \mathbb{R}^{n \times m}$.
- ***P*-radius** of a $m$-zonotope $\mathcal{Z} = p \oplus H\mathbf{B}^m$:
  $L = \max_{x \in \mathcal{Z}} \|x - p\|_P^2 = \max_{x \in \mathcal{Z}} (x - p)^\top P(x - p)$, with $P = P^\top \succ 0$

## Properties

- ***Property 1***: Affine transformation of an ellipsoid
  $A\,\mathcal{E}(c,P)+b=\mathcal{E}(Ac+b,APA^\top)$, where $A\in\mathbb{R}^{n\times n}$ and $b\in\mathbb{R}^n$.
- ***Property 2***: Sum of two ellipsoids Durieu:01 [Durieu et al., 2001]
  $\mathcal{E}_1(c_1,P_1)\cup\mathcal{E}_2(c_2,P_2)\subseteq\mathcal{E}(c,P)$, with $c=c_1+c_2$, $P=\phi_1^{-1}P_1+\phi_2^{-1}P_2$ and $\phi_1+\phi_2=1$.
- ***Property 3***: Intersection between an ellipsoid and a strip Fogel:82 [Fogel and Huang, 1982]
  $\mathcal{E}(c,P)\cap\mathcal{S}(y,d)\subseteq\mathcal{E}'(c',P')$, with $c'=c+\dfrac{\psi\delta}{1+\psi g}Pd$, $\psi\geq 0$,
  $P'=(1+\psi-\dfrac{\psi\delta^2}{1+\psi g})(P-\dfrac{\psi}{1+\psi g}Pdd^\top P)$, $g=d^\top Pd$ and $\delta=y-d^\top c$.
- ***Property 4***: Affine transformation of a zonotope Combastel:03 [Combastel, 2003]
  $A\mathcal{Z}=(Ap)\oplus(AH)\mathbf{B}^m$, with $A\in\mathbb{R}^{n\times n}$.
- ***Property 5***: The Minkovsky sum of two centered zonotopes Combastel:03 [Combastel, 2003]
  $\mathcal{Z}_1\oplus\mathcal{Z}_2=\begin{bmatrix}H_1 & H_2\end{bmatrix}\mathbf{B}^{m_1+m_2}$, with $\mathcal{Z}_1=H_1\mathbf{B}^{m_1}\subseteq\mathbb{R}^n$ and $\mathcal{Z}_2=H_2\mathbf{B}^{m_2}\subseteq\mathbb{R}^n$.
- ***Property 6***: Intersection between a zonotope and a strip Le:2012 [Le, 2012]
  $\mathcal{Z}\cap\mathcal{S}(\dfrac{d}{\sigma},\dfrac{c}{\sigma})\subseteq\hat{\mathcal{Z}}(\lambda)=\hat{p}(\lambda)\oplus\hat{H}(\lambda)\mathbf{B}^{m+1}$, with $\hat{p}(\lambda)=p+\lambda(d^\top-c^\top p)$,
  $\hat{H}(\lambda)=\begin{bmatrix}(I-\lambda c^\top)H & \sigma\lambda\end{bmatrix}$ and $\lambda\in\mathbb{R}^n$.

## System

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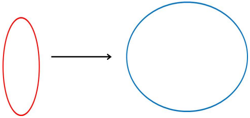
Assume that $x_0 \in \mathcal{E}_0(c_0, P_0)$, $\mathcal{W} = \mathcal{E}(0, I_n)$, $\mathcal{V} = \mathbf{B}$ and $x_k \in \mathcal{E}_k(c_k, P_k)$.

**Prediction step**:
$\bar{\mathcal{E}}_{k+1}(c_{k+1}, P_{k+1}) = \hat{\mathcal{E}}_k(Ac_k, AP_kA^\top) \cup \mathcal{E}(0, FF^\top)$
*Property 2* implies that $\exists\phi = [\phi_1, \phi_2]^\top \in \mathbb{R}^2$ such that:

$$\begin{cases} c_{k+1} = Ac_k \\ P_{k+1} = {\phi_1}^{-1}P_1 + {\phi_2}^{-1}P_2 \end{cases}$$



with $P_1 = AP_kA^\top$, $P_2 = FF^\top$ and $\phi_1 + \phi_2 = 1$.

## Minimization of the size of $\bar{\mathcal{E}}_{k+1}(c_{k+1}, P_{k+1})$

**Trace criterion**:
[Durieu et al., 2001]
$$\phi^* = \min_{\phi} tr(P_{k+1})$$

**Determinant criterion**:
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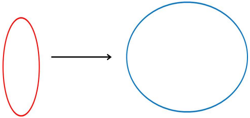

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*Property 3* implies that $\exists \psi \geq 0$ such that:

$$\hat{c}_{k+1} = c_{k+1} + \frac{\psi\delta}{1+\psi g} P_{k+1} \frac{c}{\sigma}$$

$$\hat{P}_{k+1} = (1 + \psi - \frac{\psi\delta^2}{1+\psi g})(P_{k+1} - \frac{\psi}{1+\psi g} P_{k+1} \frac{c}{\sigma} \frac{c}{\sigma}^\top P_{k+1})$$

