Robust rendezvous planning under maneuver execution errors.
A worst case approach

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Acknolewdgements

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Problems and objectives

Rendezvous

- Relative motion between a target and a chaser spacecraft;
- Impulsive rendezvous

Robustify open-loop maneuver plan to GNC systems errors:

Navigation syst. Uncertainties on the measured state

Thrusters actuation syst.

- Errors on impulses firing time
- Errors on impulses execution

Objectives

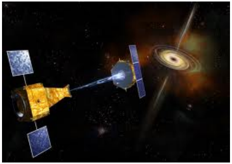
Robust and convex optimisation for worst-case approach

- Desensitize the maneuver plan to GNC errors
- Provide tractable algorithms (polynomial complexity)
- Provide deterministic and guaranteed feasability certificate

Orbital rendezvous definition

Orbital Rendezvous consists of:

- Keplerian relative motion
- Usage of ergols thrusters :
  - the control is modeled by impulsive signals,
  - Instantaneous velocity change,
  - Sequence of coasting arc limited by thruster impulses;



Rendez-vous problem

- Steering the chaser spacecraft from a state A to state B in fixed time;
- Assuming some operating constraints (Actuators bounds, safety constraints ...);

Rendezvous guidance problem $\Leftrightarrow$ Optimal control problem

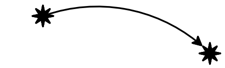
Formulation of the RdV guidance problem

Deterministic and exact final condition rendezvous exact

- Minimize a given objective $J(\cdot)$ Consumption under dynamic constraints:

$$\min_{\Delta\tilde{V},\nu_i} \quad J(\cdot)$$

$$\text{w.r.t.} \quad \begin{cases} \frac{d\tilde{X}(\nu)}{d\nu} = \tilde{A}_{TH}\tilde{X}(\nu) + \tilde{B}_{TH}\sum_i \Delta\tilde{V}_i\delta(\nu-\nu_i) \\ \tilde{X}(\nu_1) = \tilde{X}_1, \ \tilde{X}(\nu_f) = \tilde{X}_f \\ -\overline{\Delta\tilde{v}}_i \leq \Delta\tilde{V}(\nu_i) \leq \overline{\Delta\tilde{v}}_i \ , \forall \ i = 1, \cdots, N \end{cases} \tag{1}$$



ν	excentric anomaly (independant variable)
ν_i	impulses firing time
$\tilde{X} \in \mathbb{R}^6$	State vector in ν
$\Delta\tilde{V}_i \in \mathbb{R}^3$	i^{th} impulse out of N
$(\tilde{A}_{TH}, \tilde{B}_{TH})$	Tschauner-Hempel relative motion dynamics
$\overline{\Delta\tilde{v}}$	Impulse saturation

Formulation of the RdV guidance problem

Deterministic and exact final condition rendezvous exact

Direct methodology: Linear Programming [Waespy,1970]

- Fixing the number and time of impulse ν_i
- Replacing the the dynamic equation by its transition equation
- Minimizing $J(\cdot)$ linear in $\Delta\tilde{V}$

$$\begin{array}{cl} \min\limits_{\Delta\tilde{V}} & J(\cdot) \\ \text{w.r.t.} & \begin{cases} \tilde{X}_f = \Phi_{ya}(\nu_f,\nu_1)\tilde{X}_1 + \mathbf{B}\Delta\tilde{V} \\ -\overline{\Delta\tilde{v}}_i \leq \Delta\tilde{V}(\nu_i) \leq \overline{\Delta\tilde{v}}_i \;, \forall\; i = 1,\cdots,N \end{cases} \end{array} \quad (2)$$

Φ_{ya} Yamanaka-Ankersen Transition matrix

$\Delta\tilde{V}_i \in \mathbb{R}^{3N}$ Impulse vector

$Z \in \mathbb{R}^{3N}$ slack variables vector

$\mathbf{B} = \begin{bmatrix} \Phi_{ya}(\nu_f,\nu_1)B & \dots & \Phi_{ya}(\nu_f,\nu_N)B \end{bmatrix}$

Disturbed Rendezvous problem

The rendezvous condition

$$\tilde{X}(\nu_f) = \Phi_{ya}(\nu_f,\nu_1)\tilde{X}_1 + \underbrace{\begin{bmatrix}\Phi_{ya}(\nu_f,\nu_1)B & \dots & \Phi_{ya}(\nu_f,\nu_N)B\end{bmatrix}}_{\mathbf{B}}\Delta\tilde{V}$$

