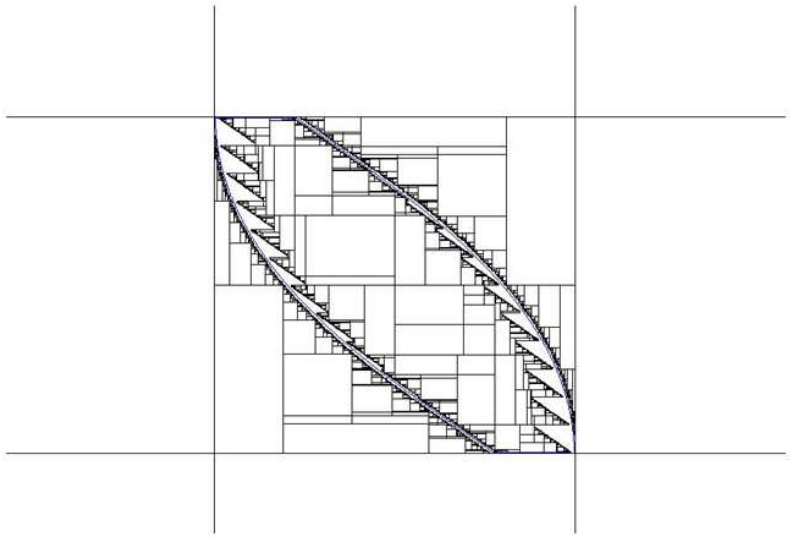
Determination of Inner and Outer Bounds of Reachable Sets



Francisco Fernandes Castro Rego

June 7, 2013

Control and Simulation Department
Faculty of Aerospace Engineering

Contents

TU Delft TÉCNICO LISBOA

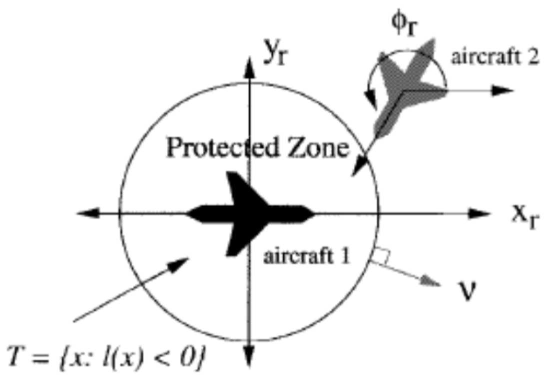
Motivation

- Motivation
- Reachable Sets
- Proposed Method
- Results
- Conclusions

Motivation

Avoid loss of separation:

- Ensuring aircraft separation is a fundamental aspect when implementing an Air Traffic Management architecture.
- Separation can be ensured by determining an outer bound of the backwards reachable set of the game of two vehicles.



From "Conflict Resolution for Air Traffic Management: A Study in Multiagent Hybrid Systems" by C. Tomlin, G. J. Pappas and S. Sastry

Motivation

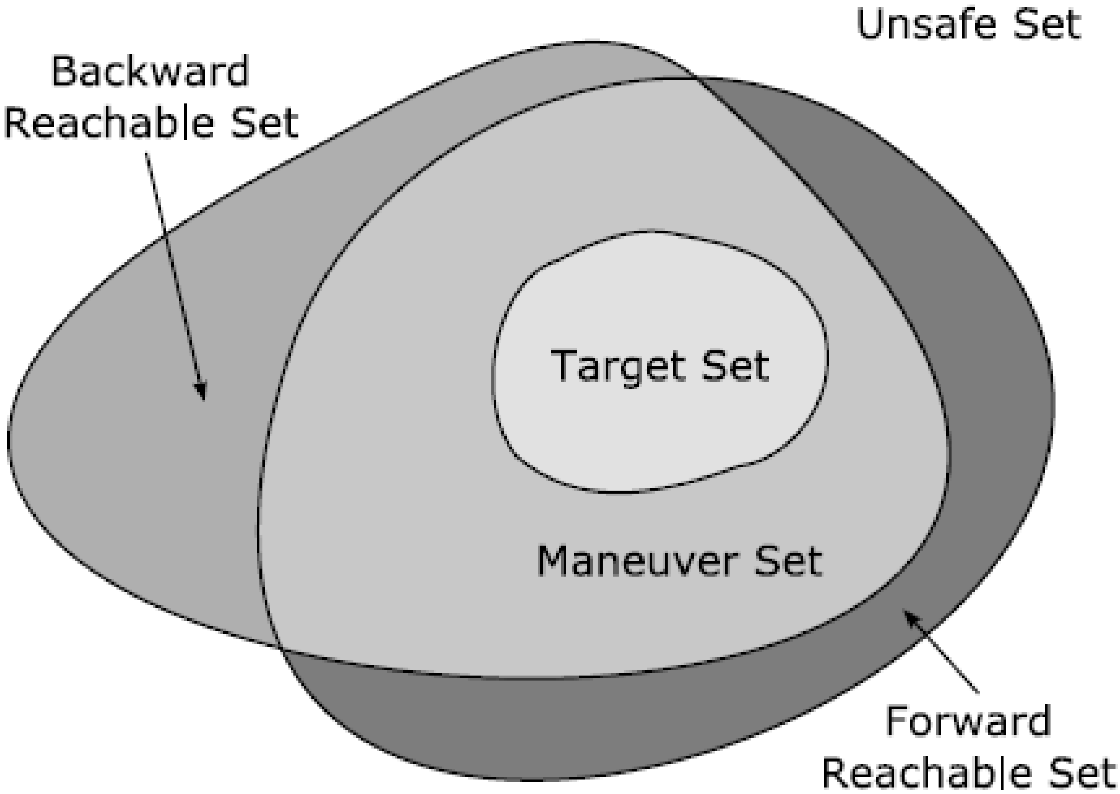
Avoid Loss-of-Control:

- Loss-of-Control of an aircraft occurs when the pilot or auto-pilot are not able to return to a trimming condition from the current flight condition.

- Loss-of-Control was the main cause of all aviation accidents (43%) during the period between 1996 and 2006.