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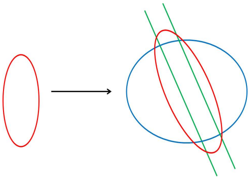

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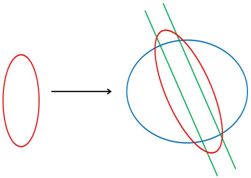

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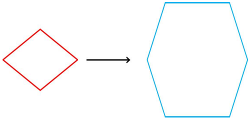

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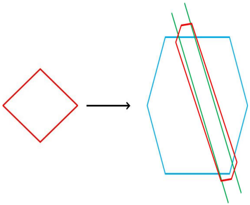

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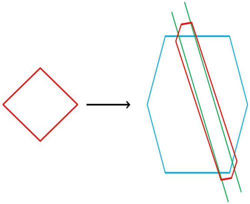

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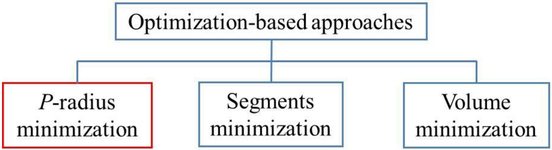


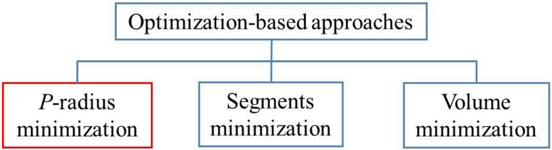

The $P$-radius of the state estimation zonotope at time $k$ is:

$$L_k = \max_{x \in \hat{\mathcal{Z}}_k(\lambda)} \|x - \hat{p}_k\|_P^2 = \max_{x \in \hat{\mathcal{Z}}_k(\lambda)} (x - \hat{p}_k)^\top P (x - \hat{p}_k).$$

The non-increasing condition of the $P$-radius is given by [Le et al., 2013]:

$$L_{k+1} \leq \beta L_k + \max_{\omega \in \mathbf{B}^n} \|F\omega\|_2^2 + \sigma^2,$$

$$0 < \beta < 1.$$

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Sufficient condition is to solve the LMI [Le et al., 2013]:
Find the smallest value of $\beta \in (0,1)$ such that:

$$\max_{\tau, P, Y} \tau$$

subject to

$$\begin{cases} \frac{(1-\beta)P}{\sigma^2 + const} \succeq \tau I \\ \begin{bmatrix} \beta P & 0 & 0 & A^\top P - A^\top c Y^\top \\ * & F^\top F & 0 & F^\top P - F^\top c Y^\top \\ * & * & \sigma^2 & Y^\top \sigma \\ * & * & * & P \end{bmatrix} \succeq 0 \end{cases}$$

with $const = \max_{\omega \in \mathbf{B}^n} \|F\omega\|_2^2$ and the decision variables are $\tau \in \mathbb{R}$, $P \in \mathbb{R}^{n \times n}$, and $Y = P\lambda \in \mathbb{R}^n$.

**Advantages of the two previous estimation approaches**:

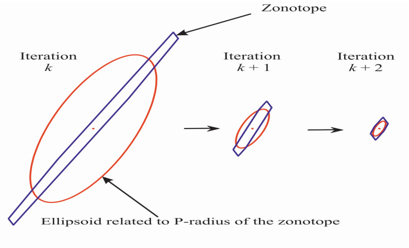
- Good estimation accuracy using the zonotopic estimation method
- Low complexity using the ellipsoidal estimation

→ The idea is to combine these two methods!

**Question**: How to make a transition from the zonotopic estimation to the ellipsoidal estimation?

Ellipsoid related to the $P$-radius of the zonotope $\hat{\mathcal{Z}}_k$:

$$\mathcal{E}(\hat{p}_k, L_k P^{-1}) = \{x_k \in \mathbb{R}^n : (x_k - \hat{p}_k)^\top (L_k P^{-1})^{-1} (x_k - \hat{p}_k) \leq 1\}.$$

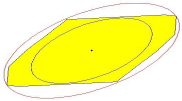



$$L_k \to L_\infty = \frac{\sigma^2 + const}{1 - \beta}$$

Scaling of the ellipsoid related to the $P$-radius of the zonotope $\hat{\mathcal{Z}}_k$:

$$\mathcal{E}(\hat{p}_k, L_k P^{-1}) = \{x_k \in \mathbb{R}^n : (x_k - \hat{p}_k)^\top \alpha (L_k P^{-1})^{-1} (x_k - \hat{p}_k) \leq 1\}.$$

with $\alpha \in (0, 1)$.



