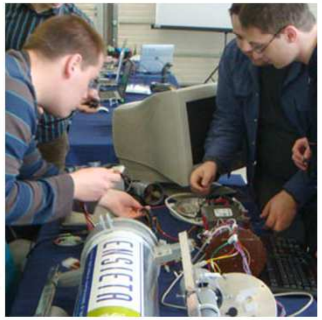


Simple set-membership methods and control algorithms applied to robots for exploration

Fabrice LE BARS

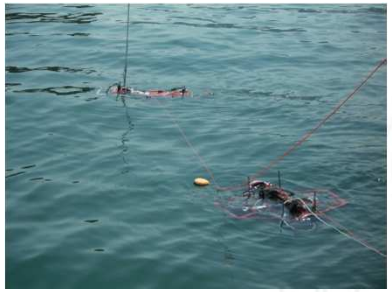
Outline

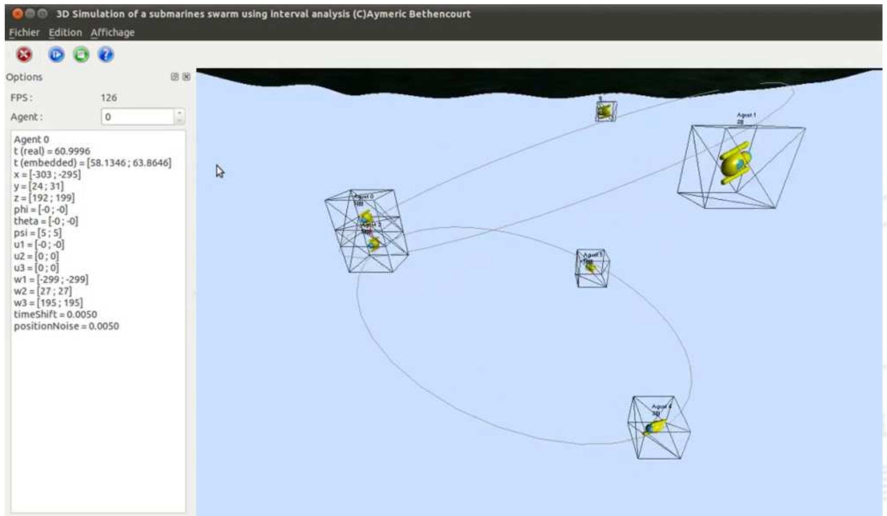
- Introduction
- A simple localization example
- Interval analysis
- Other localization scenarios
- Line following
- Additional common problems and possible methods
- Conclusion

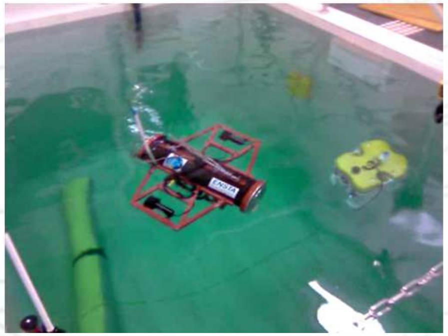
Introduction

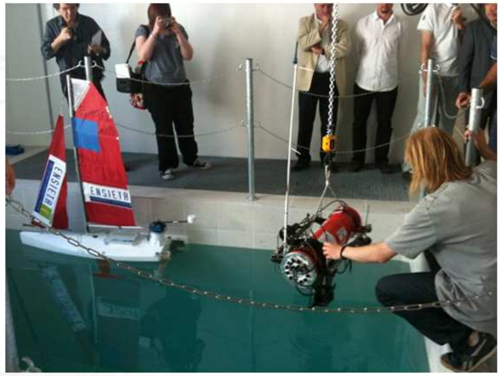
Robotics at ENSTA Bretagne

- Main research areas :
  - Autonomous marine and submarine robotics using interval methods
  - Swarm of robots



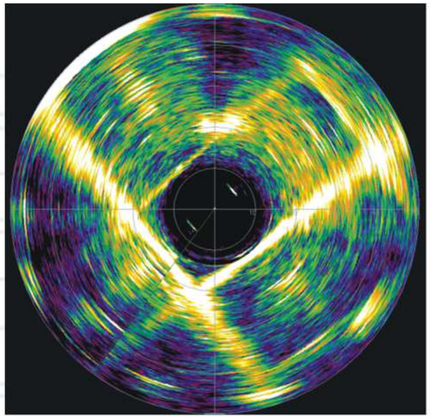


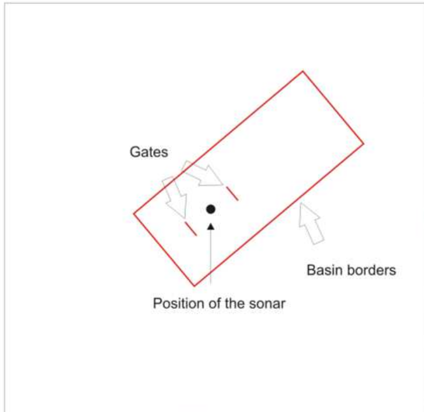


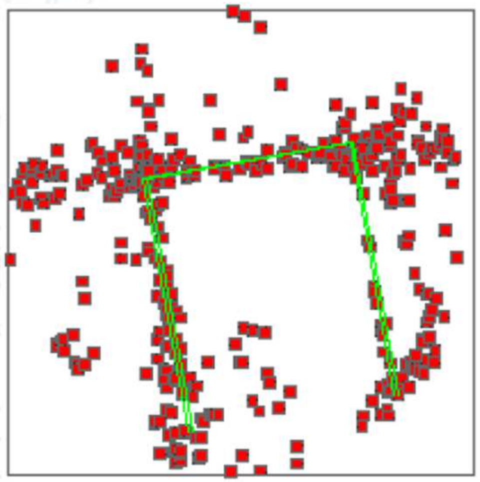


Robotics at ENSTA Bretagne

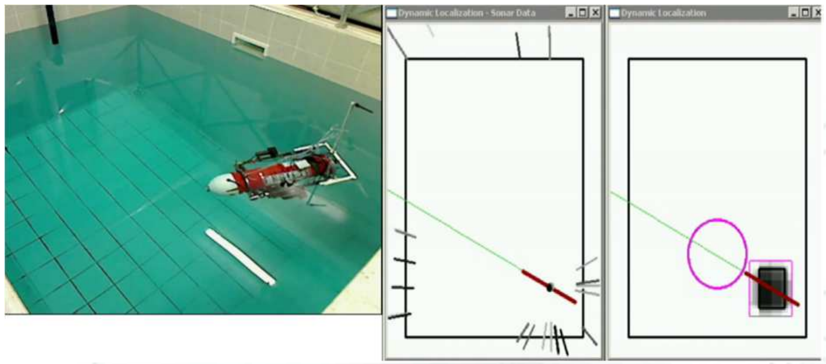
- Facts
  - **Autonomous robotics** rise new and **difficult problems**
  - There are **few demonstrations of cheap autonomous robots** able to do survey, cartography, localization tasks, especially in marine and submarine environments
  - **Current methods** : mainly **probabilistics**

