Interval Methods for the Implementation of Real-Time Capable Robust Controllers for Solid Oxide Fuel Cell Systems

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Modeling
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System Structure
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Sliding Mode Control
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Predictive Control
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Conclusions
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Contents

- Control-oriented modeling of high-temperature Solid Oxide Fuel Cell Systems (SOFC systems)
- Focus on the non-stationary thermal behavior of SOFC stack modules
- Design of alternative model-based control strategies
  - ▶ Interval-based sliding mode control
  - ▶ Sensitivity-based predictive control procedures
- Numerical simulations
- Experimental validation
- Conclusions and outlook on future work

Control-Oriented Modeling of SOFC Systems (1)

### Configuration of the SOFC test rig at the Chair of Mechatronics

- Supply of fuel gas (hydrogen and/or mixture of methane, carbon monoxide, water vapor)
- Supply of air
- Independent preheaters for fuel gas and air
- Stack module containing fuel cells in electric series connection
- Electric load as disturbance
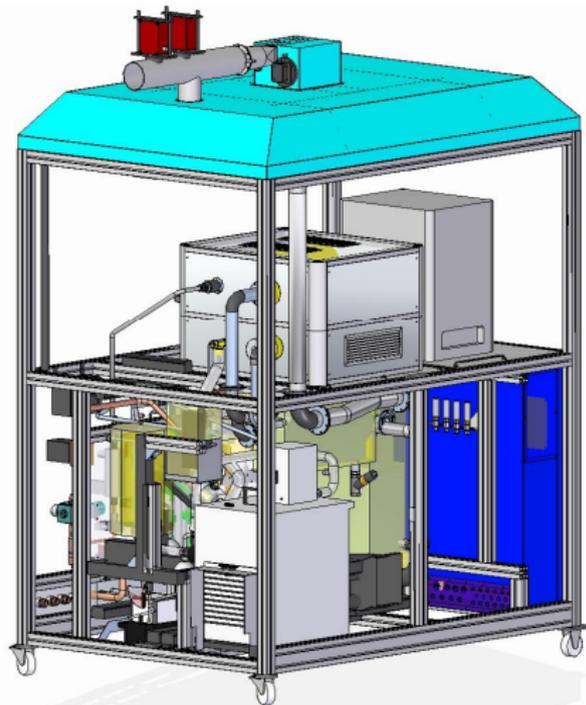
Control-Oriented Modeling of SOFC Systems (2)

Components of the stack module

1. Interconnector
2. Contacting layer (nickel)
3. Anode
4. Electrolyte
5. Cathode
6. Contacting layer (ceramic)
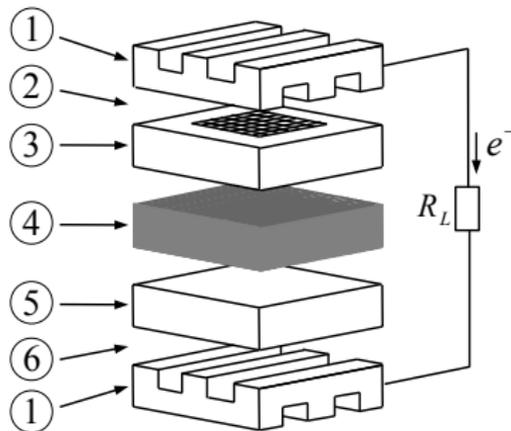
Control-Oriented Modeling of SOFC Systems (3)

Chemical reactions at each cell
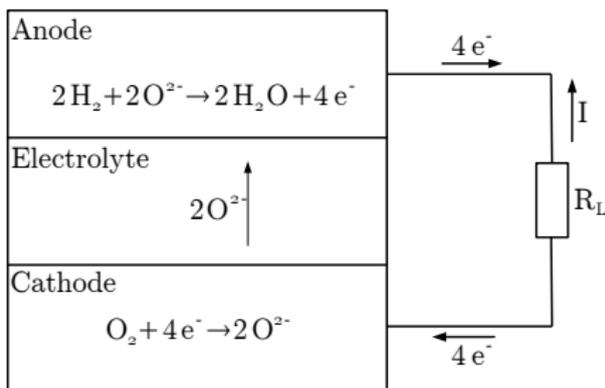
- Anode: Oxidation
- Cathode: Reduction
- Faraday's law