Motivation

Safe Flight Envelope:



Reachable Sets

- Motivation
- Reachable Sets
- Proposed Method
- Results
- Conclusions

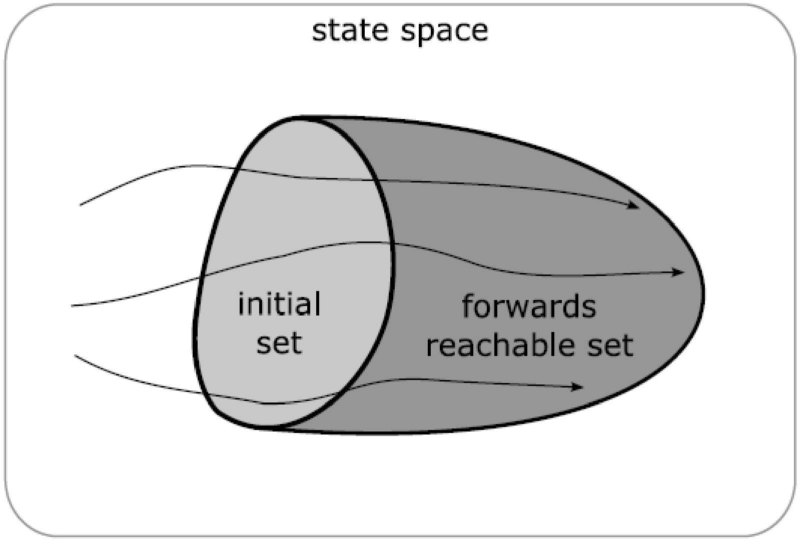
Reachable Sets

- We consider a dynamical system (u and d are control and disturbances, respectively):

$$\begin{aligned} \dot{x} &= f(x(t), u(t), d(t)) \\ & x(0) \in \mathcal{S}_0 \ (or\ x(t_f) \in \mathcal{T}_0), t \in [0, t_f] \end{aligned}$$

- f is assumed to be Lipschitz.
- Only piecewise continuous functions are admissible as $u(t)$ and $d(t)$.
- Only non-anticipative control strategies $u(t) = \gamma[d](t)$ are admissible.

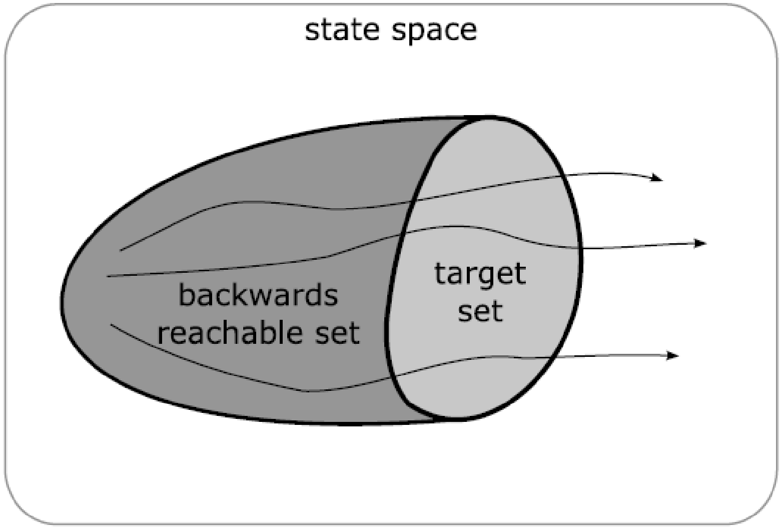
Reachable Sets



- Forwards Reachable Set $\mathcal{S}(\tau)$:

Set of all states $x(\tau)$ such that there exists an admissible strategy $u(t) = \gamma[d](t)$ for all admissible disturbance inputs $d(t)$ for which $x(\tau)$ is reachable from some $x(0) \in \mathcal{S}_0$.

Reachable Sets



- Backwards Reachable Set $\mathcal{T}(\tau)$:

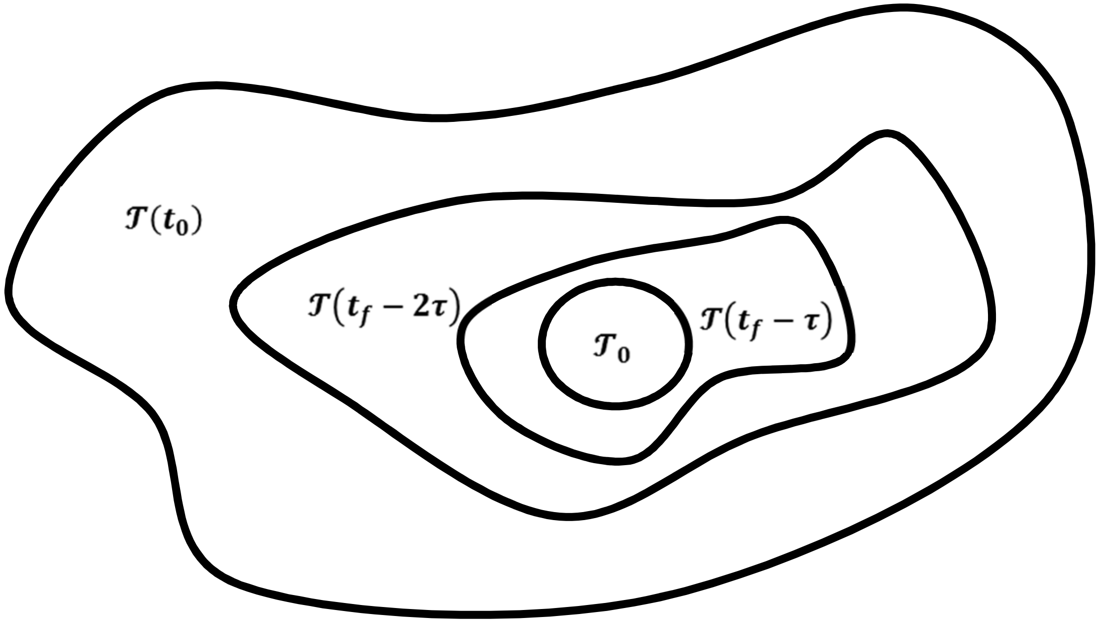
  Set of all states $x(\tau)$ such that there exists an admissible strategy $u(t) = \gamma[d](t)$ for all admissible disturbance inputs $d(t)$ for which some $x(t_f) \in \mathcal{T}_0$ is reachable from $x(\tau)$.