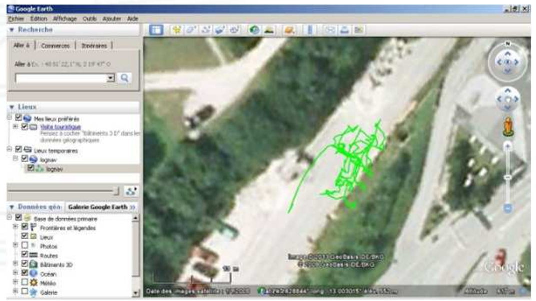






- Goals
  - Develop **observation, control methods** for **submarine, marine, ground and aerial robotics**
  - Demonstrate the use of **interval methods** through new applications in **autonomous robotics**
  - **Build** real and **convincing demonstrators**

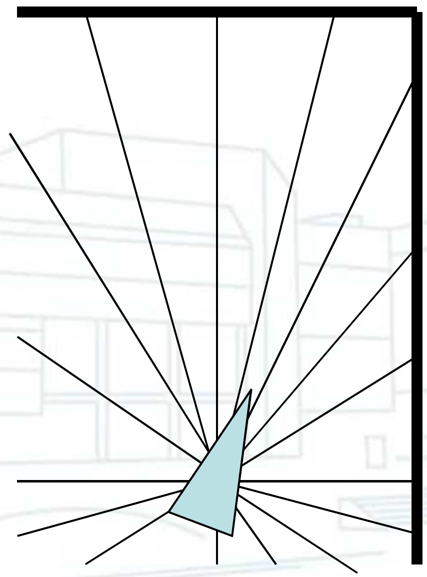




A simple localization example

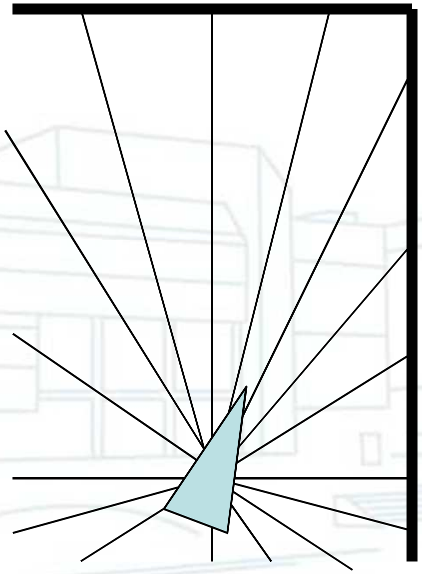
A simple localization example

- Context :
  - An underwater robot must follow a wall (not necessarily straight)
  - The environment (walls to follow) is known
  - The robot has a good compass and a sonar



A simple localization example

- Assumptions :
  - The wall to follow is on the right of the robot
  - We can approximate it as a line y=ax+b



A simple localization example

- Assumptions :
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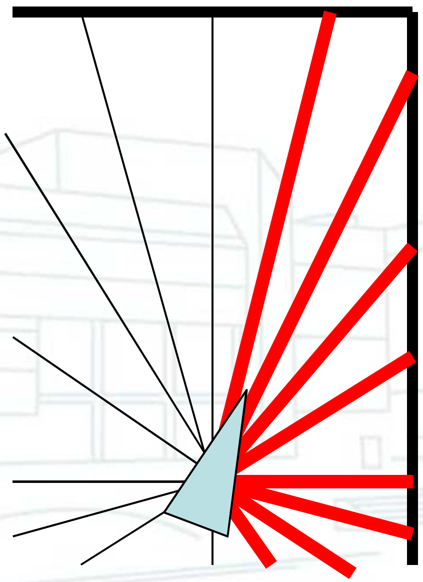

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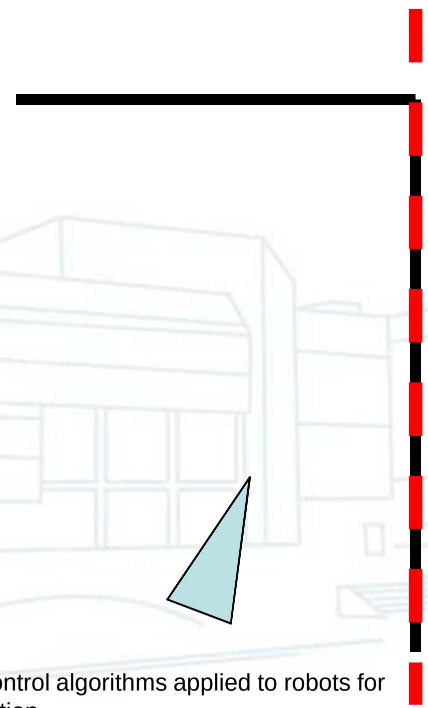

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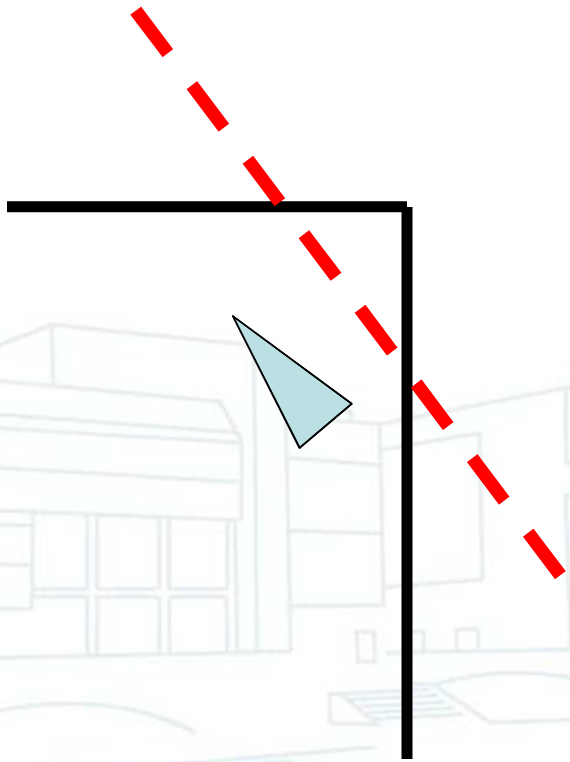

