Phase Based Localization Using Interval Analysis

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Small Workshop on Interval Methods, 2013

Outline

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What Is A Contractor[1]?

Definition

A contractor is a mapping $\mathscr{C}$ from $\mathbb{R}^n$ to $\mathbb{R}^n$ such that

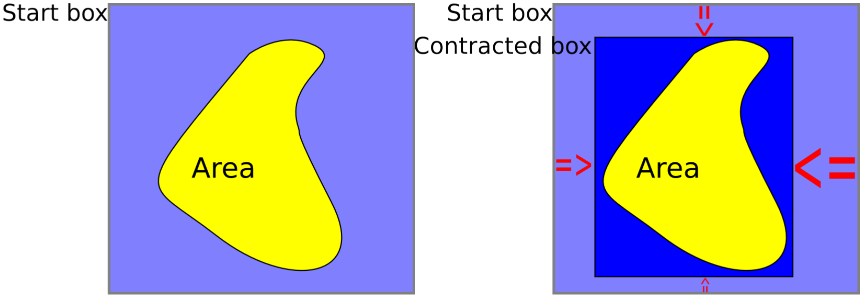
1. Contraction
$$\forall [x] \in \mathbb{R}^n, \mathscr{C}([x]) \subseteq [x]$$
2. Consistency
$$(x \in [x], \mathscr{C}(\{x\}) = \{x\}) \Rightarrow x \in \mathscr{C}([x])$$
3. Continuity
$$\mathscr{C}(\{x\}) = \emptyset \Leftrightarrow (\exists \varepsilon > 0, \forall [x] \subseteq B(x,\varepsilon), \mathscr{C}([x]) = \emptyset)$$

where $[x]$ is a box of n dimension and $B(x,\varepsilon)$ is the ball centered on x with radius ε.

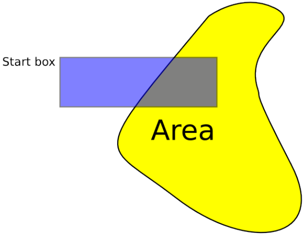
Outline

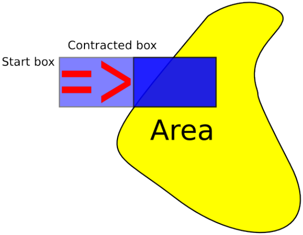
What Does A Contractor Look Like?

Let's suppose that C is a contractor from $\mathbb{R}^n$ to $\mathbb{R}^n$ and $[x]$ and $[y]$ are boxes in $\mathbb{R}^n$ as $C([x]) = [y]$.



Another Outside Contractor Example





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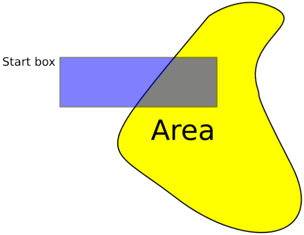
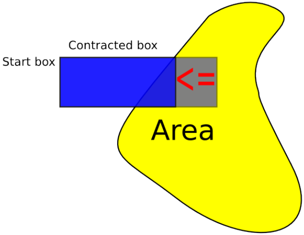
Inside Contractor





Outline

The Phase Equation.

A basic equation and yet very difficult to solve or invert.

1. Suppose that we have a source at coordinates (b_X, b_Y) emitting a continuous sine signal with a known frequency f.
2. At the coordinates (p_X, p_Y), a receiver is recording the emitted signal s with a delay. The signal equation can be described as follows:

$$s = \sin\left(2\pi f\left(t - \frac{d}{c}\right)\right)$$

   where :
   - t is the time,
   - c is the signal propagation speed and
   - d is the distance between the source and the receiver.
3. d can be written as follows:

$$d = \sqrt{(p_X - b_X)^2 + (p_Y - b_Y)^2}$$

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1. The signal equation is then:

$$s = \sin\left(2\pi f\left(t - \frac{\sqrt{(p_X - b_X)^2 + (p_Y - b_Y)^2}}{c}\right)\right)$$

2. Supposing that the state equations describing the receiver can be written:

$$\dot{X} = A$$

where:

- $X = \begin{bmatrix} p_X \\ p_Y \end{bmatrix}$ and
- $A = \begin{bmatrix} v\cos\theta \\ v\sin\theta \end{bmatrix}$ where: v is the receiver velocity and θ is its orientation.

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The Phase Localization Problem.

We want to retrieve (p_X, p_Y) knowing :

$$\dot{p_X} = v\cos\theta$$

$$\dot{p_Y} = v\sin\theta$$

$$s = \sin\left(2\pi f\left(t - \frac{\sqrt{(p_X - b_X)^2 + (p_Y - b_Y)^2}}{c}\right)\right)$$

We suppose that the following variables are measured:

- v the receiver velocity;
- θ the orientation of the receiver and
- s the signal recorded.

The Phase Localization Problem.

Or with an Euler discretization:

$$p_{X_{k+1}} = p_{X_k} + dt * v_k \cos\theta_k$$

$$p_{Y_{k+1}} = p_{Y_k} + dt * v_k \cos\theta_k$$

$$s_k = \sin\left(2\pi f\left(t - \frac{\sqrt{(p_{X_k} - b_X)^2 + (p_{Y_k} - b_Y)^2}}{c}\right)\right)$$

Outline

Use Contractor Programming

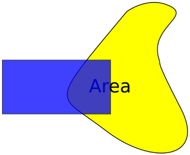
Idea

Contract and bisect a starting area until the desired precision (SIVIA-like approach).