Proposed Method

- Motivation
- Reachable Sets
- Proposed Method
- Results
- Conclusions

Proposed Method

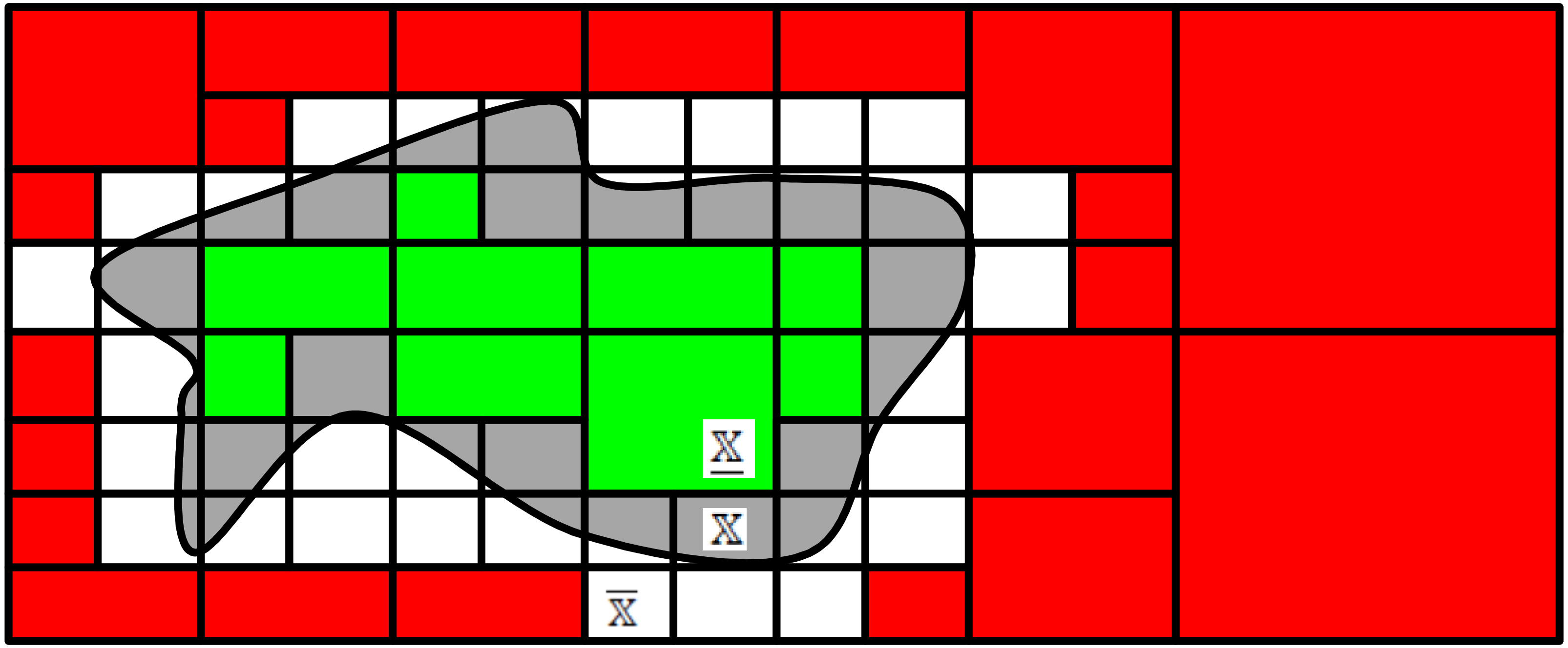
•To compute bounds on the final reachable set we begin with the target set and compute bounds at intermediate time steps.



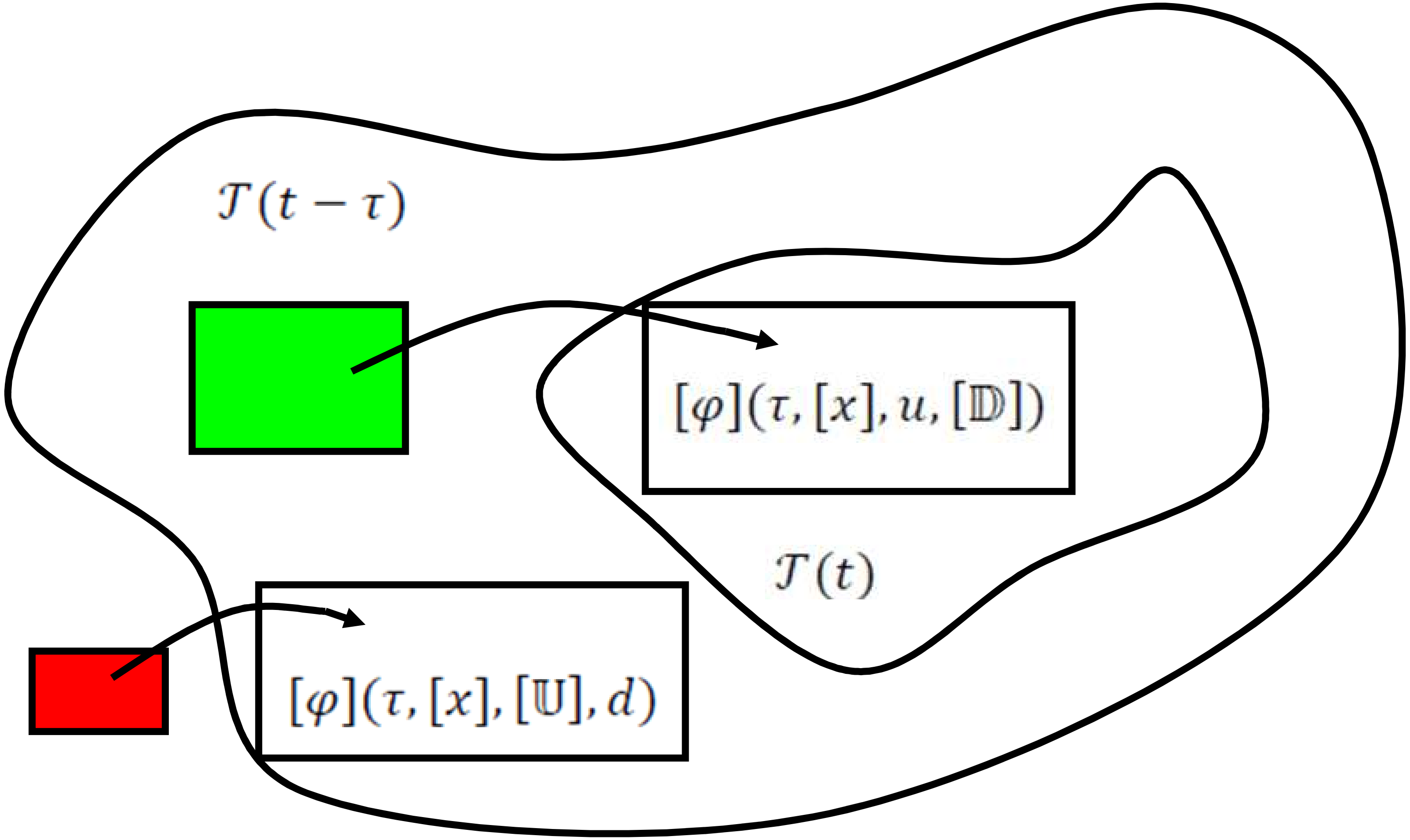
TU Delft TÉCNICO LISBOA

Proposed Method

- We can convert the reachability problem as a set inversion problem.