A simple localization example

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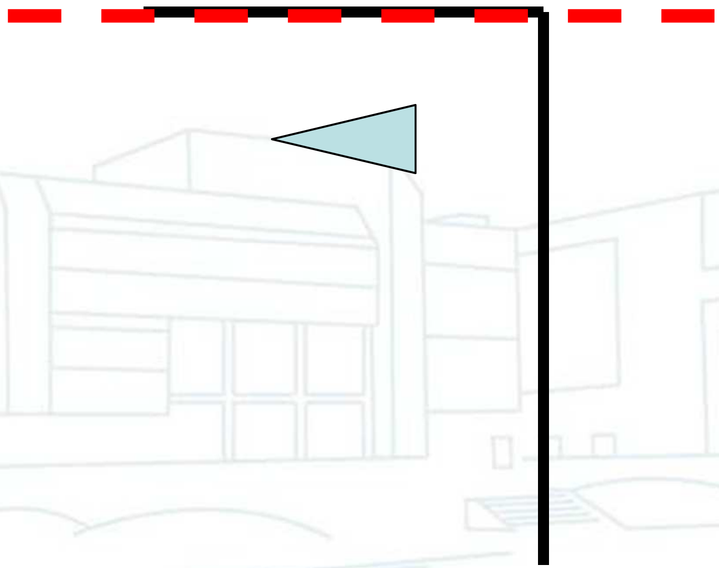
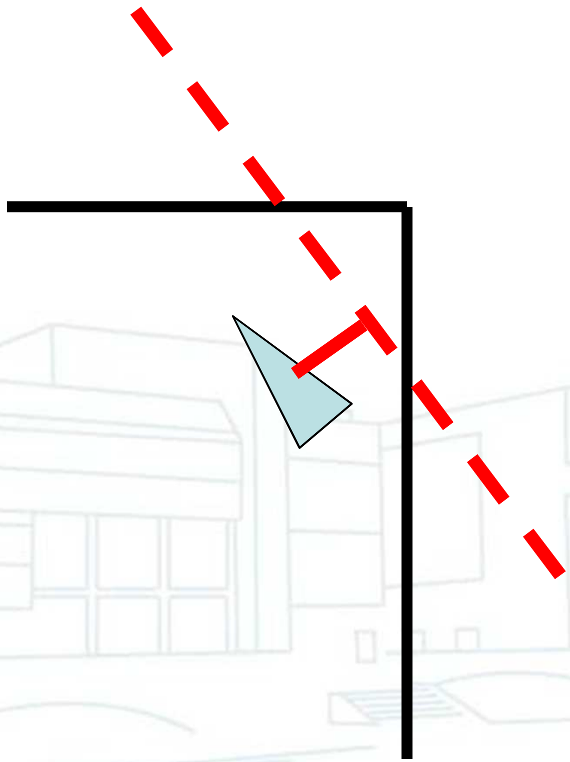


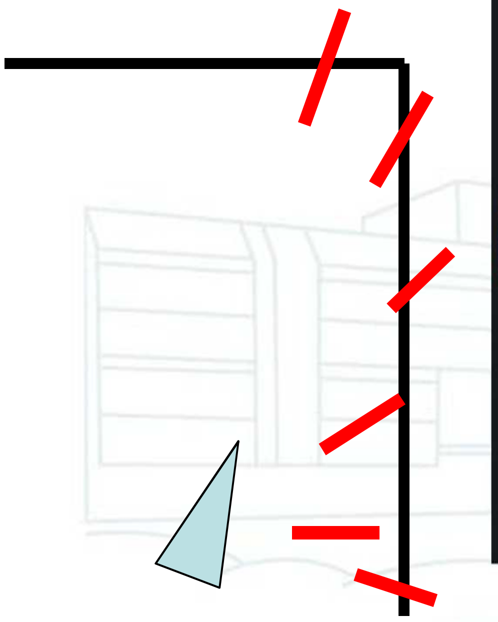
A simple localization example

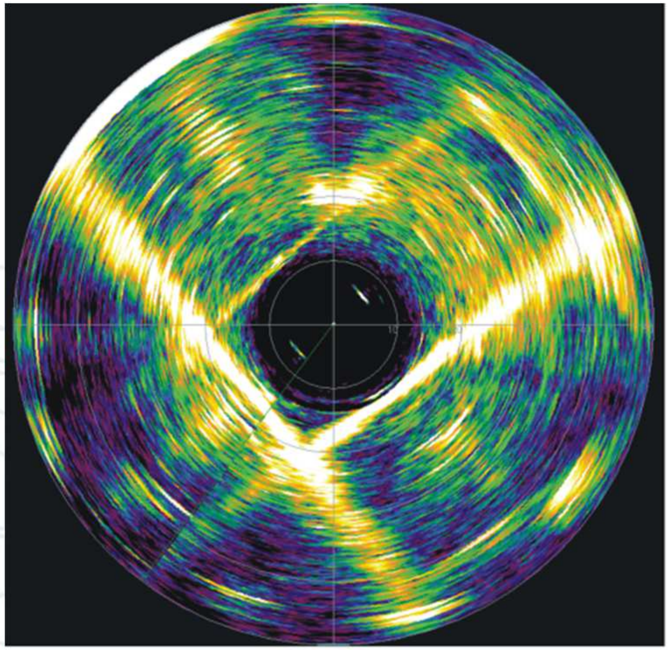
- The robot needs to find :
  - Its distance to the line
  - Its angle to the line (-> angle of the line)

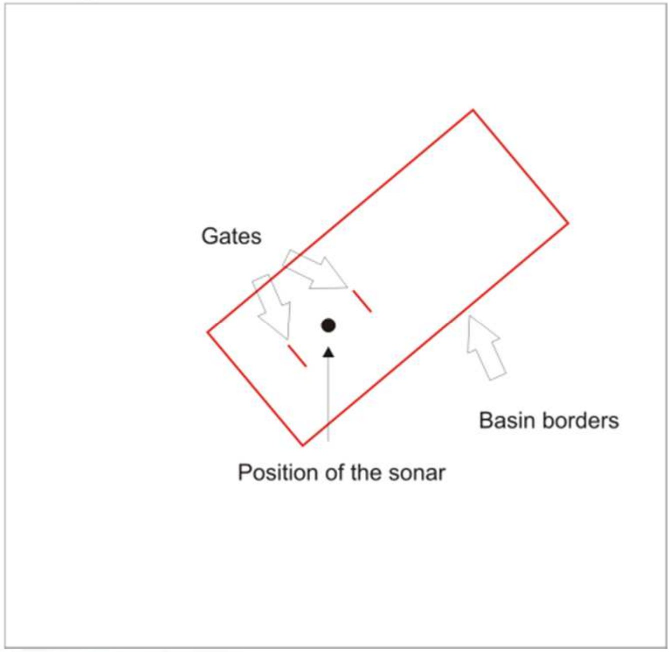


A simple localization example

- Available data :
  - The angle of the robot thanks to the compass
  - Sonar data