Under the following disturbance

Navigation errors: $\tilde{X}_1 \in \mathcal{X}_{nav}$

Impulse time errors $\nu \in \mathcal{V}$

Thrust errors: $\Delta\tilde{V} \in \mathcal{U}$

The rendezvous condition

$$\tilde{X}(\nu_f) = \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1 + \underbrace{\begin{bmatrix}\Phi_{ya}(\nu_f, \nu_1)B & \dots & \Phi_{ya}(\nu_f, \nu_N)B\end{bmatrix}}_{\mathbf{B}}\Delta\tilde{V}$$

Under the following disturbance

Navigation errors: $\tilde{X}_1 \in \mathcal{X}_{nav}$

Impulse time errors $\nu \in \mathcal{V}$

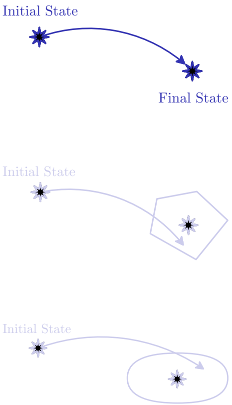
Thrust errors: $\Delta\tilde{V} \in \mathcal{U}$

$\tilde{X}(\nu_f)$ belongs to a subset $\mathcal{X}_f$ that should be included in a tolerance set $\mathcal{T}$

$$\tilde{X}(\nu_f) \in \mathcal{X}_f \subset \mathcal{T}$$

$\Rightarrow$ The rendezvous condition must be relaxed

RDV with relaxed final conditions



Exact final condition: Linear Programming

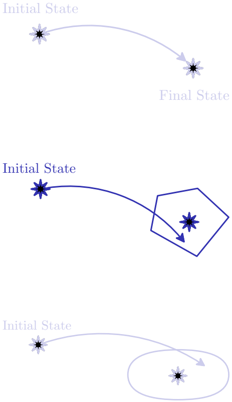
$$\min_{\Delta\tilde{V}} J(\cdot)$$
$$\begin{cases} A_{RdV}\Delta\tilde{V} = b_{RdV} \\ A_{\Delta V}\Delta\tilde{V} \leq b_{\Delta V} \end{cases}$$

where

$$\begin{aligned} A_{RdV} &= \mathbf{B} \\ b_{RdV} &= \tilde{X}_f - \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1 \end{aligned}$$

$$A_{\Delta V} = \begin{bmatrix} \mathbb{I}_{3N} \\ -\mathbb{I}_{3N} \end{bmatrix}, \; b_{\Delta V} = \begin{bmatrix} \overline{\Delta\tilde{v}I} \\ \overline{\Delta\tilde{v}I} \end{bmatrix}$$

RDV with relaxed final conditions



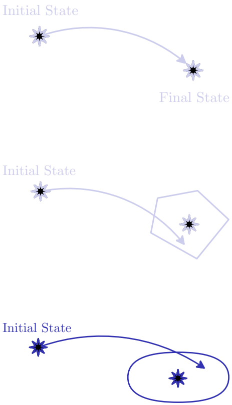
Polytopic final condition: Linear Programming

$$\min_{\Delta\tilde{V}} J(\cdot)$$
$$\text{w.r.t.} \quad A\Delta\tilde{V} \leq b$$

where

$$A = \begin{bmatrix} H\mathbf{B} \\ \mathbb{I}_{3N} \\ -\mathbb{I}_{3N} \end{bmatrix}, \quad b = \begin{bmatrix} K - H\left(\Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1\right) \\ \overline{\Delta\tilde{v}} \\ \overline{\Delta\tilde{v}} \end{bmatrix}$$

RDV with relaxed final conditions



Ellipsoidal final condition: Conic quadratic Programming

$$\min_{\Delta \tilde{V}} J(\cdot)$$

$$\text{w.r.t.} \begin{cases} \left\| A\Delta\tilde{V} + b \right\|_2 \leq 1, \\ A_{\Delta V}\Delta\tilde{V} \leq b_{\Delta V} \end{cases}$$

where

$$\begin{array}{rcl} A & = & R\mathbf{B} \\ b & = & R\left(-\tilde{X}_f + \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1\right) \end{array}$$

$$A_{\Delta V} = \begin{bmatrix} \mathbb{I}_{3N} \\ -\mathbb{I}_{3N} \end{bmatrix}, \quad b_{\Delta V} = \begin{bmatrix} \overline{\Delta\tilde{v}}I \\ \overline{\Delta\tilde{v}}I \end{bmatrix}$$