$$I = z \ F \ \dot{n}_{H_2}$$

I Current in A

z Number of electrons

F Faraday constant in $\dfrac{\text{C}}{\text{mol}}$

$\dot{n}_{H_2}$ Molar flow in $\dfrac{\text{mol}}{\text{s}}$

Anode

$2\,H_2 + 2\,O^{2-} \rightarrow 2\,H_2O + 4\,e^-$

Electrolyte

$2\,O^{2-}$

Cathode

$O_2 + 4\,e^- \rightarrow 2\,O^{2-}$

$4\,e^-$

I

R_L

$4\,e^-$

Control-Oriented Modeling of SOFC Systems (4)

## Subdivision of the SOFC model into different components

- Interconnection of 3 subprocesses
- Electrical load: Description in terms of a (time-varying) resistor R_L
- Representation of the system dynamics by a coupled set of ordinary differential equations (spatial semi-discretization)
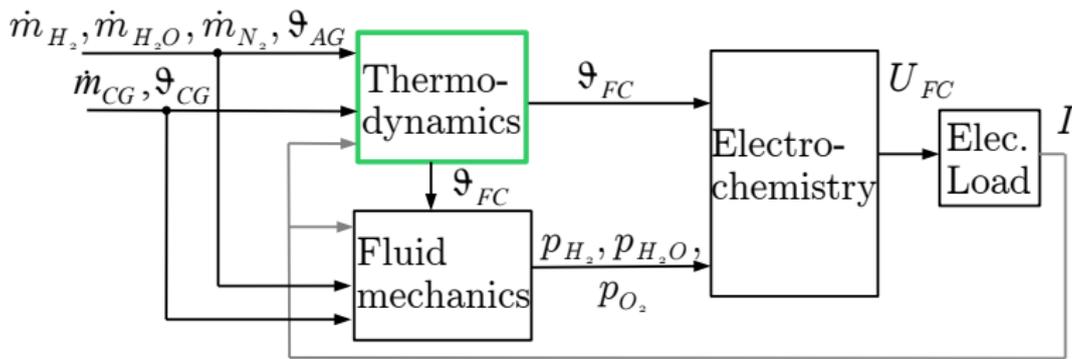
Couplings with other subprocesses can be interpreted as a disturbance from the point of view of the thermodynamic process

Control-Oriented Modeling of SOFC Systems (5)

Global energy balance for the complete stack module
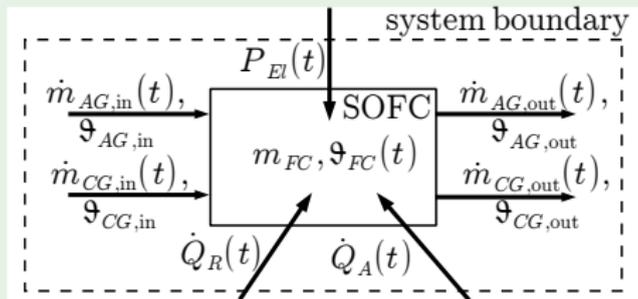
- Integral heat flow balances for non-stationary operating points
- Spatially distributed system model: Semi-discretization in terms of a finite volume model with piecewise homogeneous temperatures
- Mathematical modeling of the influence of a variable gas supply (temperature and mass flow) on both the electric load and temperature distribution

Control-Oriented Modeling of SOFC Systems (6)

- Fundamental relation between variations of the thermal energy and and the temperature in the interior of the stack module (assumption of constant parameters c_{FC} and m_{FC})

$$\frac{dE_{FC}(t)}{dt} = c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt}$$

Control-Oriented Modeling of SOFC Systems (6)