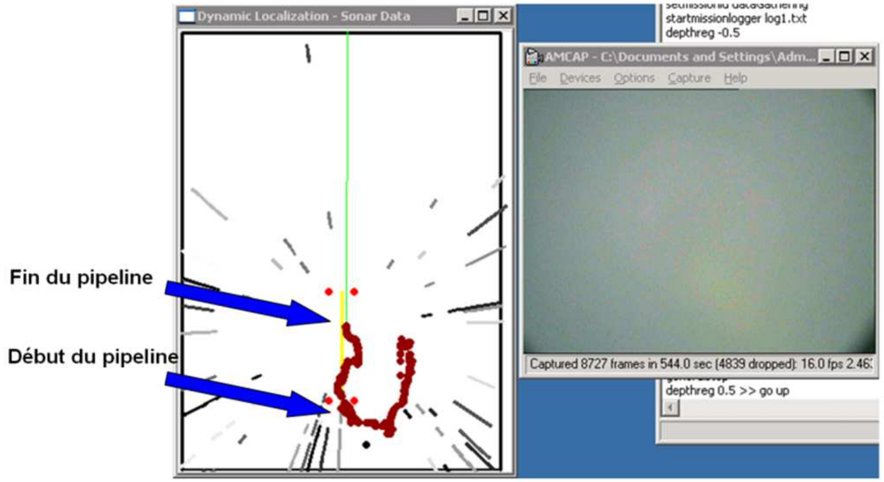


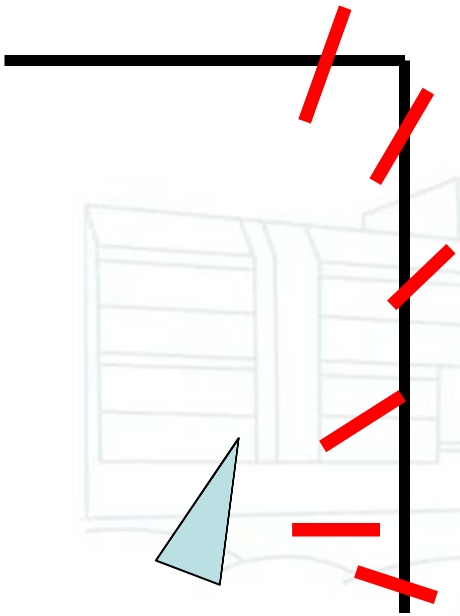


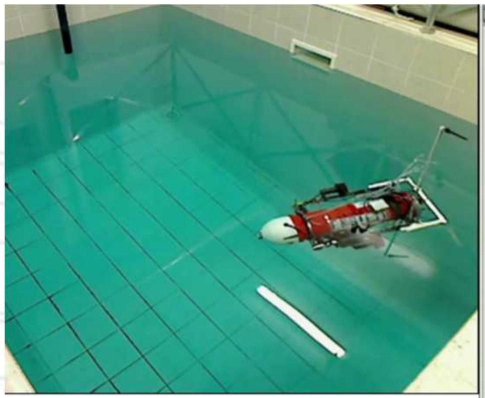


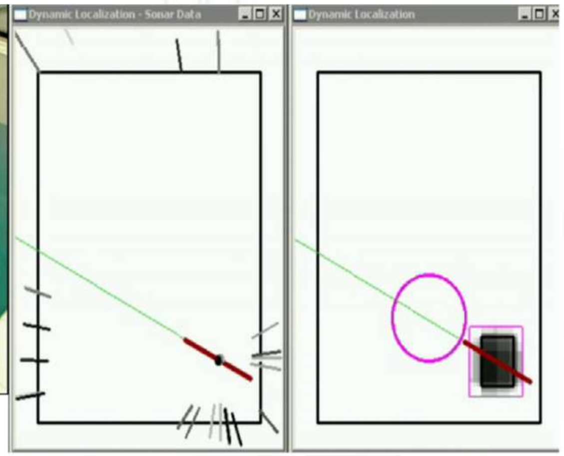
A simple localization example

- Methods :
  - Linear regression using least squares
  - What if we want to localize in a pool, a lake…?







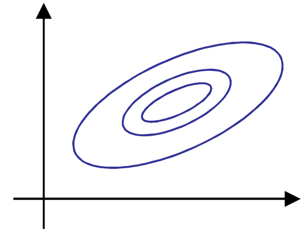


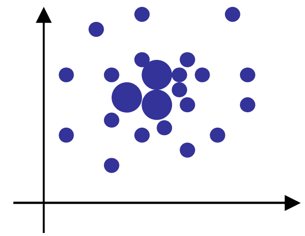
A simple localization example

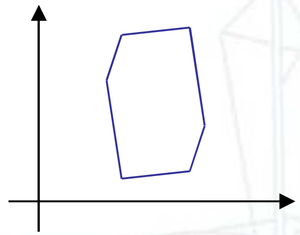
- Uncertainty representations :
  - Probabilistics
    - Gaussians
    - Particles

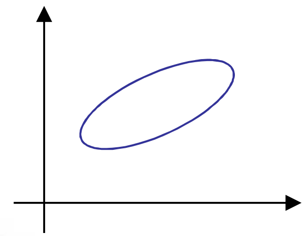
    => We try to get a probability density
  - Sets
    - Zonotopes
    - Ellipsoids
    - Intervals

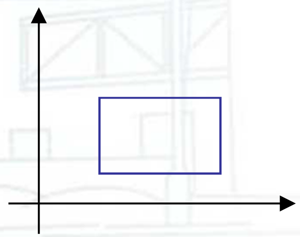
    => We try to enclose all possible solutions

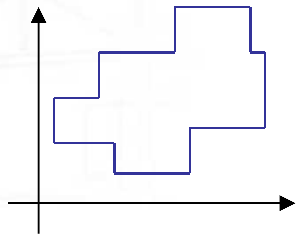








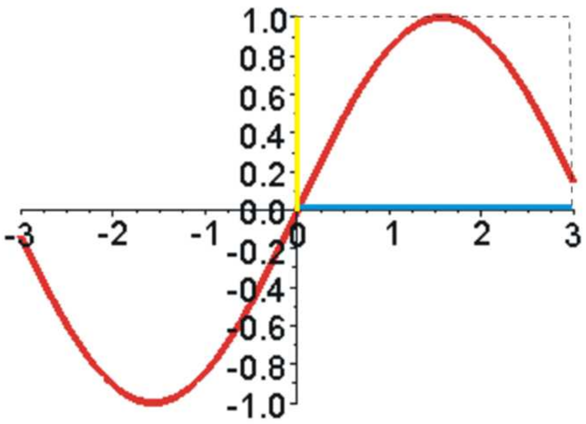




Interval analysis

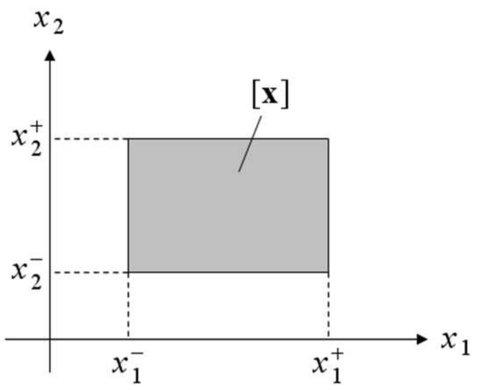
Interval analysis

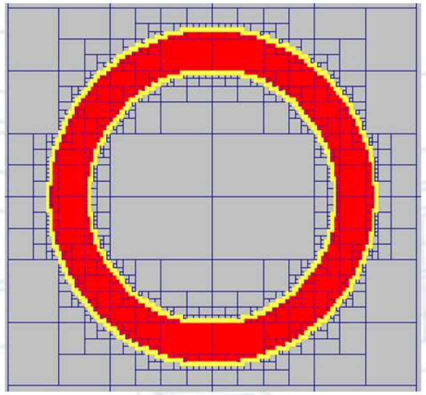
- $[-\infty, 2]$, $[-1, 4]$, $[-\infty, \infty]$ are examples of intervals
- Operations $\diamond \in \{+, -, *, /\}$
  - $[x^-, x^+] \diamond [y^-, y^+] =$ smallest interval containing the set of possible values for $x \diamond y$
  - $[-1, 4] + [2, 3] = [1, 7]$
  - $[-1, 4] * [2, 3] = [-3, 12]$
  - $[-1, 4]/[2, 3] = [-1/2, 2]$
- Multiplication by a number, intersection, union
  - $2[-1, 4] = [-2, 8]$
  - $[-1, 3] \cap [2, 4] = [2, 3]$
  - $[-1, 2] \sqcup [3, 4] = [-1, 4]$
- Image by a function
  - $\sin([0, \pi]) = [0, 1]$