Objective functions $J(\cdot)$ and budget condition

$J(\cdot)$ must be linear in the decision variable

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Robustness cost Minimize the tolerance subset

Box case $\|K\|_1$

Ellipsoid case $\log\det R^{-1}$

Tolerance subset is part of decision variables

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$J(\cdot)$ must be linear in the decision variable

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Tolerance subset is part of decision variables

ΔV Budget Constraint

- ΔV consumption has to be restricted

$$\|\Delta\tilde{V}\|_1 \leq M_{\Delta V}$$

Convex and Robust Optimisation

Uncertain program (P_Δ)

$$\begin{array}{ll} \min\limits_{x} & f_0(x) \\ \text{w.r.t.} & f_i(x, \Delta_i(u_i)) \leq 0, \quad \forall \Delta_i \in \mathcal{V}_i, \ i = 1, \dots, m \end{array} \tag{3}$$

$x \in \mathbb{R}^n$ Optimisation variables

f_0, f_i Program structure (cost and constraint functions)

Δ_i Problem data

$u_i \in \mathcal{U}_i \subset \mathbb{R}^{k_i}$ Disturbance variables

$\mathcal{V}$ Uncertainty set $\mathcal{V} = \mathcal{V}_1 \times \cdots \times \mathcal{V}_m$

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$x \in \mathbb{R}^n$	Optimisation variables
f_0, f_i	Program structure (cost and constraint functions)
Δ_i	Problem data
$u_i \in \mathcal{U}_i \subset \mathbb{R}^{k_i}$	Disturbance variables
$\mathcal{V}$	Uncertainty set $\mathcal{V} = \mathcal{V}_1 \times \cdots \times \mathcal{V}_m$
$\mathcal{U}$	Disturbance set $\mathcal{U} = \mathcal{U}_1 \times \cdots \times \mathcal{U}_m$

Paradigms

- x must be obtained by solving (3) without the exact knowledge of datas Δ_i;
- Results are valid only for datas $\Delta_i \in \mathcal{V}_i$;
- No comprimises on constraints $f_i(x,\Delta_i(u_i)) \leq 0$ are tolerated while $\Delta_i \in \mathcal{V}_i$.

Convex and Robust Optimisation

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Definitions

Robust feasible solution x is a feasible robust solution of the uncertain program $(P_{\Delta \in \mathcal{U}})$ if and only if $f_i(x, \Delta_i) \leq 0$ for all Δ's realizations

Guarenteed cost the guaranteed is the worst case cost for a given robust feasible solution x: $\max\limits_{\Delta_i}\{f_0(x) : \Delta_i \in \mathcal{V}_i, \forall i\}$.

Convex and Robust Optimisation

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Robust counterpart

$$\begin{array}{ll} \min\limits_{x} & \max\limits_{\Delta_i \in \mathcal{V}_i} f_0(x) \\ \text{sous} & f_i(x, \Delta_i(u_i)) \leq 0, \quad \forall \Delta_i \in \mathcal{V}_i,\ i = 1, \dots, m \end{array} \tag{4}$$

Robust Linear Programming

Robust counterpart to uncertain LP

$$\begin{array}{ll} \min_{x} & \max_{(A,b)\in\mathcal{V}} \gamma^T x \\ \text{sous} & Ax \leq b, \ \forall (A,b) \in \mathcal{V} \end{array}$$

- $(A,\ b)$ depend linearly in the disturbance variables u_i

$$\mathcal{V} = \left\{ [A;b] = [A^0, b^0] + \sum_{j=1}^{k} u_j [A^j, b^j], \ u \in \mathcal{U} \subset \mathbb{R}^k \right\}$$

- Disturbance variables u_i belong to convex set $\mathcal{U}$

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- Disturbance variables u_i belong to convex set $\mathcal{U}$

Robust & convex counterpart [BenTal, 2009]

Type of $\mathcal{U}$	Robust Counterpart
polytopic/interval	Linear Prog.
ellipsoidal	Conic Quadratic Prog.
ellipsoid intersection	Conic Prog.
Conic	Conic Prog.
Unstructured norm-bounded	Conic Prog.