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- Heat flows for the representation of variations of the internal energy

$$\frac{dE_{FC}(t)}{dt} = C_{AG}(\vartheta_{FC}, t) \cdot (\vartheta_{AG,in}(t) - \vartheta_{FC}(t))$$
$$+ C_{CG}(\vartheta_{FC}, t) \cdot (\vartheta_{CG,in}(t) - \vartheta_{FC}(t)) + \dot{Q}_R(t) + P_{El}(t) + \dot{Q}_A(t)$$

Control-Oriented Modeling of SOFC Systems (6)

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- Exothermic reaction between hydrogen and oxygen

$$\dot{Q}_R = \frac{\Delta_R H(\vartheta_{FC}) \cdot \dot{m}_{H_2}^R(t)}{M_{H_2}}$$

Control-Oriented Modeling of SOFC Systems (7)

- Fundamental relation between variations of the thermal energy and and the temperature in the interior of the stack module (assumption of constant parameters c_{FC} and m_{FC})

$$c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt} = C_{AG}(\vartheta_{FC}, t) \cdot (\vartheta_{AG,in}(t) - \vartheta_{FC}(t))$$
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$$+ C_{CG}(\vartheta_{FC}, t) \cdot (\vartheta_{CG,in}(t) - \vartheta_{FC}(t)) + \dot{Q}_R(t) + P_{El}(t) + \dot{Q}_A(t)$$

- Heat transfer between the stack module and the ambient medium in a locally linearized form: thermal resistance $R_{th,A}$ can be treated as an interval parameter $R_{th,A} \in \left[\underline{R}_{th,A} \; ; \; \overline{R}_{th,A} \right]$

$$\dot{Q}_A(t) = \frac{1}{R_{th,A}} \left(\vartheta_A(t) - \vartheta_{FC}(t) \right)$$

Control-Oriented Modeling of SOFC Systems (7)

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$$\dot{Q}_A(t) = \frac{1}{R_{th,A}} \left(\vartheta_A(t) - \vartheta_{FC}(t)\right)$$

- Ohmic losses in the interior of the stack module

$$P_{El}(t) = R_{El} \cdot I^2(t)$$

Control-Oriented Modeling of SOFC Systems (8)

- Fundamental relation between variations of the thermal energy and and the temperature in the interior of the stack module (assumption of constant parameters c_{FC} and m_{FC})

$$c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt} = C_{AG}(\vartheta_{FC}, t) \cdot (\vartheta_{AG,in}(t) - \vartheta_{FC}(t))$$
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- Anode gas: Approximation of the temperature dependency of specific heat capacities by 2nd-order polynomials with $c_\chi(\vartheta_{FC})$ and $\chi \in \{H_2, N_2, H_2O\}$

$$C_{AG}(\vartheta_{FC}, t) = c_{H_2}(\vartheta_{FC}) \cdot \dot{m}_{H_2}(t)$$
$$+ c_{N_2}(\vartheta_{FC}) \cdot \dot{m}_{N_2}(t) + c_{H_2O}(\vartheta_{FC}) \cdot \dot{m}_{H_2O}(t)$$

Control-Oriented Modeling of SOFC Systems (8)

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- Anode gas: Approximation of the temperature dependency of specific heat capacities by 2nd-order polynomials with $c_\chi(\vartheta_{FC})$ and $\chi \in \{H_2, N_2, H_2O\}$
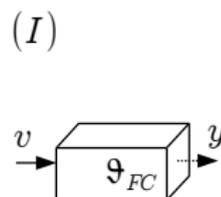
$$C_{AG}(\vartheta_{FC}, t) = c_{H_2}(\vartheta_{FC}) \cdot \dot{m}_{H_2}(t)$$
$$+ c_{N_2}(\vartheta_{FC}) \cdot \dot{m}_{N_2}(t) + c_{H_2O}(\vartheta_{FC}) \cdot \dot{m}_{H_2O}(t)$$