**Objective**: Obtain the smallest ellipsoid which is the outer bound of the zonotope $\hat{\mathcal{Z}}_k$!
**Solution**:
$\max \alpha$

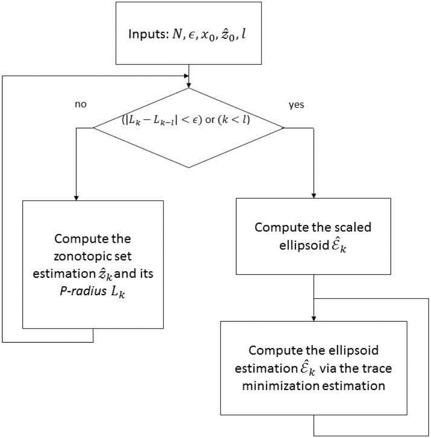
subject to

$$\begin{cases} 0 < \alpha \leq 1 \\ (x_k - \hat{p}_k)^\top \alpha P L_k^{-1} (x_k - \hat{p}_k) \leq 1, \ \ \forall x_k \in \mathcal{V}_{\hat{\mathcal{Z}}_k}, \end{cases}$$

with $\mathcal{V}_{\hat{\mathcal{Z}}_k}$ the vertices of $\hat{\mathcal{Z}}_k$.

Sofiane:2014

***Algorithm***: Combination of the two methods [Ben Chabane et al., 2014]




where $l$ is the length of the horizon of slow variation of the $P$-radius and $\epsilon$ the desired level of accuracy of the $P$-radius

## Example

Consider the following linear discrete-time invariant system:

$$\begin{cases} x_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 0.8 \end{bmatrix} x_k + \begin{bmatrix} -0.24 \\ 0.04 \end{bmatrix} \omega_k \\ y_k = \begin{bmatrix} -2 & 1 \end{bmatrix} x_k + 0.4 v_k \end{cases}$$

with $\|v_k\|_\infty \leq 1$, $\|\omega_k\|_\infty \leq 1$. The initial state belongs to the box $3\mathbf{B}^2$.

## Example

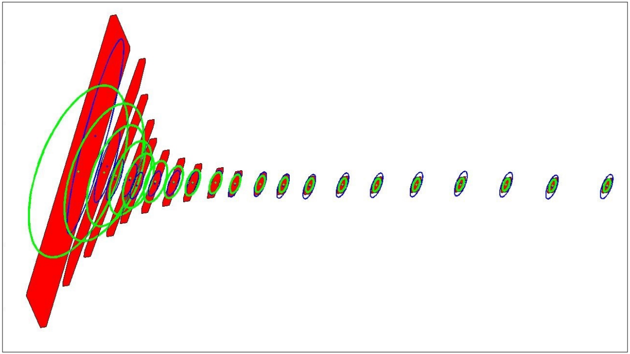


Figure: State space sets

## Example

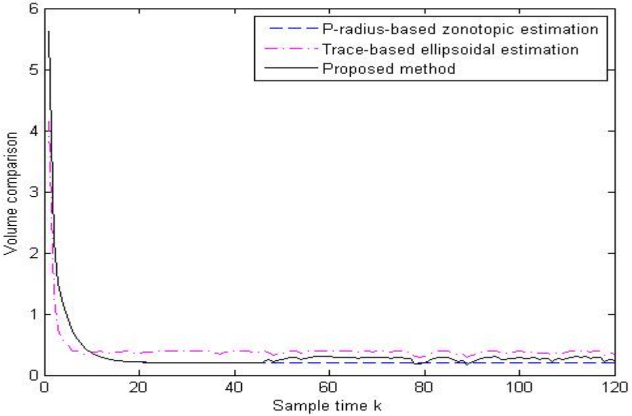



Figure: Comparison of the volumes

## Example

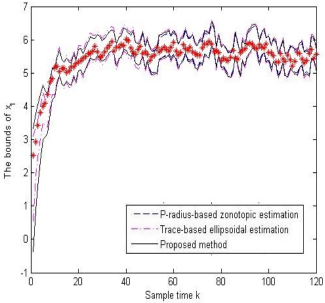



Figure: Bounds of $x_1$ using the three methods

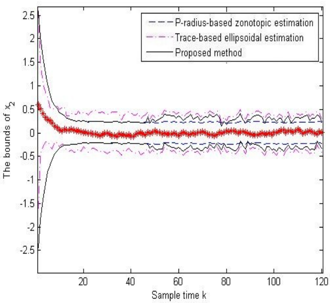



Figure: Bounds of $x_2$ using the three methods

## Example

Table: Total computation time after 120 time instants

| Algorithm | Time(second) |
|---|---|
| $P$-radius based zonotopic estimation | 0.2652 |
| Trace-based ellipsoidal estimation | 0.0808 |
| Combined method | 0.1479 |

**Conclusion**:

- Set-membership estimation based on zonotopes is more accurate than ellipsoidal estimation but with higher complexity
- A new criterion based on the $P$-radius is proposed in order to make a transition from the zonotopic estimation to the ellipsoidal estimation
- The proposed method offers a good trade-off between the accuracy and the complexity

**Perspectives**:

- Apply the proposed method to Fault Detections (FD) and Fault Tolerant Control (FTC) purposes

# THANK YOU FOR YOUR ATTENTION

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