Table: From A. Ben-Tal, L. EL Gahoui, a. Nemirovski, *Robust Optimization*, Princeton U. Press, 2009

Rendezvous guidance problem

Convex and Robust Optimisation

Handling the GNC system errors

- Errors on impulses firing time
- Thrust mis-execution
- Navigation errors

Conclusion

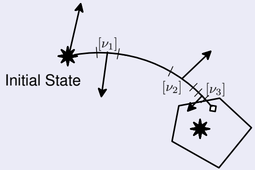
Errors on impulses firing time

- Polytopic arrival set:

$$H\left(\underbrace{\left[\Phi_{\boldsymbol{ya}}(\nu_{\boldsymbol{f}},\nu_{\boldsymbol{1}})B \quad \dots \quad \Phi_{\boldsymbol{ya}}(\nu_{\boldsymbol{f}},\nu_{\boldsymbol{N}})B\right]}_{\mathsf{B}}\Delta\tilde{V}+\Phi_{\boldsymbol{ya}}(\nu_{\boldsymbol{f}},\nu_{\boldsymbol{1}})\tilde{X}(\nu_{\boldsymbol{1}})-\tilde{X}_{\boldsymbol{f}}\right)\leq K$$

- Uncertain firing time:
  $\nu_i \in [\nu_i^* - \delta\nu_i, \nu_i^* - \delta\nu_i] = [\nu]_i$

$\Rightarrow$ Interval Analysis Computation [Moore66,Jaulin01]
$\Phi_{ya}(\nu_f, \nu_i) \in [\Phi_{ya}(\nu_i)]$



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$$H\left(\underbrace{\left[\Phi_{ya}(\nu_f,\nu_1)B \quad \dots \quad \Phi_{ya}(\nu_f,\nu_N)B\right]}_{B}\Delta\tilde{V} + \Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) - \tilde{X}_f\right) \leq K$$

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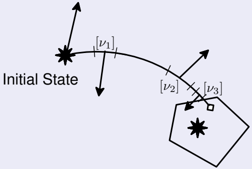

1. Uncertainties on transition matrix :

$$\Phi_{ya}(\nu_f,\nu_i) = \Phi_{ya}^*(\nu_f,\nu_i) + u_i\overline{\delta\Phi_i}, |u_i| \leq 1, \ i = 1,\dots,N$$

2. Uncertain polytopic RDV with $\|u\|_\infty \leq 1$

$$H\left(\Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) + \sum_{i=1}^{N}\{(\Phi_{ya}^*(\nu_f,\nu_i) + u_i\overline{\delta\Phi_i})B\Delta\tilde{V}(\nu_i)\}\right) \leq K$$

3. Robust polytopic RDV

$$H\left(\Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) + \sum_{i=1}^{N}\Phi_{ya}^*(\nu_f,\nu_i)B\Delta\tilde{V}(\nu_i) + |\overline{\delta\Phi_i}B\Delta\tilde{V}(\nu_i)|\right) \leq K$$

Errors on impulses firing time

- Polytopic arrival set:

$$H\left(\underbrace{\left[\Phi_{ya}(\nu_f,\nu_1)B \quad \dots \quad \Phi_{ya}(\nu_f,\nu_N)B\right]}_{B}\Delta\tilde{V} + \Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) - \tilde{X}_f\right) \leq K$$

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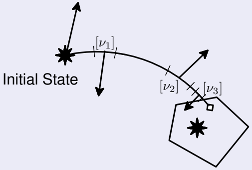


