Robust Heading Control and its Application to Ciscrea Underwater Vehicle

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Abstract—Deep inside the ocean, the earth magnetic signal is one of the merely existing information that tells the heading of robots with very good cost efficiency. Therefore, this paper focuses on the AUV (Autonomous Underwater Vehicle) heading control problem using only one magnetic compass as feedback sensor. In this application, we address AUV modeling and control issues simultaneously. Because of quadratic damping factor, underwater vehicle hydrodynamic model is nonlinear. In addition, unmodeled dynamics, parameter variations and environmental disturbances create significant uncertainties between the nominal AUV model and the reality. Finally, sensor noise, signal delay as well as unmeasured states also affect the stability and control performance of AUV motions. In order to handle these issues with improved AUV observation quality and navigation ability, we propose a CFD (Computational Fluid Dynamics) model based $H_{\infty}$ robust control scheme. Without loss of generality, the robust heading controller was implemented and validated in the sea on low-mass and complex-shaped Ciscrea AUV. Simulation and sea experimental results of both PID (Proportional Integral Derivative) and robust heading controller are analysed.

I. INTRODUCTION

Nowadays, major marine activities deploy underwater vehicles. These vehicles have a large variety of types and they are widely involved in undersea surveillance, inspection and survey missions for instance. Typically, AUVs and gliders are common with a torpedo shape for long range missions, and Human Occupied Vehicles (HOVs) as well as Remote Operating Vehicles (ROVs) are generally of a cubic shape used for hovering tasks. Note that, for some specific applications: undersea pipeline inspection, offshore infrastructure surveillance and large vessel maintenance, a small size cubic AUV is preferred. Indeed, small AUVs can be deployed to explore areas which are not accessible to HOVs and ROVs. Meanwhile, the cubic shaped AUVs enjoy more degrees of freedom than torpedo-shaped AUVs. Indeed, they can hover and enter complex underwater spaces.

Achieving good maneuverability of small AUV depends on two key factors: an accurate hydrodynamic model and an advanced control system. In [1], Yamamoto pointed out that a model-based control system is more efficient if the vehicles’ dynamics are modeled to some extent. Meanwhile, in [2], Ferreira et al. showed that an empirical linear model often fails to represent the dynamics of the AUV over a wide operating region. Indeed, obtaining hydrodynamic models of the complex-shaped cubic AUV is one of key points for better maneuverability. An accurate hydrodynamic model can reveal physical details indicating the focus of control design. In addition, inside the virtual environment, engineers can replay many dynamic and hydrodynamic phenomena with seldom limit of time, space or cost. In this work, we adopt our previously published CFD (Computational Fluid Dynamics) model [3] and [4].

Regardless of modeling issues, the value of a model-based control approach depends on how robust and efficient the control scheme can adopt the hydrodynamic model. Potential trends of current methods focus on faster controllers to assist the pilot or the autopilot with better accuracy. Optimal controllers can reduce propelling actions to save the battery power as well as to increase the propeller lifespan. Meanwhile, regulated AUV motions are expected to be less complex for high level guidance, navigation and swarm research. However, underwater vehicles are generally designed to operate in the ocean environments. Therefore, numerous uncertainties should be considered, including parameter variations, nonlinear hydrodynamic damping effects, sensor transmit delays and ocean current disturbances. Owing to these unpredictable problems, traditional control methods, such as PID and LQG (Linear Quadratic Gaussian), are less efficient and they can’t achieve both stability and high performance, see [5]. In SAUC-E [6] and euRathlon [7] competitions, we concluded that PID yaw controller was less efficient for low mass AUV. Consequently, advanced control algorithms should be involved, such as the adaptive control scheme in [8], robust control scheme in [9] and interval analysis approach in [10]. Note that robust control schemes are shown to be successfully in [11] and [12] for torpedo-shaped AUVs.

In this work, we appointed the semi-AUV CISCREA, as shown in Figure 1, and characterized in Table I.

Fig. 1. CISCREA
This paper is organized as follows; AUV main notions, Ciscrea model and its derivative equations for control design are presented in Section II; Proposed $H_\infty$ controller is described in Section III; Section IV demonstrates the Matlab simulation results of $H_\infty$ and PID controllers. In addition, the improved $H_\infty$ scheme adaption and its Ciscrea sea test are presented. Finally, conclusions are drawn in section V.