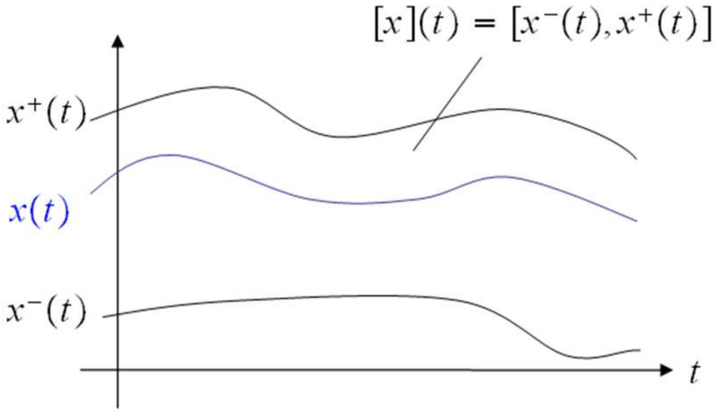


Interval analysis

- Real intervals can be generalized
  - Vectors intervals (boxes)
  - Sets intervals
  - Functions intervals (tubes)
  - Any set with a lattice structure

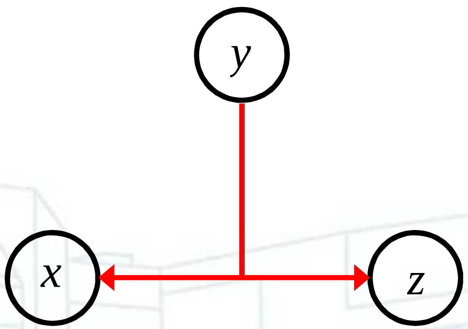






Interval analysis

- Contraction
  - If $z^2 = \exp(x) + y$ and $x \in [1,4]$, $y \in [3.1, 3.2]$, $z \in [4,7]$, then
    - $x = \ln(z^2 - y) \Rightarrow x \in [x] \cap \ln\left([z]^2 - [y]\right) = [2.5, 3.9]$

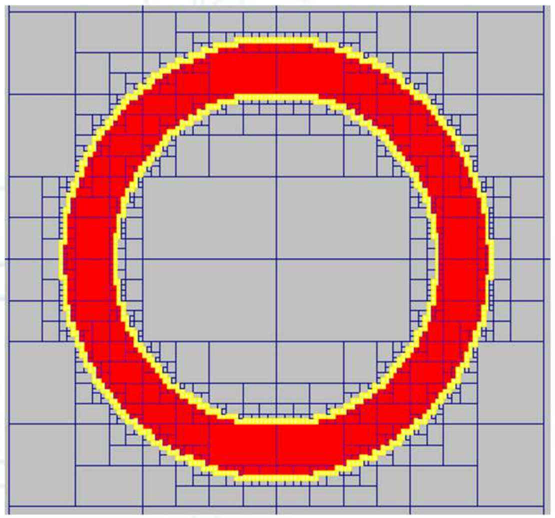


Interval analysis

- Contraction and propagation
  - We call *contractor* an operator that reduce the domain of variables
  - A *propagation* is a repeated call to contractors
  - We can repeat contractions until a fix point

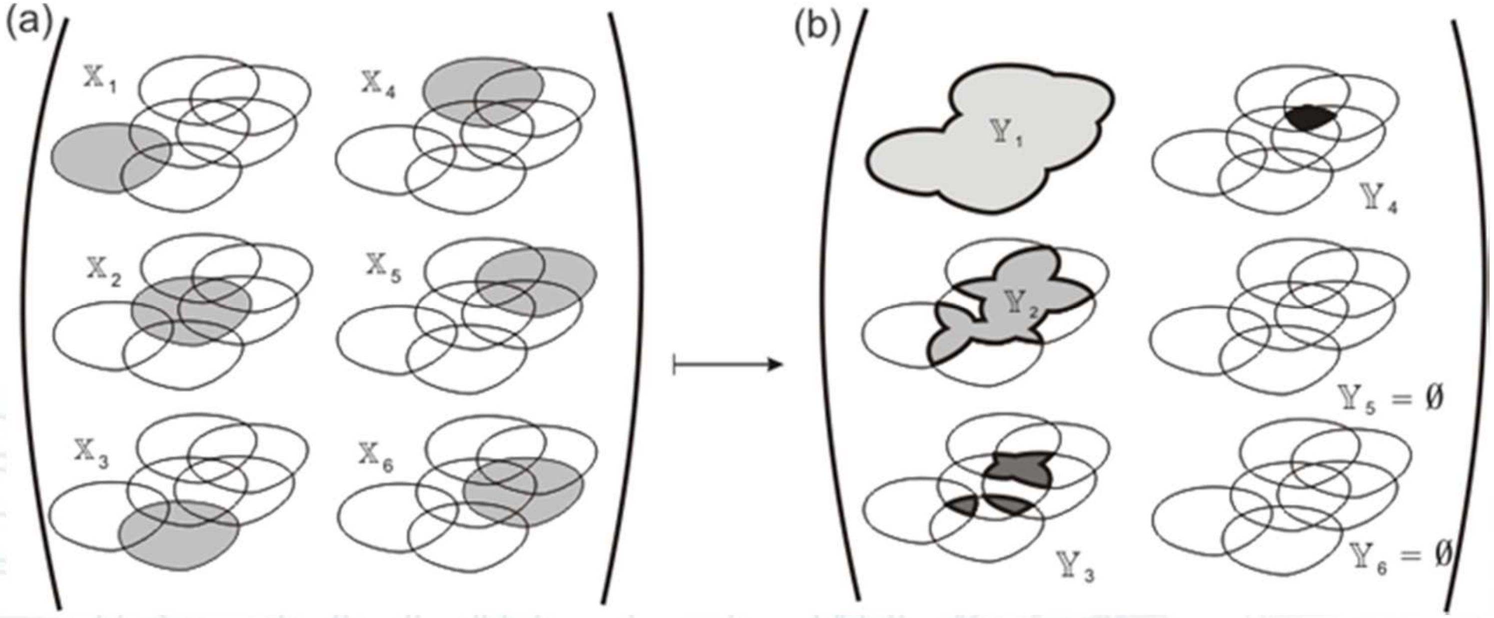
Interval analysis

- Other techniques such as bisections can also be done



Interval analysis

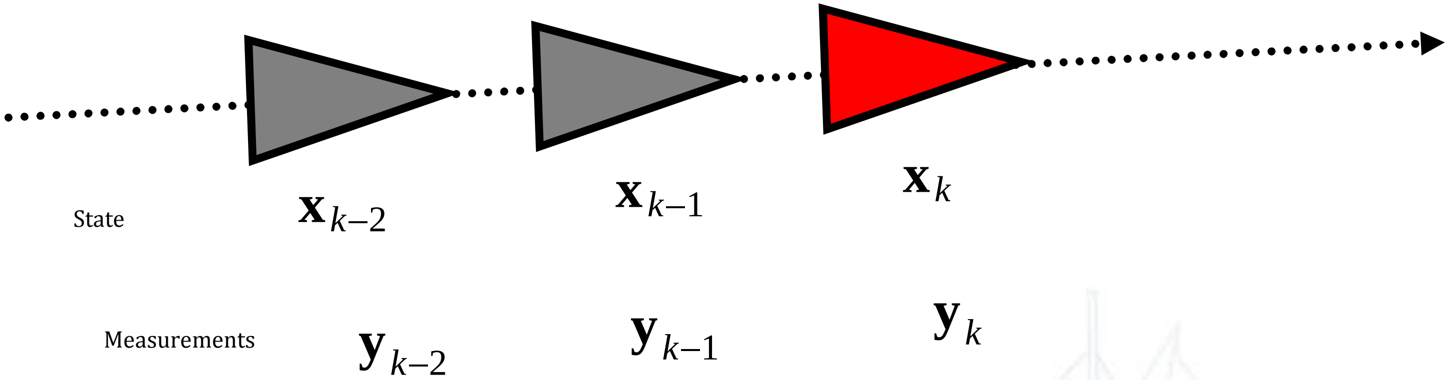
- Handling outliers : relaxed intersection (q-intersection)