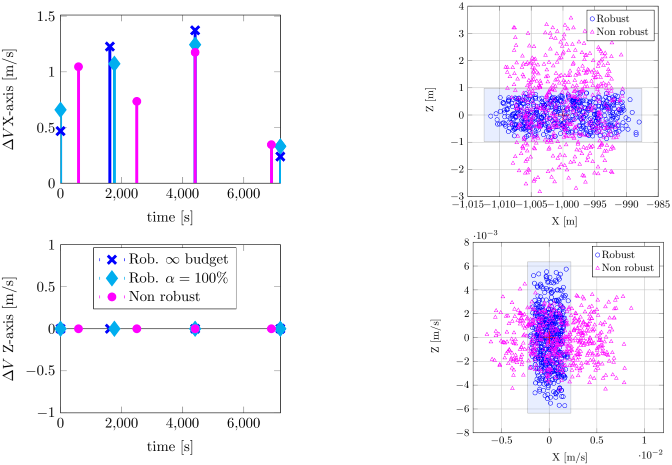
Robust Counterpart, LP [Ben-Tal & Nemirovski 00]

$$\min_{\Delta\tilde{V},Z} \quad \sum_{i=1} K_i$$

$$\text{w.r.t.} \begin{cases} H\Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) + H\sum_{i=1}^{N}\Phi_{ya}^*(\nu_f,\nu_i)B\Delta\tilde{V}(\nu_i) + |\overline{\delta\Phi_i}B|Z_i \leq K \\ \left.\begin{matrix} -Z_i \leq \Delta\tilde{V}_i \leq Z_i, \quad -\Delta\tilde{V}_i \leq Z_i \\ [Z_{3i+1},\ Z_{3i+2},\ Z_{3i+3}]^T \leq \overline{\Delta\tilde{v}}_{i+1}\ ,\forall\ i \end{matrix}\right\} \quad \text{Saturation} \\ \|\Delta\tilde{V}\|_1 \leq \sum_{i=1}^{3N} Z_i \leq M_{\Delta V} \qquad \Delta V\text{-Budget} \end{cases} \quad (5)$$

Errors on impulses firing time: numerical examples

ATV mission: 1 second errors



Semi-major axis	Eccentricity	Initial anomaly	mission duration
6763 km	0.0052	0	7200 s
$X_1 = [-30000 \;\; 5000 \;\; 8.154 \;\; 0]^T$		$X_1 = [-1000 \;\; 0 \;\; 0 \;\; 0]^T$	

Errors on impulses firing time: numerical examples

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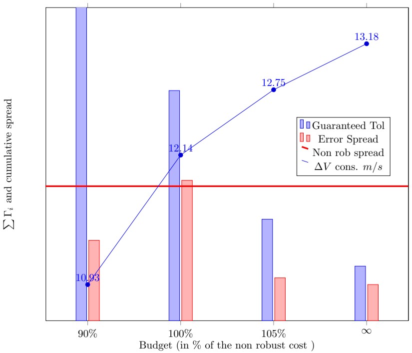

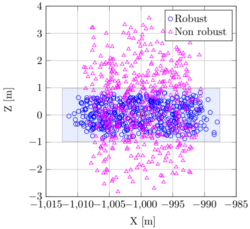

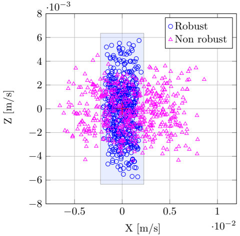

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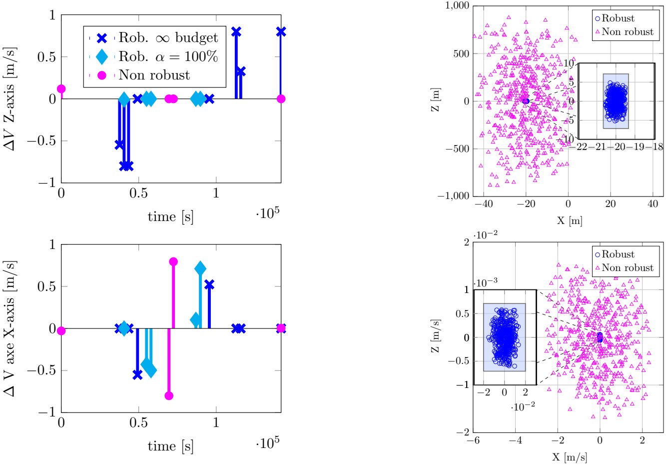
Errors on impulses firing time: numerical examples

