SLAM with fleeting detections

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Outline

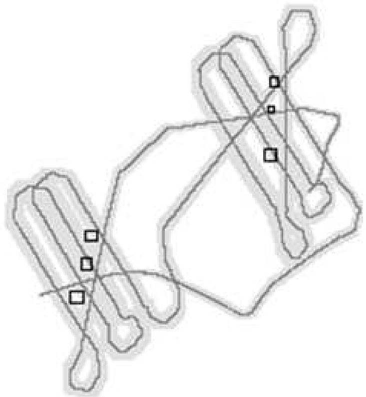
- Introduction
- Problem formalization
- CSP on tubes
- Contraction of the visibility relation
- Conclusion

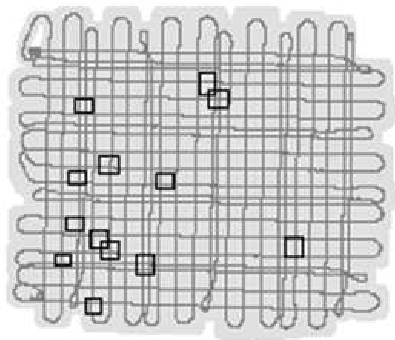
Introduction

- Context:
  - Offline SLAM for submarine robots
  - Fleeting detections of marks
  - Static and punctual marks
  - Known data association
  - Without outliers
- Tools:
  - Interval arithmetic and constraint propagation on tubes

Introduction

- Experiments with Daurade and Redermor submarines with marks in the sea





Problem formalization

Problem formalization

- Equations

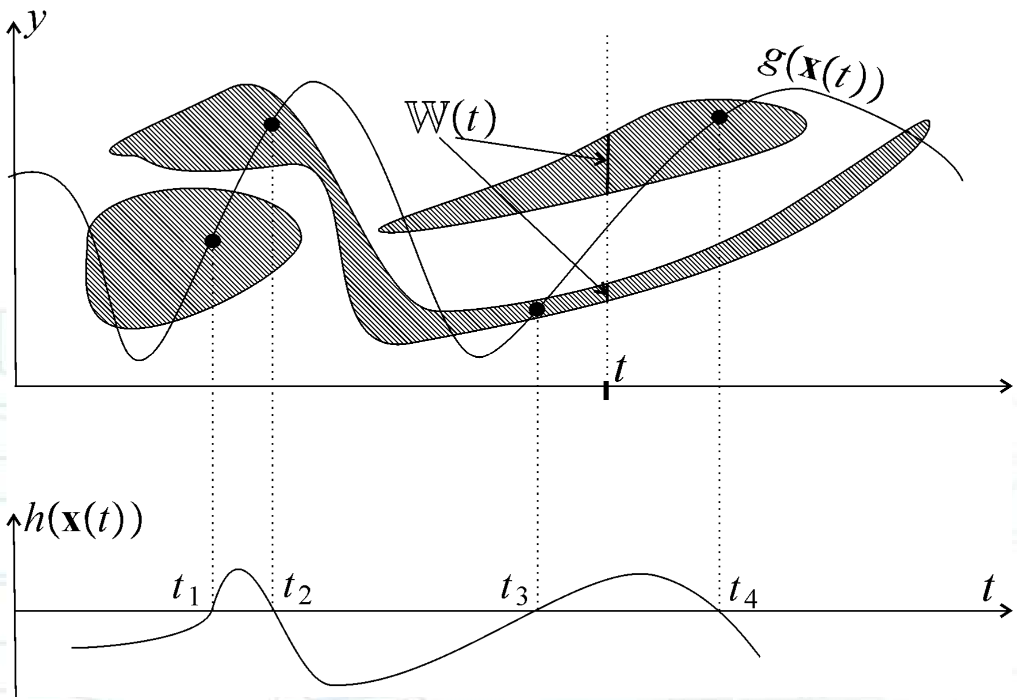
$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) & \text{(evolution equation)} \\ \mathbf{y} = \mathbf{g}(\mathbf{x}) & \text{(measurement equation independent from waterfall)} \\ \mathbf{z} = \mathbf{h}(\mathbf{x},\mathbf{u},\mathbf{m}) & \text{(marks detection equations)} \\ v(\mathbf{x},\mathbf{u},\mathbf{m}) = 0 \Rightarrow \mathbf{z} \in \mathbb{W} & \text{(marks visibility condition on the waterfall)} \end{cases}$$

Problem formalization

- Fleeting data
  - A fleeting point is a pair $(t, \mathbf{z})$ such that $v(\mathbf{x}(t), \mathbf{u}(t), \mathbf{m}) = 0$
  - It is a measurement that is only significant when a given condition is satisfied, during a short and unknown time.