Proposed Method



Proposed Method

If $\mathcal{T}^-(t)$ *and* $\mathcal{T}^+(t)$ *are such that* $\mathcal{T}^-(t) \subseteq \mathcal{T}(t) \subseteq \mathcal{T}^+(t)$*, if* $[x] \in \mathbb{IR}^n$ *is a box of states, if* $u \in \mathcal{U}$ *and if* $d \in \mathcal{D}$ *then*

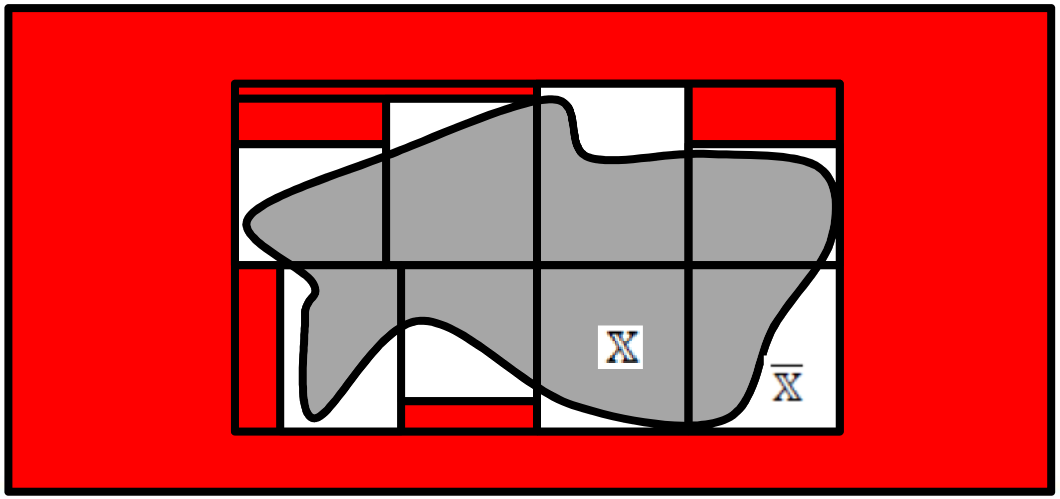
- $[\varphi](\tau, [x], u, [\mathbb{D}]) \subseteq \mathcal{T}^-(t) \Longrightarrow [x] \subseteq \mathcal{T}(t-\tau)$
- $[\varphi](\tau, [x], [\mathbb{U}], d) \cap \mathcal{T}^+(t) = \emptyset \Longrightarrow [x] \cap \mathcal{T}(t-\tau) = \emptyset$

Proposed Method

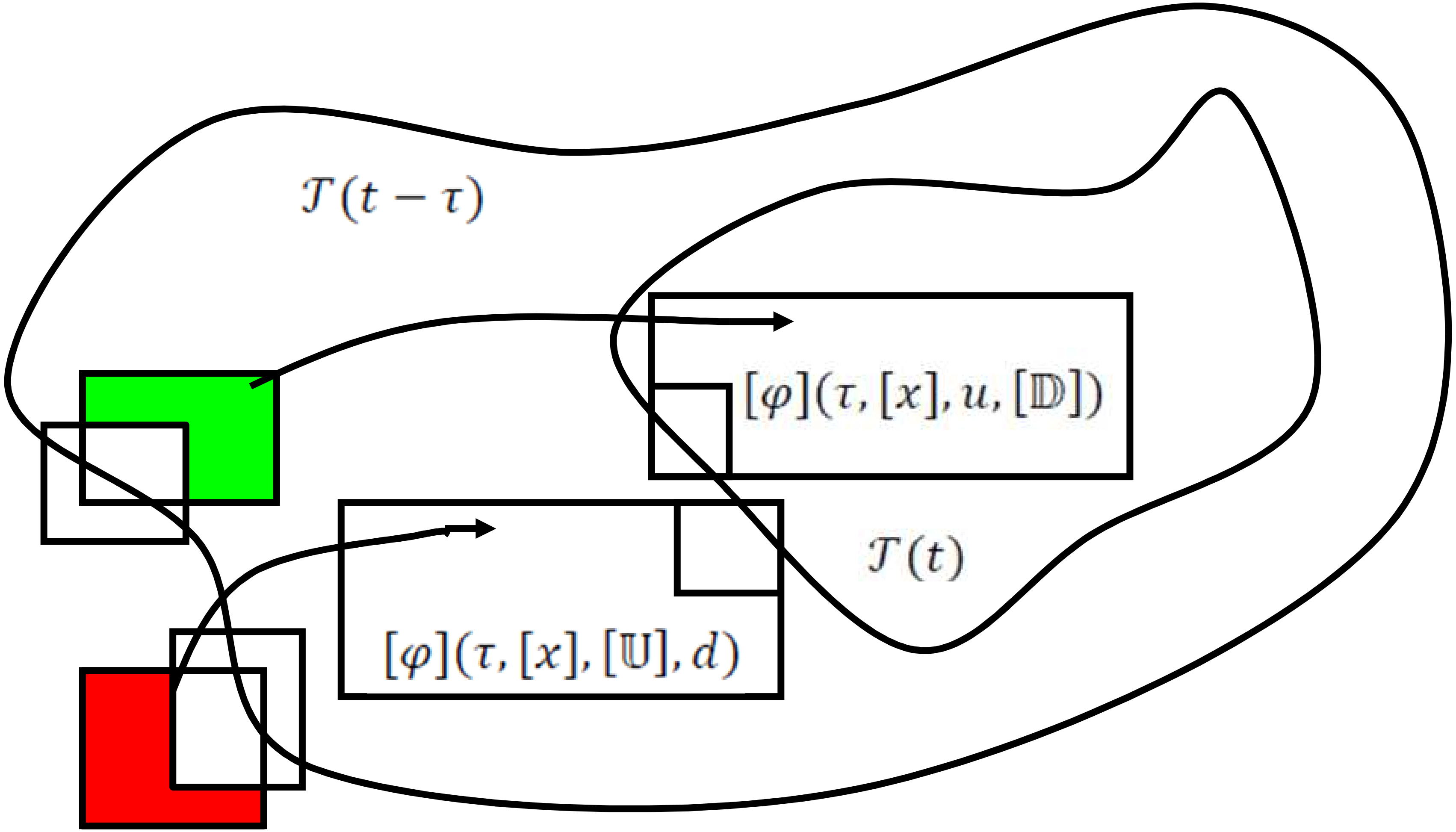
- Another method of computing approximations (by overestimation) is to use contractors:

$$\forall [x], \mathcal{C}_{\mathbb{S}}([x]) \subseteq [x] \qquad \text{(contractance)},$$

$$\forall [x], [x] \cap \mathbb{S} \subseteq \mathcal{C}_{\mathbb{S}}([x]) \qquad \text{(correctness)},$$



Proposed Method



Proposed Method

If $\mathcal{T}^-(t)$ *and* $\mathcal{T}^+(t)$ *are such that* $\mathcal{T}^-(t) \subseteq \mathcal{T}(t) \subseteq \mathcal{T}^+(t)$, *if* $[x] \in \mathbb{IR}^n$ *is a box of states, if* $u \in \mathcal{U}$ *and if* $d \in \mathcal{D}$ *then*

- *The interval function*
  $\mathcal{C}_{\mathcal{T}^c(t-\tau)}([x]) = [x] \cap [\varphi_{inv}](\tau, [[\varphi](\tau, [x], u, [\mathbb{D}]) \cap \mathcal{T}^{-c}(t)], u, [\mathbb{D}])$
  Is a contractor for the constraint $x \in \mathcal{T}^c(t-\tau)$

- *The interval function*
  $\mathcal{C}_{\mathcal{T}(t-\tau)}([x]) = [x] \cap [\varphi_{inv}](\tau, [[\varphi](\tau, [x], [\mathbb{U}], d) \cap \mathcal{T}^+(t)], [\mathbb{U}], d)$
  Is a contractor for the constraint $x \in \mathcal{T}(t-\tau)$

Proposed Method
Outer bound

Algorithm: $SubReach$ (out: $\mathcal{T}^+(0)$,in: $[x]_0, \varepsilon, \mathcal{I}, \mathbb{U}, \mathbb{D}, \mathcal{T}_0{}^+, t_f, ns, Nstp$)

$\tau := t_f / Nstp$
Compute the sampling set for the disturbance input
$\mathcal{T} := \mathcal{T}_0{}^+$
$j := 1$
Do
    $\mathcal{T} = B\&P(test(\cdot), [x]_0, \varepsilon, \mathcal{I})$ or $\mathcal{T} = B\&C(\mathcal{C}_{\mathbb{T}}(\cdot), [x]_0, \varepsilon, \mathcal{I})$
While $j \leq Nstp$
$\mathcal{T}^+(0) := \mathcal{T}$

Proposed Method
Inner bound

Algorithm: $SubReach$ (out: $\mathcal{T}^-(0)$,in: $[x]_0$, ε, $\mathcal{J}$, $\mathbb{U}$, $\mathbb{D}$, ${\mathcal{T}_0}^{c+}$, t_f, ns, $Nstp$)

$\tau := t_f/Nstp$

Compute the sampling set for the disturbance input

$\mathcal{T} := {\mathcal{T}_0}^{c+}$

$j := 1$

Do

$\quad\quad\mathcal{T} = B\&P(test(\cdot), [x]_0, \varepsilon, \mathcal{J})$ or $\mathcal{T} = B\&C(\mathcal{C}_{\mathbb{T}}(\cdot), [x]_0, \varepsilon, \mathcal{J})$

Insert on the subpaving $\mathcal{T}$ the boxes resulting from $exclude([x]_0, \mathbb{R}^n)$

While $j \leq Nstp$

$\mathcal{T}^-(0) := \mathcal{T}^c$

Results

- Motivation
- Reachable Sets
- Proposed Method
- Results
- Conclusions

Results

Double Integrator

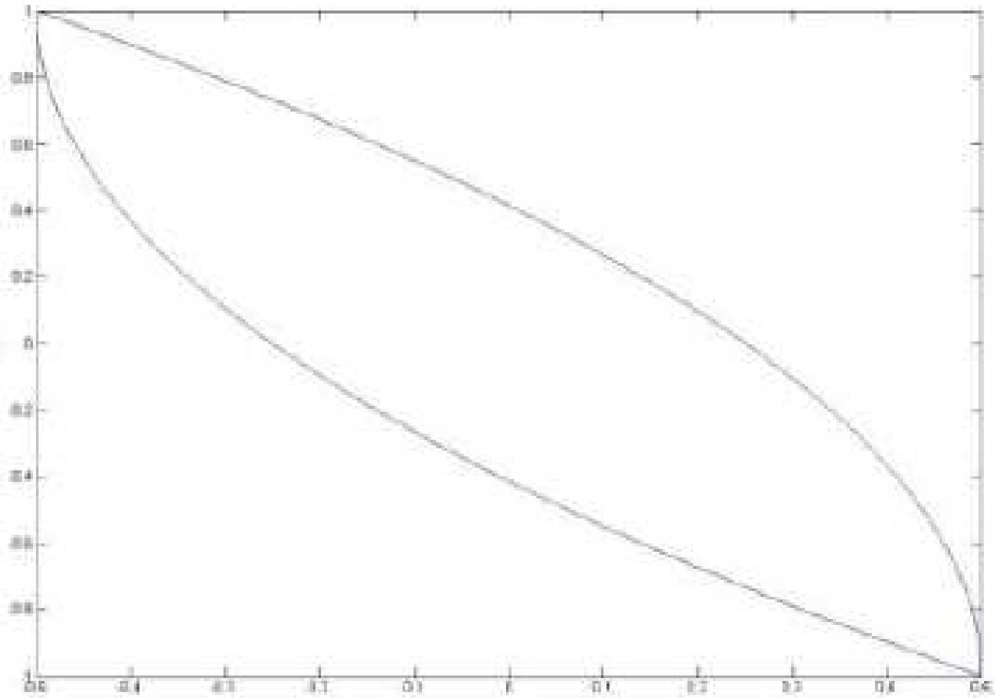
$$\dot{x} = \begin{bmatrix} x_2 \\ u \end{bmatrix} = f(x, u)$$