II. AUV MODELING

This section is dedicated to describe the AUV modeling notions as well as the dynamic and hydrodynamic parameters of Ciscrea AUV. A yaw model is derived in this section for robust heading control design. Note that, modeling data in this section comes from our previous CFD works [3], [4].

A. AUV Modeling Notions

CISCREA dynamics are represented by Fossen’s marine vehicle formulation in [13], [14]. Positions, angles, linear and angular velocities, force and moment definitions are reflected in Tab II. The position vector $\eta$, velocity vector $\nu$ and force vector $\tau$ are defined as follows:

$$\eta = [x, y, z, \phi, \theta, \psi]^T; \quad \nu = [u, v, w, p, q, r]^T; \quad \tau = [X, Y, Z, K, M, N]^T$$

### TABLE II. THE NOTATION OF SNAME FOR MARINE VESSELS

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Linear and Angular Velocities</th>
<th>Forces and Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>$x$</td>
<td>$u$</td>
</tr>
<tr>
<td>Sway</td>
<td>$y$</td>
<td>$v$</td>
</tr>
<tr>
<td>Heave</td>
<td>$z$</td>
<td>$w$</td>
</tr>
<tr>
<td>Roll</td>
<td>$\phi$</td>
<td>$p$</td>
</tr>
<tr>
<td>Pitch</td>
<td>$\theta$</td>
<td>$q$</td>
</tr>
<tr>
<td>Yaw</td>
<td>$\psi$</td>
<td>$r$</td>
</tr>
</tbody>
</table>

Depending on [13], rigid-body hydrodynamic forces and moments can be linearly superimposed. Therefore, the overall non-linear underwater model can be characterized by two parts, the rigid-body dynamic (1) and hydrodynamic formulations (2) (hydrostatics included). Tab III describes the parameters of this model.

$$M_{RB}\ddot{v} + C_{RB}(v)\nu = \tau_{env} + \tau_{hydro} + \tau_{pro}$$

(1)

$$\tau_{hydro} = -M_A\ddot{v} - C_A(v)\nu - D(|\nu|)\nu - g(\eta)$$

(2)

For Ciscrea AUV, the rigid-body mass inertia matrix $M_{RB}$ is simplified because of its symmetric properties as well as $r_G = 0$, i.e., $O_b \equiv CG$. Here, $r_G = [x_G, y_G, z_G]^T$ is the vector from $O_b$ (origin of B-frame) to CG (center of gravity). $C_{RB}$ and $C_A$ contribute to the centrifugal force. In our case, these two matrices can be neglected, thanks to the fact that the vehicle speed is low enough to be considered, $C(v) \approx 0$. For an AUV with neutral buoyancy, the weight $W$ is approximately equal to the buoyancy force $B$. For Ciscrea, CB (the buoyancy center) and CG are located using trial and error methods by adding and removing the payload and floats. The marine disturbances, such as the wind, waves and currents are related to the environmental effect $\tau_{env}$. However for a deep sea underwater vehicle, only current should be considered since wind and waves have negligible effects. Two hydrodynamic parameters added mass, $M_A \in \mathbb{R}^{6 \times 6}$, and damping, $D(|\nu|) \in \mathbb{R}^{6 \times 6}$, should be carefully involved in the AUV model. Added mass is a virtual conception representing the hydrodynamic forces and moments. Any accelerating emerged-object would encounter this $M_A$ due to the inertia of the fluid. For a cubic-shaped AUV, added mass in some directions are generally larger than the rigid-body mass [3]. Damping in the fluid consists of four parts: Potential damping $D_P(|\nu|)$, skin friction $D_S(|\nu|)$, vortex shedding damping $D_M(|\nu|)$. For Ciscrea, quadratic damping is the main dynamic nonlinearity of the system [3].

B. Ciscrea model

Mass inertia matrix $M_{RB}$ is calculated using PRO/ENGINEER, and the results of CISCREA around CG ($O_b$) are listed in $M_{RB}$ (3) (Mass: $kg$, Inertia: $kg \cdot m^2$).

$$M_{RB} = \begin{bmatrix}
15.643 & 0 & 0 & 0 & 0 & 0 \\
0 & 15.643 & 0 & 0 & 0 & 0 \\
0 & 0 & 15.643 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2473 & 0 & 0.0029 \\
0 & 0 & 0 & 0.3698 & 0 & 0.0319 \\
0 & 0 & 0 & 0.0029 & 0 & 0.3578 \\
\end{bmatrix} \quad (3)$$