Other localization scenarios

Other localization scenarios

- Dynamic localization

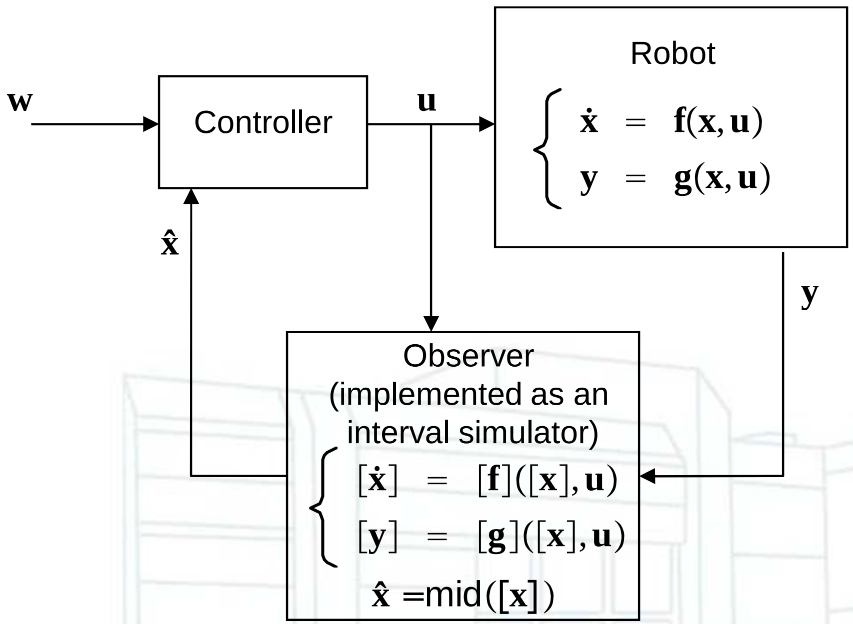


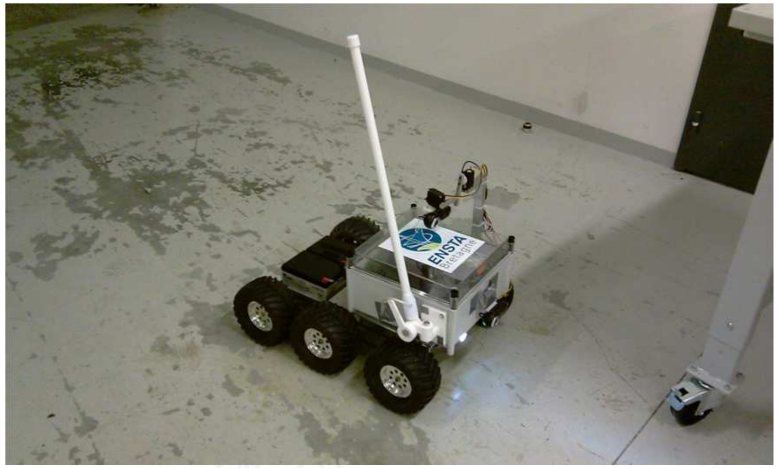
<u>For each k</u>

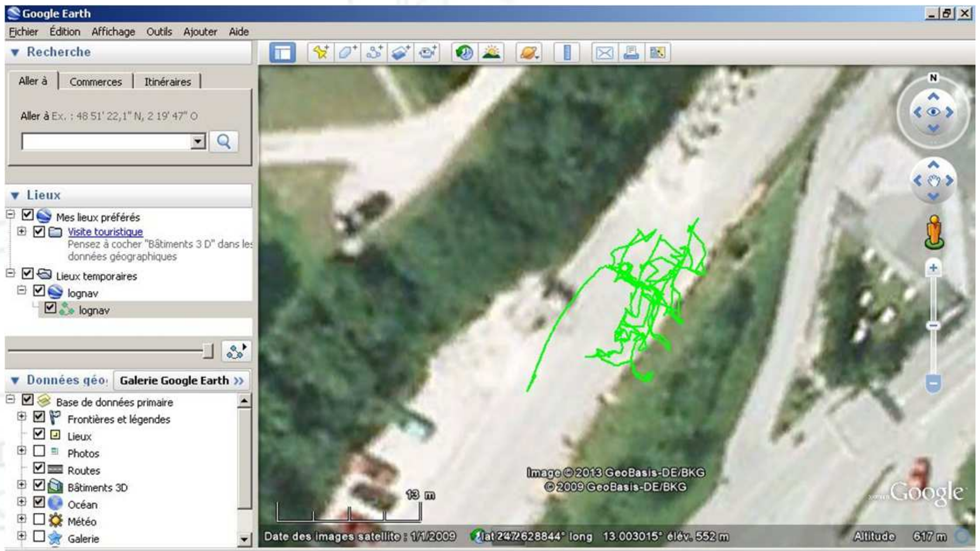
$$\boxed{\begin{aligned} \mathbf{x_{k+1}} &= \mathbf{f_k}(\mathbf{x_k}) \\ \mathbf{y_k} &= \mathbf{g_k}(\mathbf{x_k}). \end{aligned}} \longrightarrow \left\{ \begin{aligned} &\mathbf{g}_k(\mathbf{x}_k) = \mathbf{y}_k \\ &\mathbf{g}_{k-1} \circ \mathbf{f}_{k-1}^{-1}(\mathbf{x}_k) = \mathbf{y}_{k-1} \\ &\ldots \\ &\mathbf{g}_{k-n-1} \circ \mathbf{f}_{k-n-1}^{-1} \circ \ldots \circ \mathbf{f}_{k-1}^{-1}(\mathbf{x}_k) = \mathbf{y}_{k-n} \\ &\mathbf{x}_k \in \mathbb{R}^m, \mathbf{y}_i \in [\mathbf{y}_i], i \in \{k-n, .., k\}. \end{aligned} \right.$$

Other localization scenarios

- EURATHLON 2013
  - Task "Search and rescue in a smoke-filled underground structure“