$$\mathcal{T}_0 = (0,0)$$

$$t_f = 1$$

$$u \in [-1,1]$$

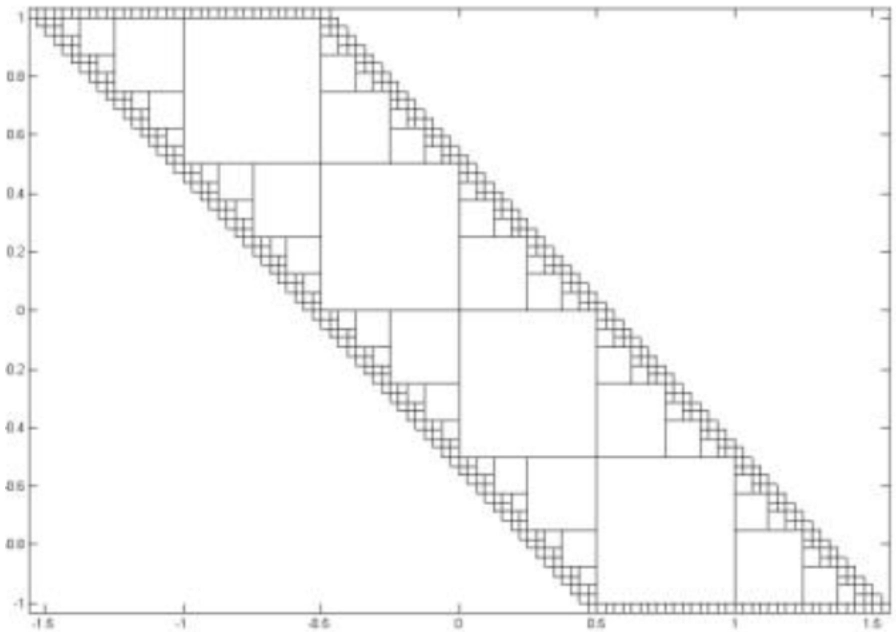
$\mathcal{T}(0)$



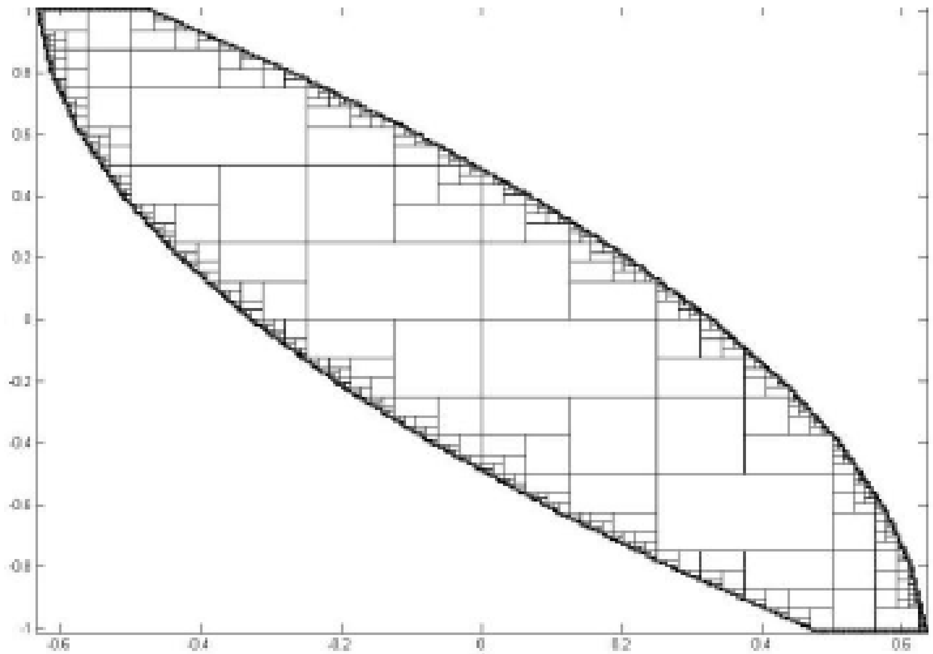
Results

Branch and Prune

Outer approximation of reachable set



1 Time Step

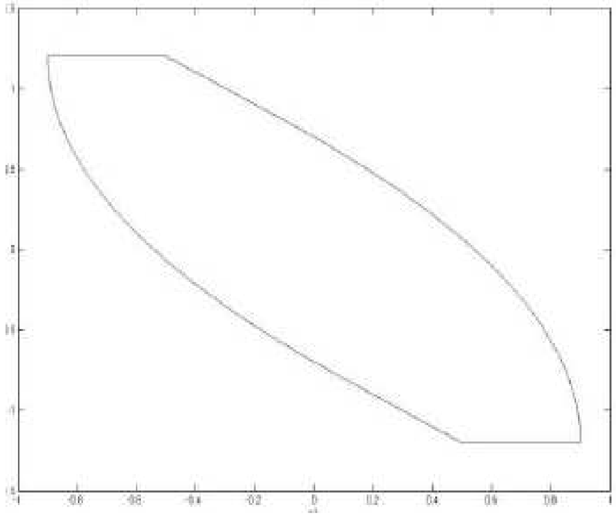


10 Time Steps

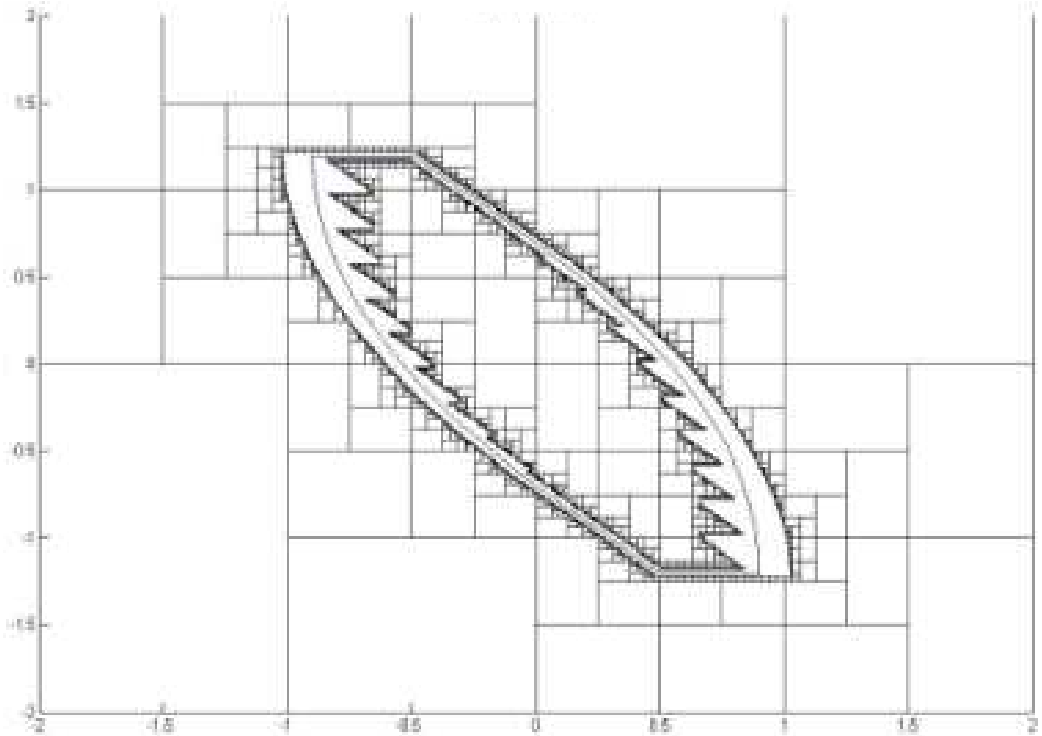
Results

Branch and Prune

$$\mathcal{T}_0 = ([-0.2,0.2],[-0.2,0.2])$$



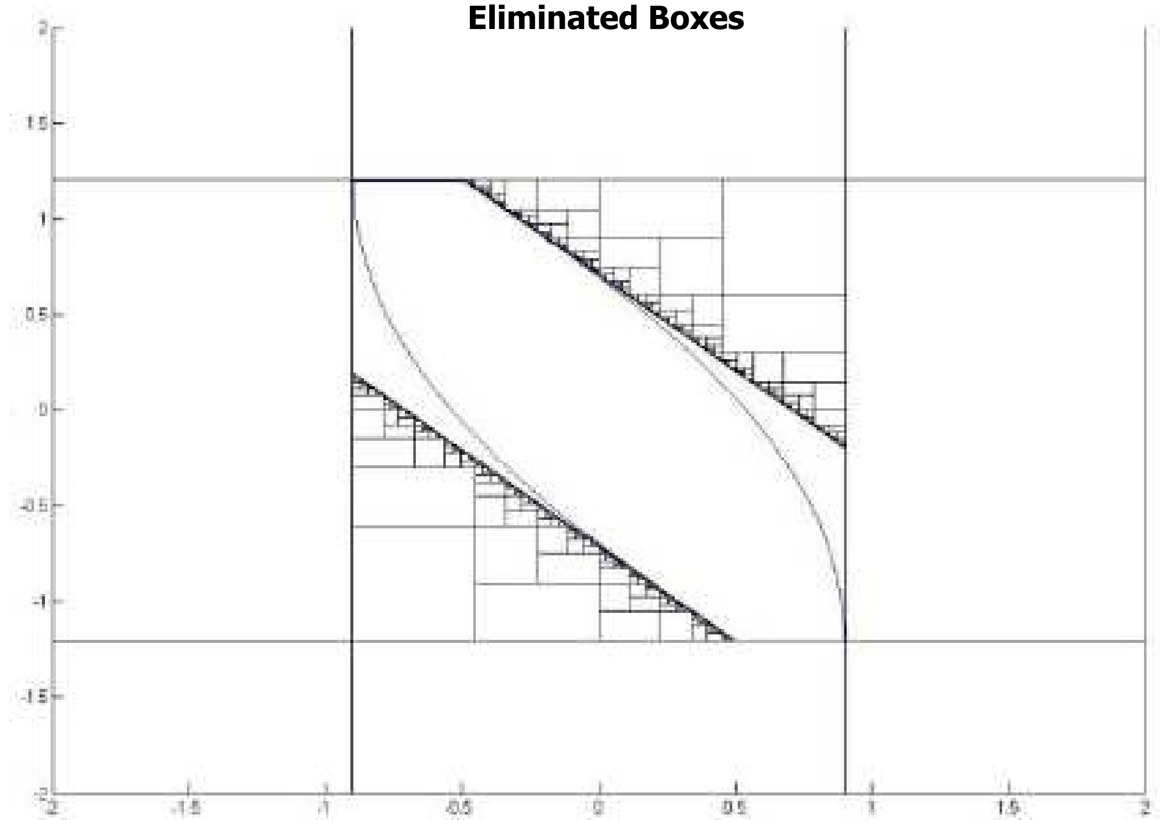