Added mass matrix $M_A$ is calculated using radiation/diffraction program WAMIT™. Results of CISCREA are listed in $M_A$ (4) (Mass: $kg$, Inertia: $kg \cdot m^2$).

$$M_A = \begin{bmatrix}
11.985 & -0.091 & -0.105 & 0.039 & 0.308 & 0.012 \\
0.149 & 20.261 & -0.147 & 0.085 & -0.013 & -0.758 \\
0.111 & -0.129 & 67.141 & -0.133 & 2.530 & 0.064 \\
0.122 & 0.319 & -0.056 & 0.385 & 0.003 & -0.011 \\
0.047 & -0.001 & 2.543 & -0.002 & 0.791 & 0.002 \\
-0.003 & -0.758 & 0.064 & -0.003 & 0.004 & 0.138 \\
\end{bmatrix} \quad (4)$$

### TABLE IV. EXPERIMENTAL RESULTS CURVE FITTING (BENCHMARK: STAR-CCM+ , VELOCITY x: m/s, rad/s, DAMPING y: N · m or N)

| Assumed nominal model | Surge $y = 25x^{2.1} + 5.379x$ | Sway $y = 57.48x^{2.7} + 4.88x$ | Heave/dive $y = 80.37x^{2.5}$ | Yaw $y = 0.2496x^{2.1} + 0.021x$ |

CFD software STAR-CCM™ and real world experiments are conducted to estimate the relationship among damping forces, damping moments, vehicle velocities and angular velocities. In [3], [4], second order polynomial lines are implemented to approximate the relationship between damping and velocities. In Tab IV, We appointed a nominal model with damping parameters that equal to the average of experiments and CFD results.
C. Yaw model

Without loss of generality, we demonstrate the robust controller in yaw direction. The rotational model is simplified as equation (5) (neglecting buoyancy and gravity). Definitions and parametric values, such as inertia and damping coefficients, are listed in Table V. Note that, all the parameters have uncertainties, as they are either measured or numerically calculated. The uncertainties will be carefully discussed and treated using $H_\infty$ solution in section III.

$$\begin{align*}
(I_{YRB} + I_{YA}) \ddot{x}_r + D_{YN} |x| \dot{x} + D_{YL} \dot{x}_r = \tau_i
\end{align*}$$

TABLE V. ROTATIONAL MODEL NOTIONS OF YAW DIRECTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{YRB}$</td>
<td>Rigid-body inertia</td>
<td>0.3578 kg m$^2$</td>
</tr>
<tr>
<td>$I_{YA}$</td>
<td>Added mass inertia</td>
<td>0.1388 kg m$^2$</td>
</tr>
<tr>
<td>$D_{YN}$</td>
<td>Nominal quadratic damping factors</td>
<td>Ideal 0.2496</td>
</tr>
<tr>
<td>$D_{YL}$</td>
<td>Nominal linear damping factors</td>
<td>Ideal 0.021</td>
</tr>
<tr>
<td>$\dot{x}_r$</td>
<td>Angular Velocity</td>
<td>0 to 4 rad/s</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>Torque input</td>
<td>0 to 6 N · m</td>
</tr>
<tr>
<td>$\tau_{com}$</td>
<td>Compensation Torque</td>
<td>0 to 6 N · m</td>
</tr>
<tr>
<td>$x_0$</td>
<td>Equilibrium velocity</td>
<td>0 to 4 rad/s</td>
</tr>
<tr>
<td>$D_{YND}$</td>
<td>CFD quadratic damping factors</td>
<td>0.1479</td>
</tr>
<tr>
<td>$D_{YLD}$</td>
<td>CFD linear damping factors</td>
<td>0.0013</td>
</tr>
<tr>
<td>$D_{YLA}$</td>
<td>Artificial linear factors</td>
<td>&lt; Motor limit (select 1.2)</td>
</tr>
</tbody>
</table>

III. $H_\infty$ CONTROLLER

In this section, $H_\infty$ theory is presented and adapted to CISCREA for heading control. We propose a framework to change the nonlinear yaw model into a linear system with uncertainties based on previous modeling works. By then, we solve a $H_\infty$ controller for the linear system.