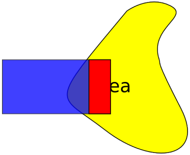
```
push initial_box to stack;
while (stack not empty){
    pop box from stack;
    contract box;
    if (box's width > precision){
        bisect box into two boxes;
        push boxes to stack;
    }
}
```

Outline

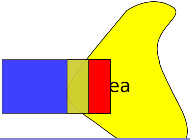
Implementation.



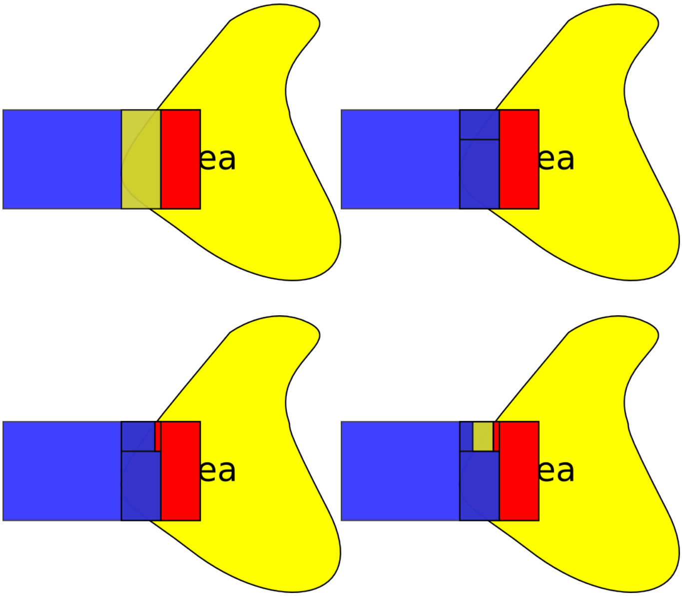
Apply an Inside Contractor to determine the boxes inside the solution;



Apply an Outside Contractor to remove the boxes that are not part of the solution.



Implementation.



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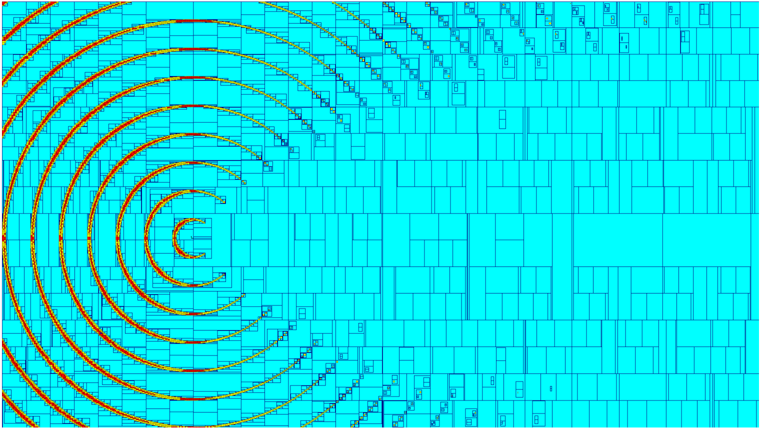
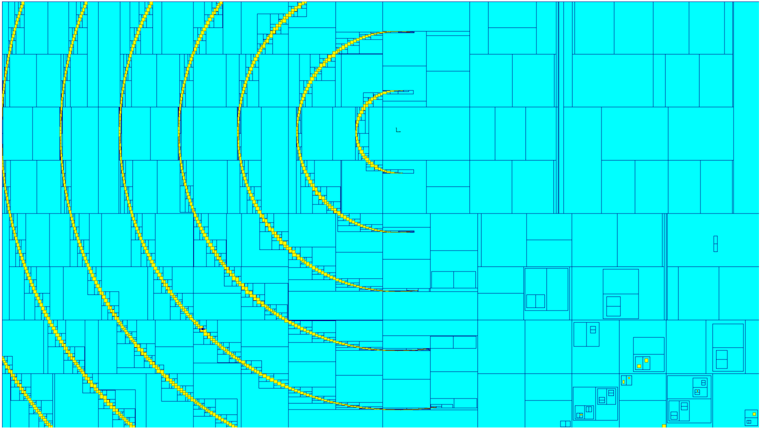
Simulation

- Velocity of $20m/s$;
- Sampling rate $1ms$;
- Buffer size 15000;
- Frequency $100Hz$.



Simulation

- Velocity of $3m/s$;
- Sampling rate $1ms$;
- Buffer size 15000;
- Frequency $100Hz$.



Conclusions

- The contraction was able solve the problem without having to invert or linearize the equations or another other information.
- A new complementary contractor has been defined that is used in the inside contractor part.
- The inside contractor is not necessary to localize the receiver as the outside is enough to give all the possible solutions.

- Future work
  - Use a second receiver.
  - Use a second source.
  - Add an observer to only make computations around the possible solution boxes.

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