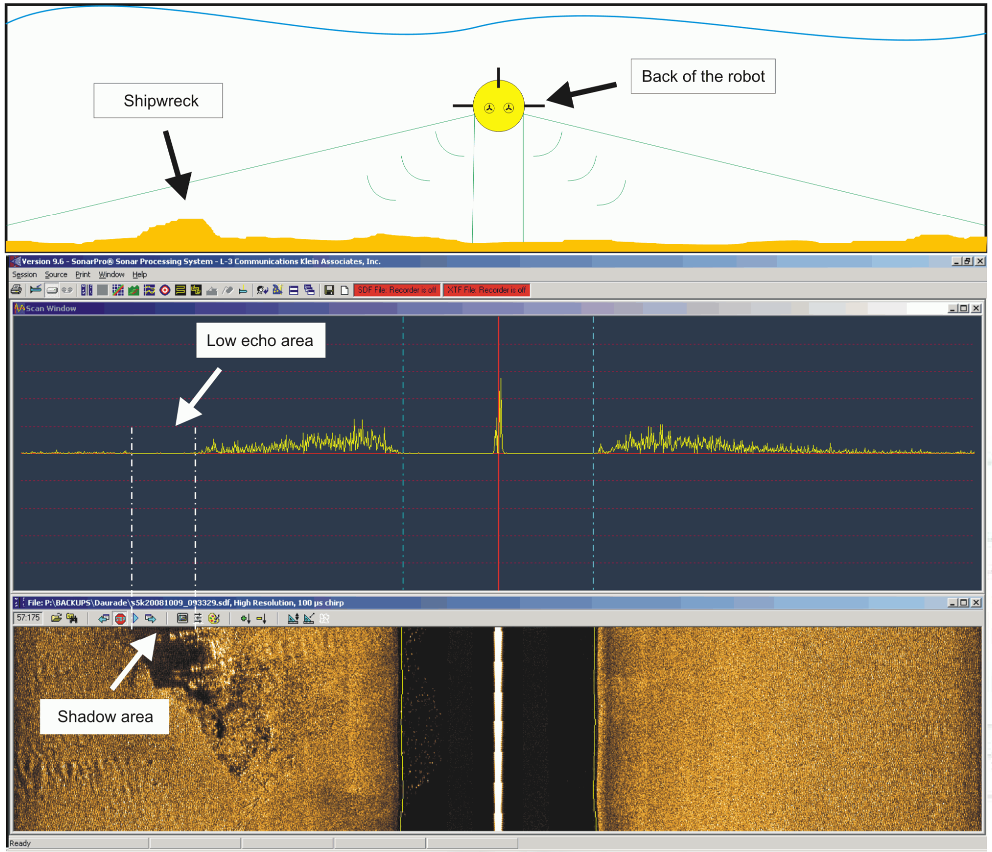
Problem formalization

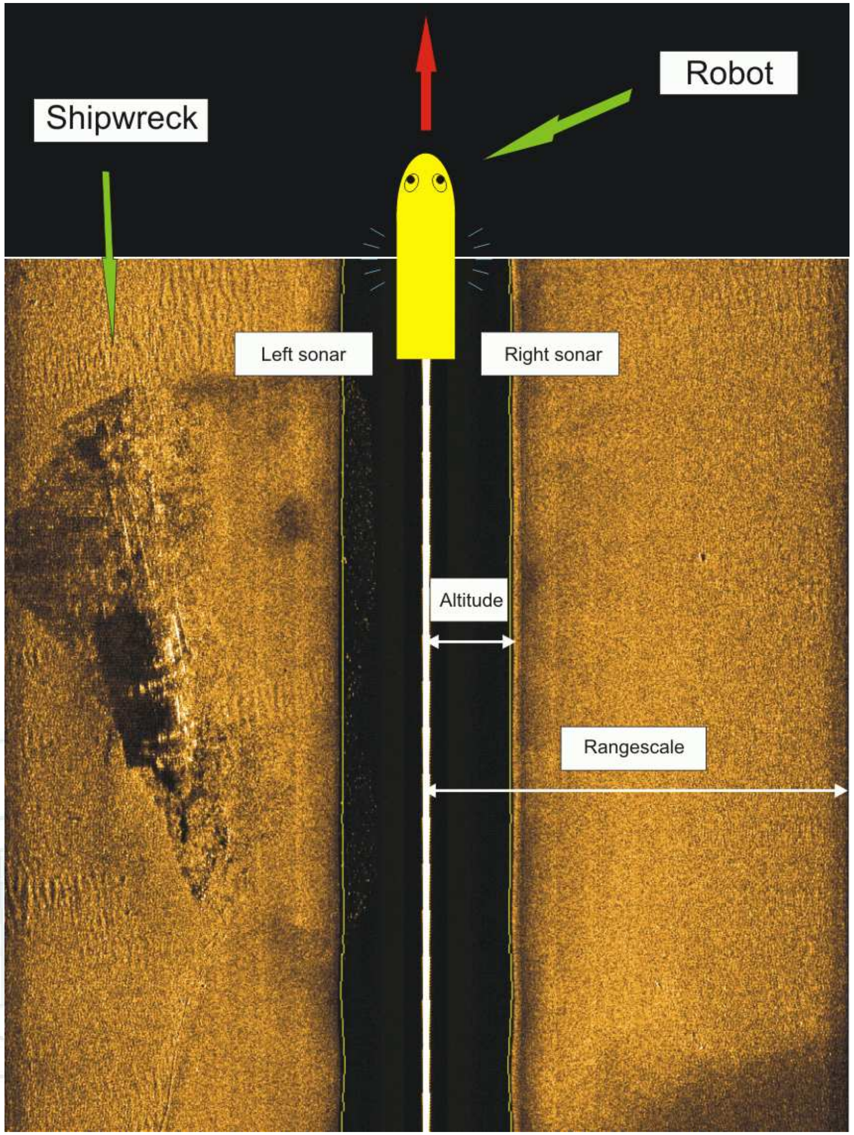
- Waterfall
  - $\mathbb{W}$ is a function that associates to a time $t \in \mathbb{R}$ a subset $\mathbb{W}(t) \in \mathbb{R}$
  - $\mathbb{W}$ is called the waterfall (from the lateral sonar community).