Proba 3 mission: 1 second errors

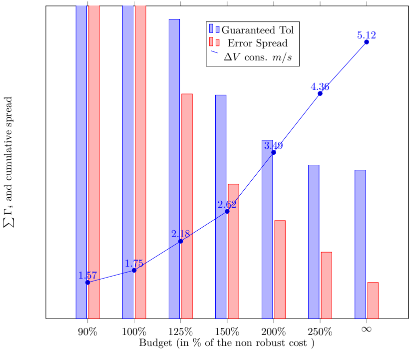


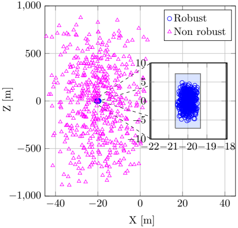
Semi-major axis	Eccentricity	Initial anomaly	mission duration
37039 km	0.8	117 °	31872 s
$X_1 = [-5000 \;\; 0 \;\; 0 \;\; 0]^T$		$X_1 = [-20 \;\; 0 \;\; 0 \;\; 0]^T$	

Table: Proba3 mission

Errors on impulses firing time: numerical examples

Proba 3 mission: 1 second errors





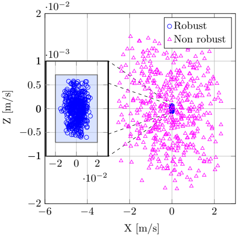


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Table: Proba3 mission

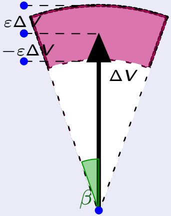
Impulsion mis-execution

Errors description

- Uncertain orientation: where M_{rot} is The Cardan rotation matrix with small angles $(|\psi|,\ |\theta|,\ |\phi| \leq \beta)$

$$M_{rot} = \begin{bmatrix} 1 & -\psi_i & \theta_i \\ \psi_i & 1 & -\phi_i \\ -\theta_i & \phi_i & 1 \end{bmatrix}$$

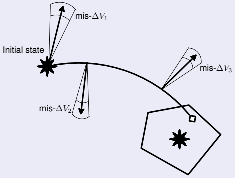
- Uncertain amplitude: $|\lambda_i| \leq \varepsilon$



$$\Delta\tilde{V}_i = (1 + \lambda_i) M_{rot} \Delta V_i^*$$

Impulsion mis-execution

- Mis-impulsion : $\Delta\tilde{V}_i = (1+\lambda_i) M_{rot} \Delta V_i^*$
- Polytopic arrival set:
  $H\left(\mathbf{B}\Delta\tilde{V} + \Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) - \tilde{X}_f\right) \leq K$



1. Uncertainties on the **B** matrix
   $\mathbf{B} = \left[\Phi_{ya}(\nu_f,\nu_1)BM_1 \quad \ldots \quad \Phi_{ya}(\nu_f,\nu_N)BM_N\right]$ where $M_i = (1+v_i)M_{rot}$

$$M_i \in [M] = \mathbb{I}_3 + [-\overline{\delta M},\ \overline{\delta M}] \text{ with } \overline{\delta M} = \begin{bmatrix} \varepsilon & (1+\varepsilon)\beta & (1+\varepsilon)\beta \\ (1+\varepsilon)\beta & \varepsilon & (1+\varepsilon)\beta \\ (1+\varepsilon)\beta & (1+\varepsilon)\beta & \varepsilon \end{bmatrix}$$

2. Uncertain polytopic RDV with $\|u\|_\infty \leq 1$:

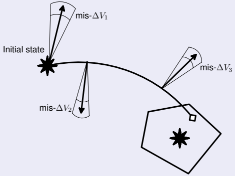
$$H\Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) + H\mathbf{BM}(u))\Delta\tilde{V} \leq K$$

3. Robust polytopic RDV condition

$$\Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) + H\mathbf{B}\Delta\tilde{V} + \sum_{i=1}^{N}\sum_{j=1}^{4}\left|H\Phi_{ya}(\nu_f,\nu_i)BM^j\Delta\tilde{V}_i\right| \leq K$$

Impulsion mis-execution

- Mis-impulsion : $\Delta\tilde{V}_i = (1+\lambda_i)M_{rot}\Delta V_i^*$
- Polytopic arrival set:
  $H\left(\mathbf{B}\Delta\tilde{V} + \Phi_{\boldsymbol{ya}}(\nu_{\boldsymbol{f}},\nu_{\mathbf{1}})\tilde{X}(\nu_{\mathbf{1}}) - \tilde{X}_{\boldsymbol{f}}\right) \leq K$