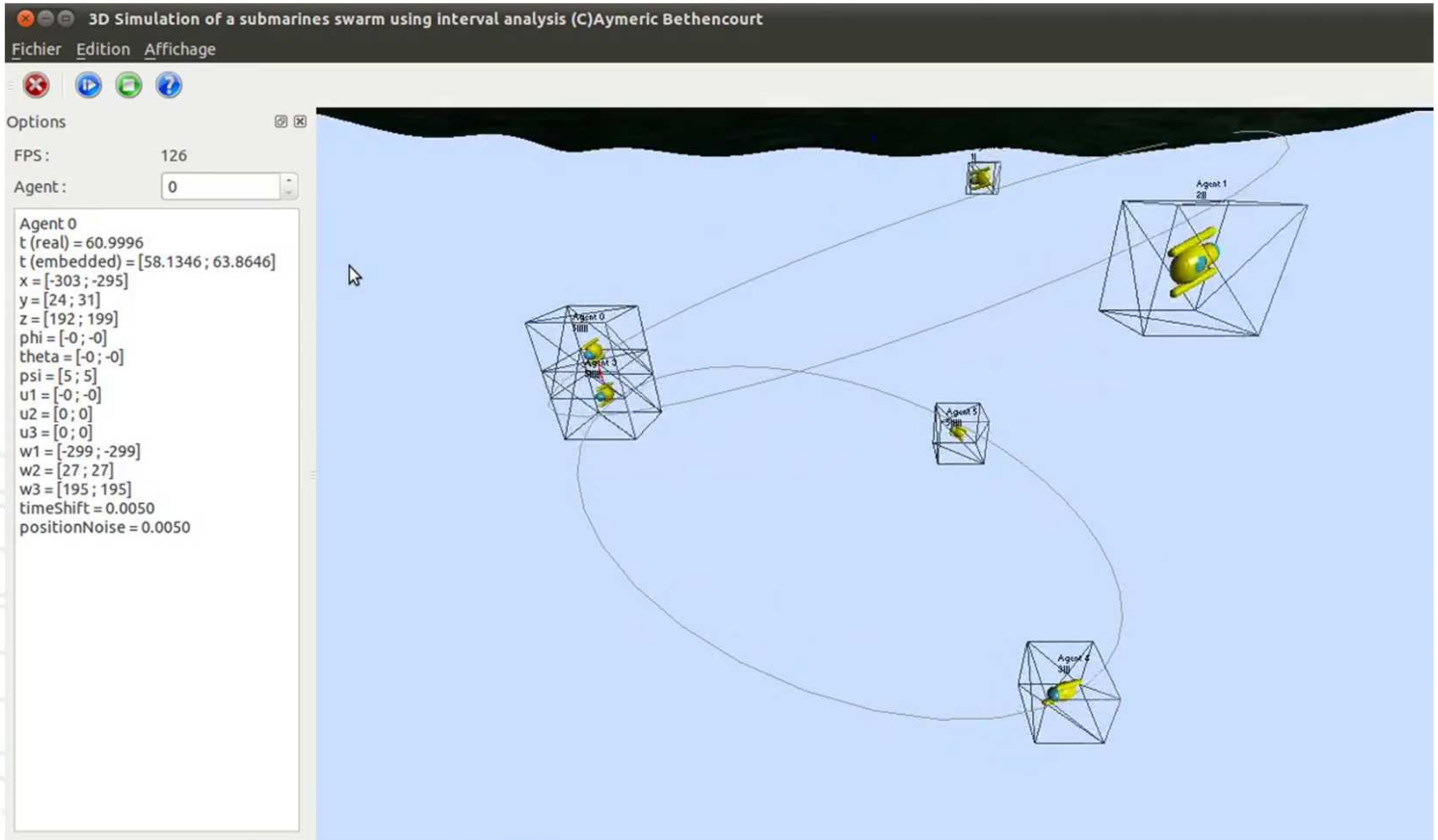






Other localization scenarios

- Localization of a swarm of robots with acoustic communication



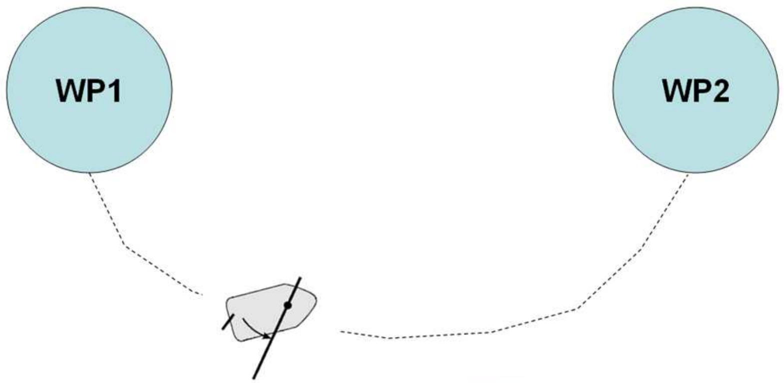
Line following

Line following

- Purpose
  - Cover autonomously an area as accurately as possible

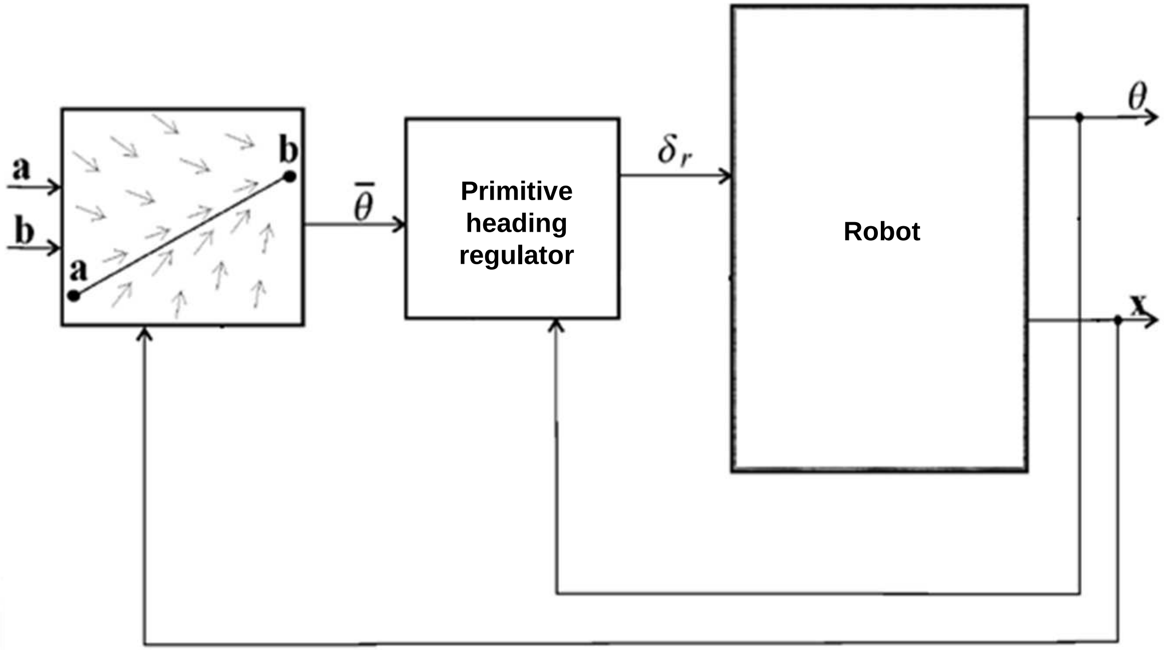
Line following

- From waypoints following to line following
  - Primitive heading control loop
  - Existing approaches : basic waypoint following
    - The robot follows a heading in direction of its waypoint
    - Waypoint reached when in a predefined radius
    - Problem : nothing prevent the drift between waypoints (because of currents...)