Eliminated Boxes



10 Time Steps

Results

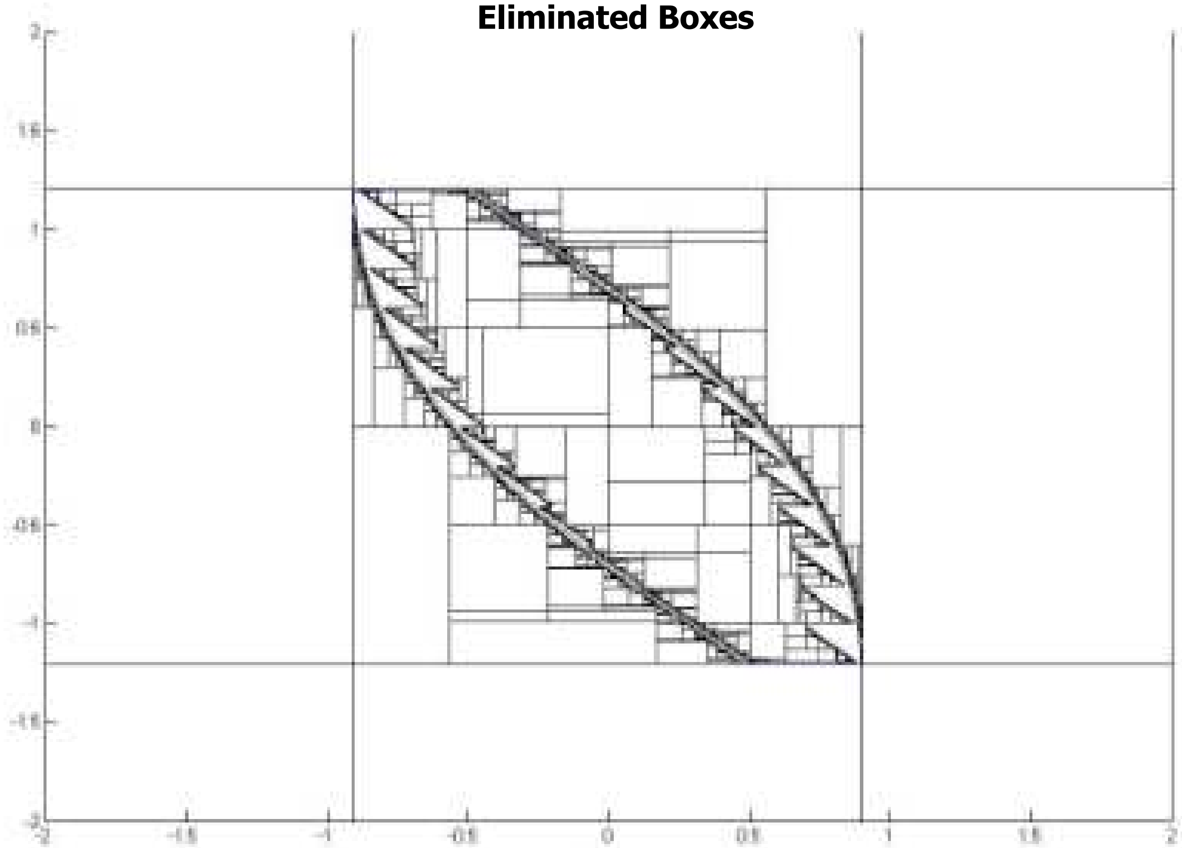
Branch and Contract



1 Time Step

Results

Branch and Contract

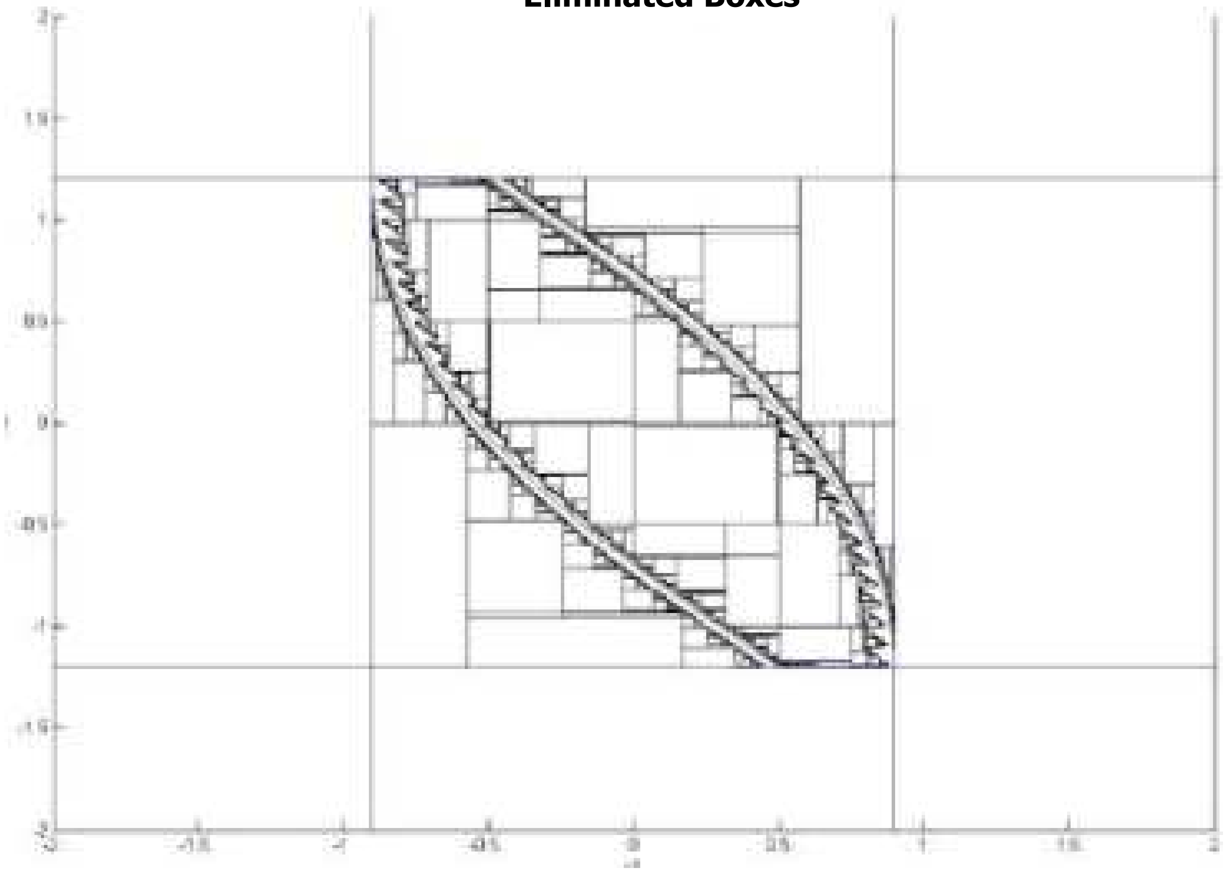


10 Time Steps

Results

Branch and Contract

Eliminated Boxes



20 Time Steps

TU Delft TÉCNICO LISBOA

Results

Method (10 Time steps)	Overestimation	Underestimation
Branch & Prune	17.3%	21.1%
Branch & Contract	3.6%	13.0%

Results

Time steps B&C	Overestimation	Underestimation
1	17.8%	-
10	3.6%	13.0%
20	7.3%	8.9%

Conclusions

- Motivation
- Reachable Sets
- Proposed Method
- Results
- Conclusions

Conclusions

- Successful approach of application of subpavings to enclose reachable sets.
- Use of contractors drastically reduced overestimation and underestimation.
- More time steps reduced overestimation up to a point. After that point accumulated overestimation at each time step became important.

Thank you!