Problem formalization

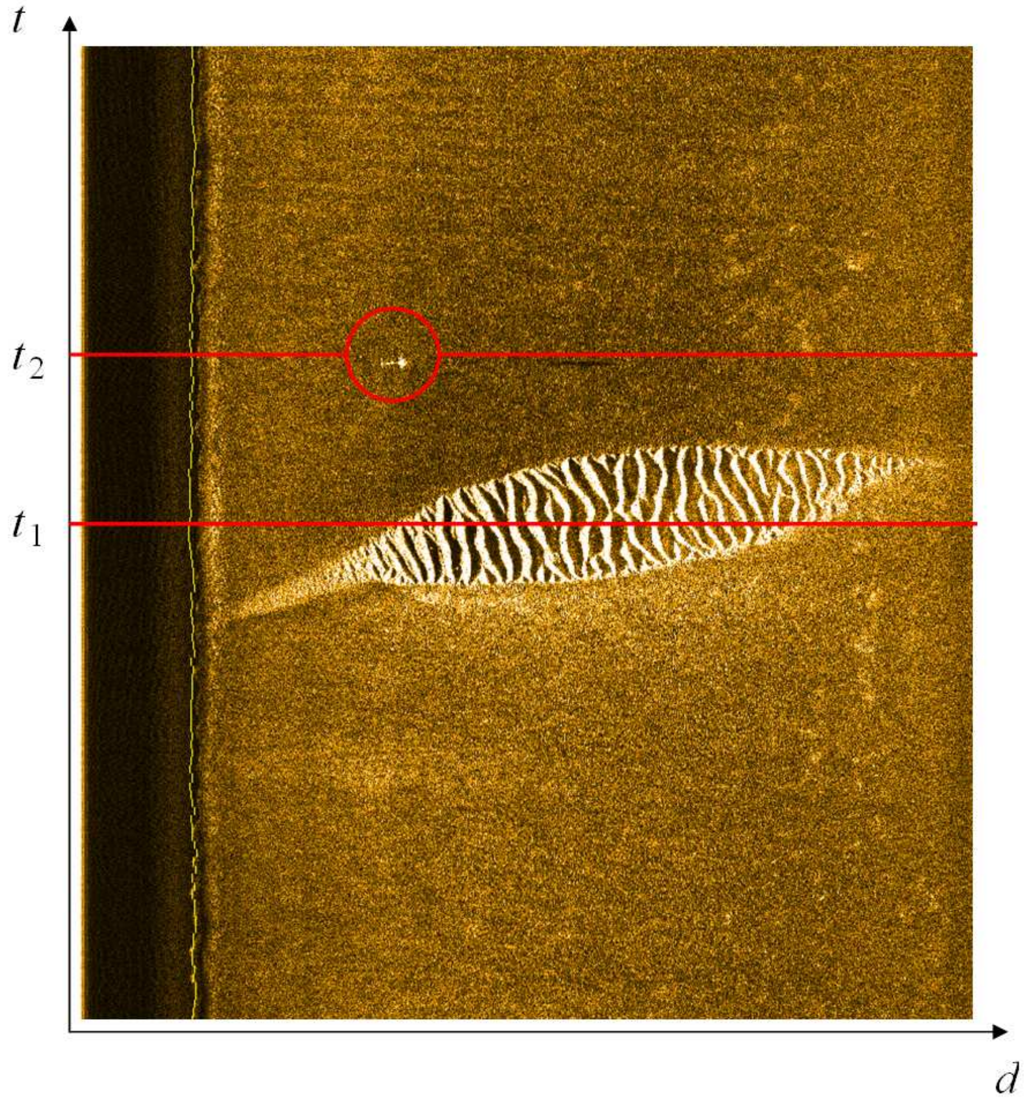
- Waterfall
  - Example: lateral sonar image





Problem formalization

- Waterfall
  - On a waterfall, we do not detect marks, we get areas where we are sure there are no marks



CSP on tubes

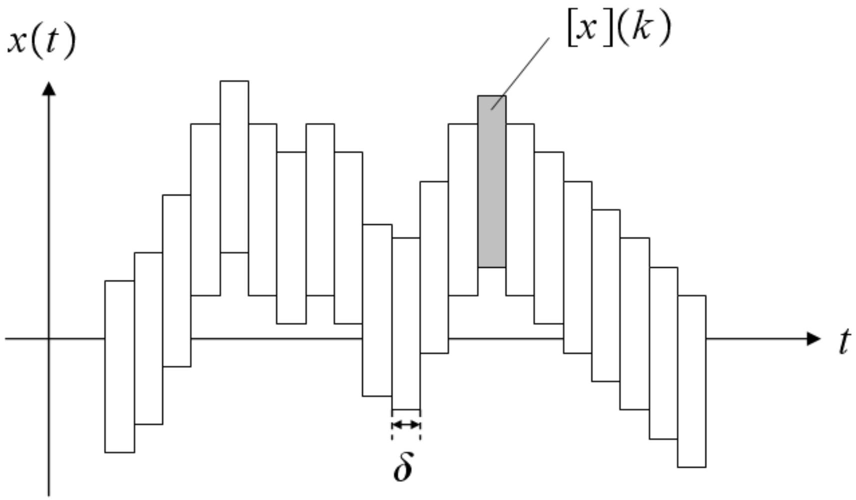
CSP on tubes

- CSP
  - Variables are trajectories (temporal functions) $\mathbf{x}(t), \dot{\mathbf{x}}(t)$ and real vector $\mathbf{m}$
  - Domains are interval trajectories (tubes) $[\mathbf{x}](t)$, $[\dot{\mathbf{x}}](t)$ and box $[\mathbf{m}]$
  - Constraints are:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} = \mathbf{g}(\mathbf{x}) \\ \mathbf{z} = \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{m}) \\ v(\mathbf{x}, \mathbf{u}, \mathbf{m}) = 0 \Rightarrow \mathbf{z} \in \mathbb{W} \end{cases}$$