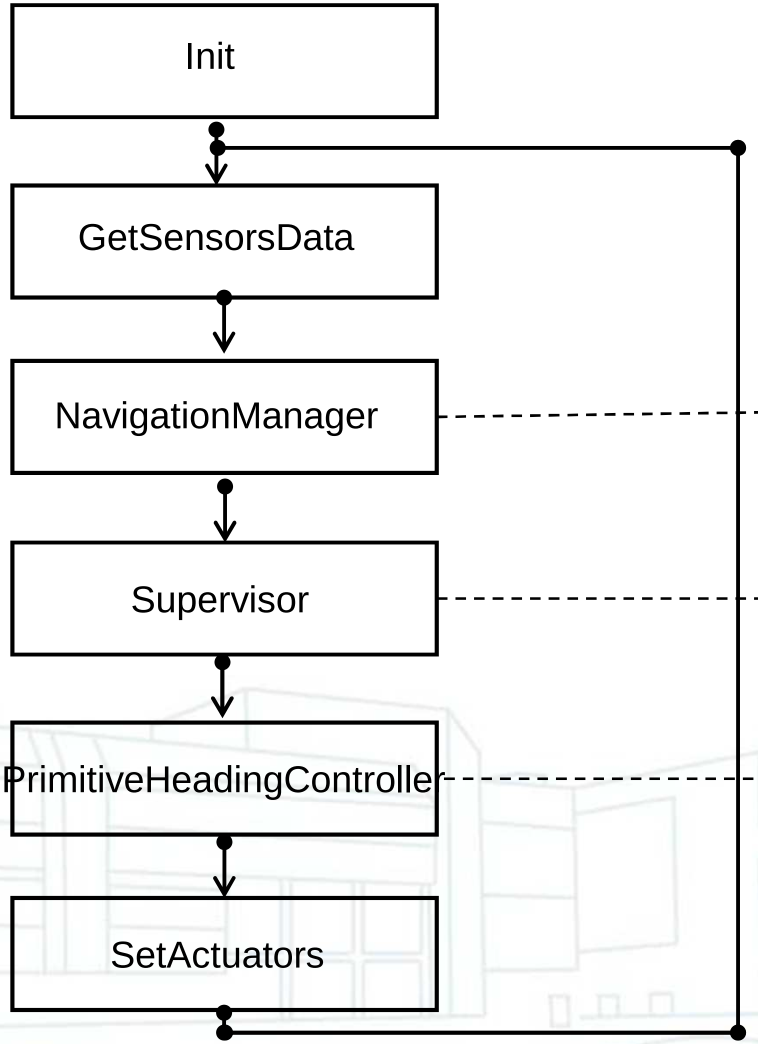


  - => Line following (use the distance to the line)

Line following



Line following



$$\langle \mathbf{b}_j - \mathbf{a}_j, \mathbf{m} - \mathbf{b}_j \rangle \geq 0$$

$$\begin{aligned} e &= \det\left(\frac{\mathbf{b}-\mathbf{a}}{\|\mathbf{b}-\mathbf{a}\|}, \mathbf{m}-\mathbf{a}\right) \\ \varphi &= \text{atan2}(\mathbf{b}-\mathbf{a}) \\ \bar{\theta} &= \varphi - \frac{2.\gamma_\infty}{\pi}.\text{atan}\left(\frac{e}{r}\right) \end{aligned}$$

$$\delta_r = \begin{cases} \delta_r^{\max}.\sin\left(\theta - \bar{\theta}\right) & \text{if } \cos\left(\theta - \bar{\theta}\right) \geq 0 \\ \delta_r^{\max}.\text{sign}\left(\sin\left(\theta - \bar{\theta}\right)\right) & \text{otherwise} \end{cases}$$

Line following

- Line following
  - Primitive controller stage for heading control
    - Rudder control

$$\delta_r = \begin{cases} \delta_r^{\max}.\sin\left(\theta - \bar{\theta}\right) & \text{if } \cos\left(\theta - \bar{\theta}\right) \geq 0 \\ \delta_r^{\max}.\text{sign}\left(\sin\left(\theta - \bar{\theta}\right)\right) & \text{otherwise} \end{cases}$$

  - Navigation manager sends lines to supervisor and validates lines
    - Validation condition

$$\langle \mathbf{b}_j - \mathbf{a}_j, \mathbf{m} - \mathbf{b}_j \rangle \geq 0$$

Line following

- Line following
  - Desired heading is the line made by the 2 current waypoints with an attractiveness angle to the line depending on the distance to the line

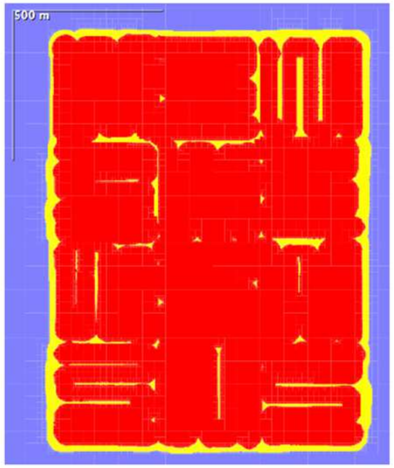
$$
\begin{aligned}
e &= \det\left(\frac{\mathbf{b}-\mathbf{a}}{\|\mathbf{b}-\mathbf{a}\|}, \mathbf{m}-\mathbf{a}\right) \\
\varphi &= \mathrm{atan2}(\mathbf{b}-\mathbf{a}) \\
\bar{\theta} &= \varphi - \frac{2.\gamma_\infty}{\pi}.\mathrm{atan}\left(\frac{e}{r}\right)
\end{aligned}
$$

Additional common problems and possible methods

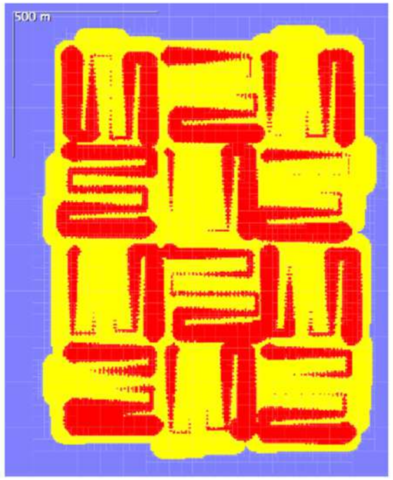
Additional common problems and possible methods

- Evaluate the guaranteed covered area depending on localization accuracy

  - With communication&ranging



  - Without communication&ranging



- Obstacle avoidance using vector fields

Conclusion

Conclusion

- Interval methods can be efficient for parameters and state estimation problems
- Advantages : can give an estimation of the error together with the state, parameters
- Can also be used when there are outliers
- Line following can improve easily the accuracy of a trajectory following especially in marine environment
- Vector fields are interesting for obstacle avoidance or other higher level trajectory planning

Questions?

- Contact
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