A. $H_\infty$ design

Given a linear invariant system,

$$
\begin{bmatrix}
\dot{x} \\
z \\
y
\end{bmatrix} = \begin{bmatrix}
A & B_1 & B_2 \\
C_1 & D_{11} & D_{12} \\
C_2 & D_{21} & D_{22}
\end{bmatrix}
\begin{bmatrix}
x \\
w \\
u
\end{bmatrix}
$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^{m_2}$ the control input, $y \in \mathbb{R}^{m_1}$ system output, $w \in \mathbb{R}^{m_1}$ the external input vector, $z \in \mathbb{R}^{2}$ the error vector. The robust design process is to find a feedback controller $K$, such that the close-loop system remains stable and achieves certain performance in presence of uncertainties [15], [16]. Generally, cost functions for finding $K$ are represented by $H_\infty$ norms of the close-loop transfer functions from $w$ to $z$, as seen in equation 7. Here, $K$ is the robust controller in Figure 2, and $G$ is the linear nominal yaw model. The nominal model is derived from the linear fractional transformation (LFT) technique, which separates uncertainties into an individual block [15].

$$
\min_{K_{stable}} \left\| \begin{bmatrix} W_p(I + GK)^{-1} & W_c(I + GK)^{-1} & W_eK(I + GK)^{-1} \end{bmatrix} \right\|_\infty < \gamma
$$

In $H_\infty$ theory, weighting functions are also introduced for setting control specifications. Generally, it is difficult to get the accurate frequency characteristics of external input signals. Therefore, weighting functions are sometimes the upper bound that covers original signals. For example, the weighting function $W_p$, which represents the frequency characteristics of the external disturbance, is used to describe output disturbance rejection ability. Satisfying the above norm inequality indicates that the closed-loop system indeed reduces the disturbance effects to a prescribed level.

Finding appropriate weighting functions is critical and difficult, trials are necessary for a successful robust control design. In this application, we choose a structure with three weighting functions, see Figure 2. $W_p$ is chosen as a reference tracking error requirement, $W_e$ represents the input disturbance rejection. $W_c$, which restricts the output disturbance, is the same specification with $W_e$, but with different objectives.

To solve the $H_\infty$ problem, one can use the Raccati method or Linear Matrix Inequality (LMI) approach [16]. Usually, we prefer to choose LMI approach, as it requires less initial conditions [15].

![Fig. 2. Weighting Functions for Robust Synthesis](image)

B. Proposed yaw controller

In this part, we discuss the nonlinear problem without concern of parametric uncertainties, such as inertia and damping coefficient errors. As found in [3], damping is a major nonlinear component in the AUV model. Therefore, in Figure 3, we propose to compensate nonlinear behaviors using the CFD yaw model, as feedback for real world propellers.

![Fig. 3. Robust Controller & Nonlinear Compensator](image)

The nonlinear compensation is given in equation (8).

$$\tau_{com} = (D_{YLA} - D_{YLD} - D_{YND} |\dot{x}_r|) \ddot{x}_r$$

$D_{YLA}$ is the artificial linear factor given in Tab V. $D_{YND}$ and $D_{YLD}$ are CFD damping estimations. The linear model result of compensation is given in equation (9).

$$
(D_{YLA} + (D_{YN} |\dot{x}_r| - D_{YND} |\dot{x}_r| + D_{YL} - D_{YLD}) |\dot{x}_r| + (I_{YRB} + I_{YA}) \ddot{x}_r = \tau_i
$$

The term $\delta = D_{YN} |\dot{x}_r| - D_{YND} |\dot{x}_r| + D_{YL} - D_{YLD}$ is calculated as an uncertainty added to $D_{YLA}$. Generally, this $\delta$ is small comparing to $D_{YLA}$. If we calculate $\delta$ using that:
we can consider that $D_{YLA}$ is a nominal parameter which has a dynamic uncertainty of 23.7%:

$$(I_{YRB} + I_{YA})\ddot{x}_r + (D_{YLA} + \delta)\dot{x}_r = \tau, \quad \delta \in [-0.4265, 0.4265]$$

(10)

At the end, the proposed model, equation (10), is a linear system with uncertainties. Therefore, $H_\infty$ approach is feasible for this model.

IV. SIMULATION AND EXPERIMENTS

A. Heading control simulation

In the simulation, $I_{YRB} + I_{YA}$ and $D_{YLA}$ are considered to be two varying parameters, which have respectively 30% and 40% (> 23.7%) of variations. The bode plots of the uncertain systems family are shown in Figure 4.