CSP on tubes

- Tubes
  - The set of functions from $\mathbb{R}$ to $\mathbb{R}^n$ is a lattice so we can use interval of trajectories
  - A tube $[\mathbf{x}](t)$, with a sampling time $\delta > 0$ is a box-valued function constant on intervals $[k\delta, k\delta + \delta], k \in \mathbb{Z}$
  - The box $[k\delta, k\delta + \delta] \times [\mathbf{x}](t_k)$, with $t_k \in [k\delta, k\delta + \delta]$ is the k^{th} slice of the tube $[\mathbf{x}](t)$ and is denoted $[\mathbf{x}](k)$



CSP on tubes

- Tubes
  - The integral of a tube can be defined as:

$$\int_{t_0}^{t} [\mathbf{x}](\tau)d\tau = \sum_{k \in \kappa([t_0,t])} \delta.[\mathbf{x}](k)$$

  - We have also:

$$\mathbf{x}(t) \in [\mathbf{x}](t) \Rightarrow \int_{t_0}^{t} \mathbf{x}(\tau)d\tau \in \int_{t_0}^{t} [\mathbf{x}](\tau)d\tau$$

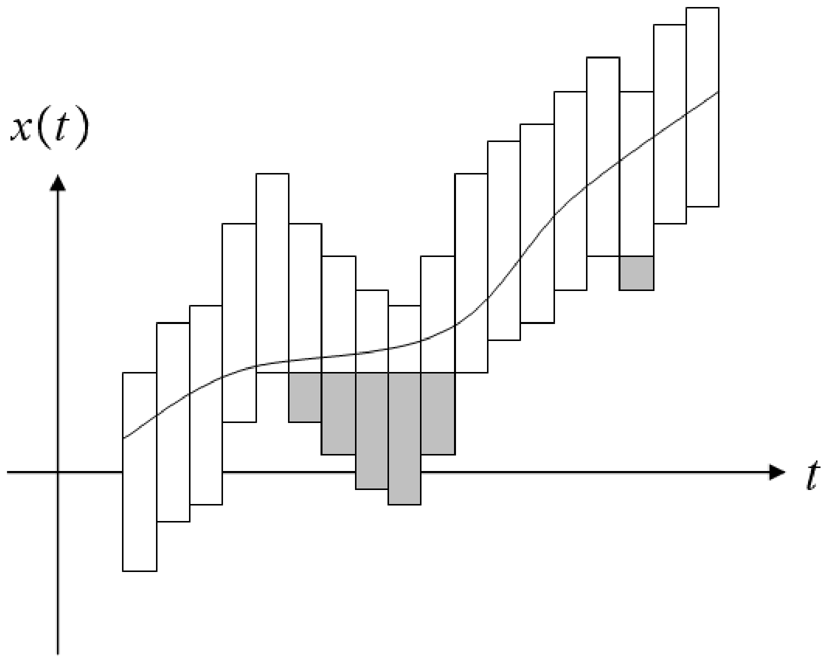
CSP on tubes

- Tubes
  - However, we cannot define the derivative of a tube in the general case, even if in our case we will have analytic expressions of derivatives:

$$[\dot{\mathbf{x}}](t) = [\mathbf{f}]([\mathbf{x}](t), [\mathbf{u}](t))$$

CSP on tubes

- Tubes
  - Contractions with tubes: e.g. increasing function constraint



Contraction of the visibility relation

Contraction of the visibility relation

- Problem: contract tubes $[v](t)$, $[y](t)$ with respect to a relation:

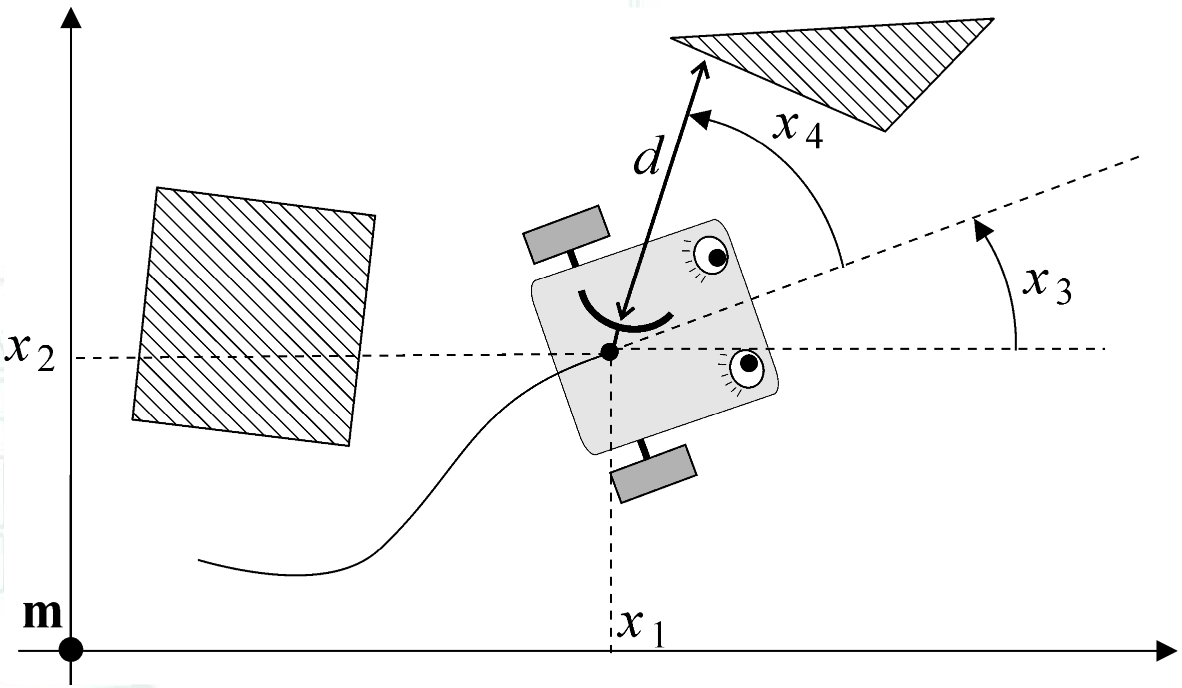
$$v(t) = 0 \Rightarrow y(t) \in \mathbb{W}(t)$$

- 2 theorems: 1 to contract $[y](t)$ and the other for $[v](t)$

Contraction of the visibility relation

- Example 1:
  - Dynamic localization of a robot with a rotating telemeter in a plane environment with unknown moving objects hiding sometimes a punctual known mark

$$
\begin{cases}
\dot{x}_1 = \cos x_3 + b_1 \\
\dot{x}_2 = \sin x_3 + b_2 \\
\dot{x}_3 = u + b_3 \\
\dot{x}_4 = \omega + b_4.
\end{cases}
$$



Contraction of the visibility relation

- Example 1
  - $\mathbf{m}$ is at $(0,0)$
  - Telemeter accuracy is $\pm 0.01\,\mathrm{m}$, scope is $[s] = [s^-, s^+] = [1, 10]\,\mathrm{m}$
  - Visibility function h and observation function g are:

$$\begin{cases} h(\mathbf{x}) & = & x_1 \sin(x_3 + x_4) - x_2 \cos(x_3 + x_4) \\ g(\mathbf{x}) & = & -x_1 \cos(x_3 + x_4) - x_2 \sin(x_3 + x_4). \end{cases}$$

Contraction of the visibility relation

- Example 1
  - We have:

$$\left(h(\mathbf{x}) = 0 \text{ and } g(\mathbf{x}) \in [s] \cap [-\infty, d]\right) \Rightarrow d = g(\mathbf{x})$$

which translates to:

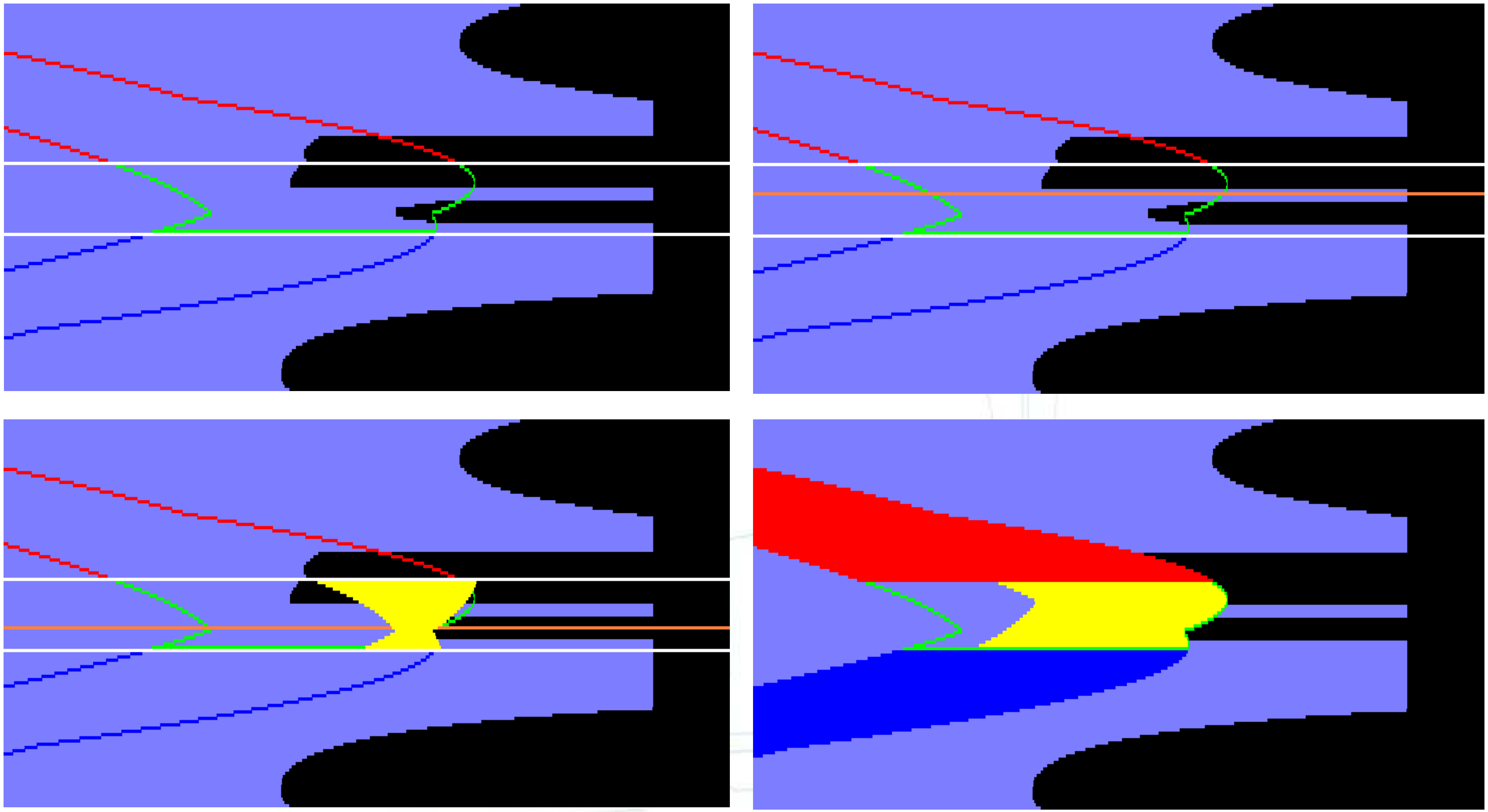
$$h(\mathbf{x}(t)) = 0 \Rightarrow g(\mathbf{x}(t)) \in \mathbb{W}(t)$$

with:

$$\mathbb{W}(t) = [-\infty, s^-] \cup [s^+, \infty] \cup [d^-, \infty].$$

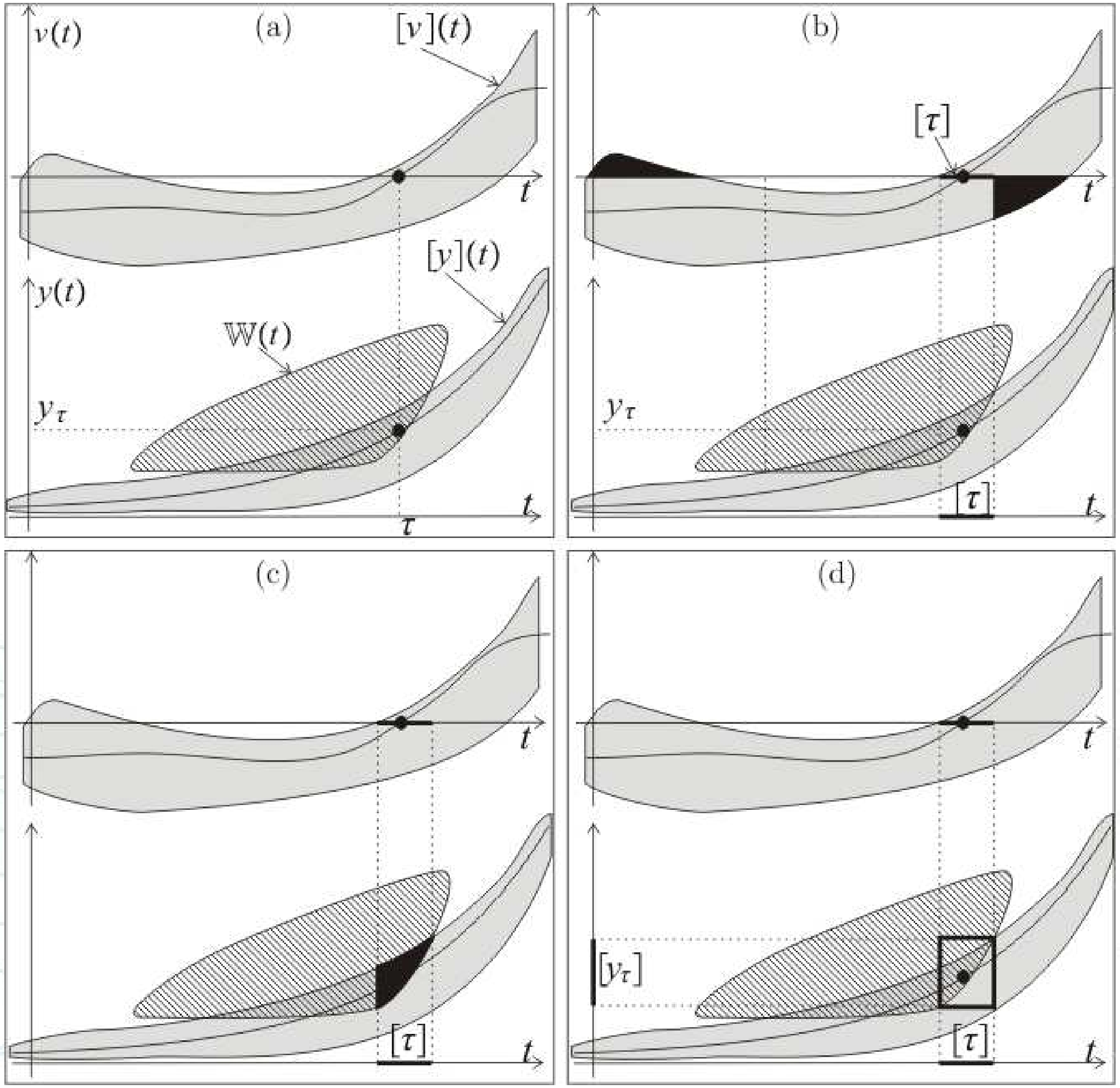
Contraction of the visibility relation

- Example 1: contraction of $[y](t)$



Contraction of the visibility relation

- Example 2: contraction of $[v](t)$



Conclusion

Conclusion

- In this talk, we presented a SLAM problem with fleeting detections
- The main problem is to use the visibility relation to contract positions
- We proposed a method based on tubes contractions

Questions?

- Contacts
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