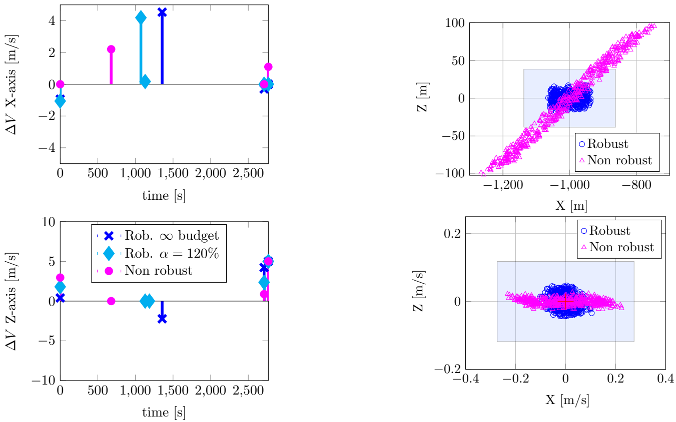
Robust counterpart, LP

$$\min_{\Delta\tilde{V}} \quad \sum_{i=1}^{6} K_i$$

$$\text{w.r.t.} \begin{cases} \Phi_{ya}(\nu_f,\nu_1)\tilde{X}(\nu_1) + H\mathbf{B}\Delta\tilde{V} + \sum_{i=1}^{N}\sum_{j=1}^{4}\left|H\Phi_{ya}(\nu_f,\nu_i)BM^j\right| Z_i \leq K \\ \left.\begin{array}{c} -Z_i \leq \Delta\tilde{V}_i \leq Z_i, \quad -\Delta\tilde{V}_i \leq Z_i \\ [Z_{3i+1},\ Z_{3i+2},\ Z_{3i+3}]^T \leq \overline{\Delta\tilde{v}}_{i+1}\ ,\forall\ i \end{array}\right\} \quad \text{Saturation} \\ \|\Delta\tilde{V}\|_1 \leq \sum_{i=1}^{3N} Z_i \leq M_{\Delta V} \qquad \Delta V\text{-Budget} \end{cases}$$

Impulses mis-execution : numerical examples

ATV mission: Magnitude errors 0.1% Orientation errors 1°

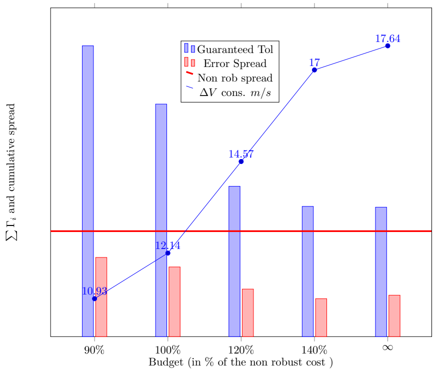


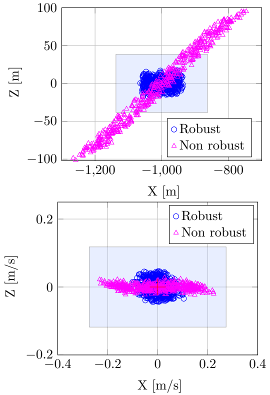
Semi-major axis	Eccentricity	Initial anomaly	mission duration
6763 km	0.0052	0	2767 s
$X_1 = [-30000 \;\; 5000 \;\; 8.154 \;\; 0]^T$		$X_1 = [-1000 \;\; 0 \;\; 0 \;\; 0]^T$	

Table: ATV mission

Impulses mis-execution : numerical examples

ATV mission: Magnitude errors 0.1% Orientation errors 1°



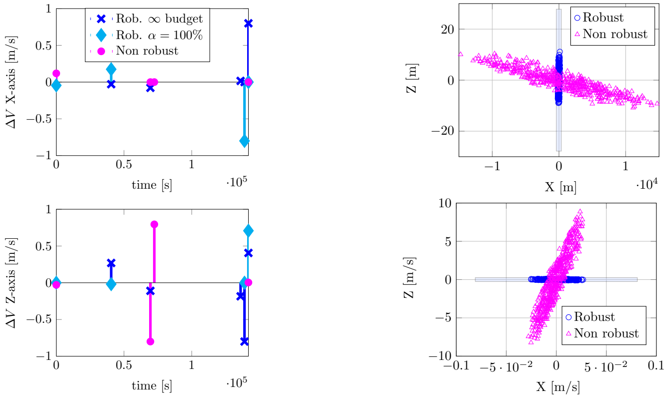


Semi-major axis	Eccentricity	Initial anomaly	mission duration
6763 km	0.0052	0	2767 s
$X_1 = [-30000 \quad 5000 \quad 8.154 \quad 0]^T$		$X_1 = [-1000 \quad 0 \quad 0 \quad 0]^T$	