During $H_\infty$ synthesis, weighting function parameters are selected according to equations (11) to (13). $W_e$ is selected to be a very small scalar ($Gu = 0.01$) for simply disturbance rejection. We choose $W_e$ and $W_p$ according to [11] and [15], carefully considered the robust margins, tracking error (1%) and fast response.

Figure 5, shows that feedback controller $K$ is found (five states, one input and one output) which satisfies the $H_\infty$ norm inequality with $\gamma$ equals to 1.075. Close-loop sensitivity functions are well covered by the inverse of chosen weightings. Here, $T(s) = (I + GK)^{-1}$ stands for the sensitivity function, and $Tk(s) = K(I + GK)^{-1}$ is the complementary sensitivity function.

$$W_p(s) = \frac{0.95s^2 + 1.8s + 10}{s^2 + 8s + 0.01}$$

(11)

$$W_e(s) = \frac{0.5s + 0.92}{s + 0.0046}$$

(12)

$$W_u(s) = 0.01$$

(13)

In Figure 6, step responses of three scenarios are represented: PID control, damping compensated $H_\infty$ approach and bare $H_\infty$ control. From the simulation comparison, we can conclude that compensated $H_\infty$ controller handles the non-linearity with the fastest response. Compensated $H_\infty$ controller has no overshoot and oscillations during the rotation process. Tracking error achieves the specification less than 1%. To emphasize the speed and robustness of our approach, we inject a small disturbance of 0.5 rad on the output at 50s. Figure 7 demonstrates the robust performance of our controller handling a yaw model with 30% inertia variations.

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B. Ciscrea Sea Test

In real world applications, it is necessary to adapt the proposed $H_\infty$ approach, see Figure 8. First, underwater vehicles might not be equipped with enough sensors to detect all the states, such as the angular velocity $\dot{x}_r$. In addition, the magnetic compass may encounter serious signal delay and noise injection. Therefore, we propose to use a CFD model based kalman filters, numerically estimating unmeasured as well as noisy states. In addition, model based compensation algorithms are recommended to deal with the sensor delay.

Fig. 8. Ciscrea $H_\infty$ heading control sea experiment (Brest Port)

In order to reveal the magnetic compass delay as well as noise injections on the rotational motion of CISCREA AUV. A less tuned $H_\infty$ heading controller was demonstrated in Figure 9. Among the Kalman angle estimation and magnetic compass output, an obvious 0.5s delay was observed. In this case, the delay lead to distinct heading control oscillations. Meanwhile, there exist noise efforts on the control output to propellers.

Fig. 9. Ciscrea $H_\infty$ heading control delay and noise problem

For Ciscrea heading control application, a classic Smith compensator was introduced by [17] to compensate the magnetic compass delay, see Figure 10 and equation (14). The main idea is to estimate current delay free output $y$ from the nominal model $G_0(s)$ and real output $y e^{(-0.5+\delta)s}$. Figure 11 shows the $H_\infty$ heading control simulation result using smith predictor compensation. In addition, as robust controller is insensitive to compensation errors, we are enlighten to propose another compensation scheme using kalman angular velocity estimation ($\dot{x}_r$), see equation (16). A $K_c = 0.57$ was tuned which has an efficient compensation result in Figure 12.

Fig. 10. Smith Predictor (Delay Compensation)

Fig. 11. Improved Ciscrea $H_\infty$ heading controller (Smith Compensation)

Fig. 12. Improved Ciscrea $H_\infty$ heading controller (Kalman Compensation)

\[
P(s) = G_0(s) - G_0(s)e^{-0.5s}
\]  
\[y = P(s)u + ye^{(-0.5+\delta)s}
\]  
\[y = K_c\dot{x}_r + ye^{(-0.5+\delta)s}, K_c > 0
\]
The adaption of our $H_{\infty}$ heading control scheme has been validated on Ciscra in sea tests, and its results are compared with a traditional PID approach, respectively shown in Figure 13 and 14. First, $H_{\infty}$ heading controller is faster than PID scheme (even with low battery conditions). Second, there is no nonlinearity induced oscillations in the control output, and the tracking accuracy is better. Third, from the propeller thrust signal, we can determine that the magnetic compass noise and disturbances are well rejected, while PID is less efficient to handle those uncertainties. Finally, the characteristics of our controller result in an optimal and smooth propulsion, which save the battery energy, and increase the working range.

**References**