Table: ATV mission

Impulses mis-execution : numerical examples

Proba 3 mission: Magnitude errors 0.1% Orientation errors 1°

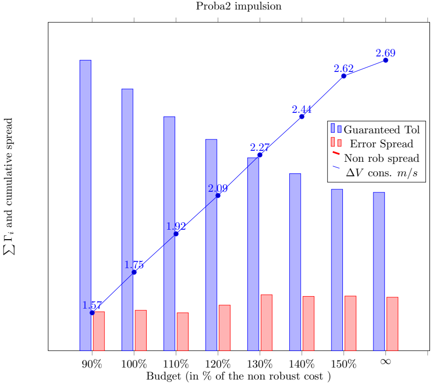


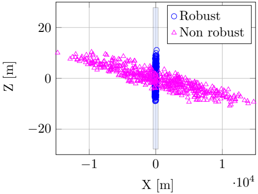
Semi-major axis	Eccentricity	Initial anomaly	mission duration
37039 km	0.8	0	141888 s
$X_1 = [-5000 \ \ 0 \ \ 0 \ \ 0]^T$		$X_1 = [-20 \ \ 0 \ \ 0 \ \ 0]^T$	

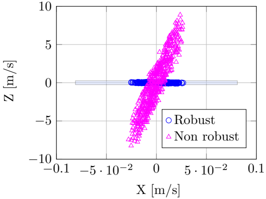
Table: Proba3 mission

Impulses mis-execution : numerical examples

Proba 3 mission: Magnitude errors 0.1% Orientation errors 1°





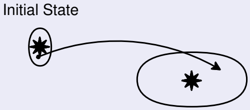


Semi-major axis	Eccentricity	Initial anomaly	mission duration
37039 km	0.8	0	141888 s
$X_1 = [-5000 \;\; 0 \;\; 0 \;\; 0]^T$		$X_1 = [-20 \;\; 0 \;\; 0 \;\; 0]^T$	

Table: Proba3 mission

Navigation errors

- Elliposidal uncertainties on initial state: $\tilde{X}_1 = \tilde{X}_1^* + P_u u,\ u^T u \leq 1$
- Arrival ellipsoidal tolerance subset: $\left\| R \left(\mathbf{B}\Delta\tilde{V} - \tilde{X}(\nu_f) + \Phi_{ya}(\nu_f,\nu_1)\tilde{X}_1 \right) \right\|_2 \leq 1$



Minimal ellipsoidal tolerance subset

- The tolerance subset of minimum volume is independant from the optimized plan;
- The minimal tolerance subset can be computed from the uncertainties set on initial state and the propagation time:

$$Q^{-1} = P_u^T \Phi_{ya}(\nu_f,\nu_1)^T \Phi_{ya}(\nu_f,\nu_1) P_u$$

- The optimized plan only control the center of the propagated domain.
- The tolerance can be optimized by a feedback MPC approach [Deaconu13][1]

[1] [Deaconu13], G. Deaconu *et al.*, Minimizing the effects of the navigation uncertainties on the spacecraft rendezvous precision Journal of Guidance, Control, and Dynamics , Vol. 37, No. 2 (2014), pp. 695-700. doi: 10.2514/1.62219

Concluding remarks

Used tools

- Interval analysis for error description and spread effect analysis
- Robust optimisation for setting tractable robust counterparts;

Obtained results

- Desensitized maneuver plans for thrusters system errors

$$\tilde{X}(\nu_f) = \Phi_{ya}(\nu_f, \nu_1)\tilde{X}_1 + \mathbf{B}\Delta\tilde{V}$$

- Certified tolerance set with respect to a fuel budget

Comments

- LP tools (tests on embedded CPU);
- Algorithms provide certified results for risk management ;
- Robustness price may be expensive.
- Submission to Journal of Guidance, Control and Dynamics

Thank You For Your Attention