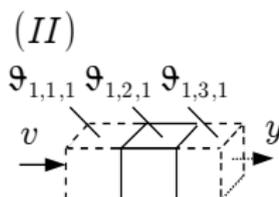- Cathode gas: 2nd-order polynomial for $c_{CG}(\vartheta_{FC})$

$$C_{CG}(\vartheta_{FC}, t) = c_{CG}(\vartheta_{FC}) \cdot \dot{m}_{CG}(t)$$

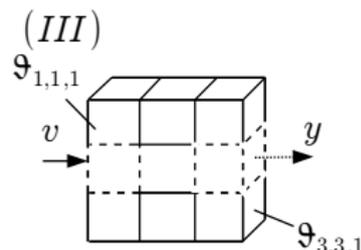## Different Variants of the Finite Volume Model

$$k=1,\ldots,N$$
$$j=1,\ldots,M$$
$$i=1,\ldots,L$$

(I)

(II)
$\vartheta_{1,1,1} \; \vartheta_{1,2,1} \; \vartheta_{1,3,1}$

(III)
$\vartheta_{1,1,1}$

$\vartheta_{3,3,1}$

$x = \vartheta_{FC}$ $\quad$ $\mathbf{x}^T = [\vartheta_{1,1,1}, \vartheta_{1,2,1}, \vartheta_{1,3,1}]$ $\quad$ $\mathbf{x}^T = [\vartheta_{1,1,1}, \ldots, \vartheta_{3,3,1}]$

- System input: Enthalpy flow of cathode gas (here, case (II))

$$v(t) = \dot{m}_{CG}(t) \cdot \big(\vartheta_{CG}(t) - \vartheta_{FC}(t)\big) \quad \text{with} \quad \vartheta_{FC}(t) = \vartheta_{1,1,1}(t)$$

- Vector representation of the input

$$\mathbf{u}(t) = \begin{bmatrix} \dot{m}_{CG}(t) \\ \vartheta_{CG}(t) - \vartheta_{FC}(t) \end{bmatrix}$$

- System output: Temperature at a predefined segment (i, j, k)

Transformation into Nonlinear Controller Normal Form (1)

Lie derivatives of the output $y = h(\mathbf{x})$ in the case (II), $M = 3$

$$y^{(i)} = L_{\mathbf{f}}^i h(\mathbf{x}) = L_{\mathbf{f}}\left(L_{\mathbf{f}}^{i-1}h(\mathbf{x})\right), \quad i = 0,...,\delta - 1$$

with $y = h(\mathbf{x}) = L_{\mathbf{f}}^0 h(\mathbf{x})$ for $i = 0$ and the relative degree $\delta = M$

Assumption

Negligible influence of variations of $\dot{m}_{CG}$ on the time derivatives of the system output according to

$$\frac{\partial L_{\mathbf{f}}^i h(\mathbf{x})}{\partial v} = 0$$

Modeling
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System Structure
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Sliding Mode Control
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Predictive Control
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Conclusions
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Transformation into Nonlinear Controller Normal Form (2)

Introduction of the new state vector

$$\mathbf{z}^T = \begin{bmatrix} h(\mathbf{x}) & L_{\mathbf{f}}h(\mathbf{x}) & \dots & L_{\mathbf{f}}^{M-1}h(\mathbf{x}) \end{bmatrix}^T \in \mathbb{R}^M$$

Transformation into Nonlinear Controller Normal Form (2)

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New set of state equations (input-affine system model)

$$\dot{\mathbf{z}} = \begin{bmatrix} L_{\mathbf{f}} h(\mathbf{x}) \\ \vdots \\ L_{\mathbf{f}}^{M-1} h(\mathbf{x}) \\ L_{\mathbf{f}}^{M} h(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} z_2 \\ \vdots \\ z_M \\ \tilde{a}(\mathbf{z}, \mathbf{p}, d) \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \tilde{b}(\mathbf{z}, \mathbf{p}) \end{bmatrix} \cdot v$$

with the additive bounded disturbance $d \in [d]$, $d \in \mathbb{R}$, and the interval parameters $\mathbf{p} \in [\mathbf{p}]$, $\mathbf{p} \in \mathbb{R}^{n_p}$

Modeling
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System Structure
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Sliding Mode Control
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Predictive Control
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Conclusions
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Transformation into Nonlinear Controller Normal Form (3)

Goal: Accurate trajectory tracking and stabilization of the error dynamics despite the interval uncertainties $d \in [d]$ and $\mathbf{p} \in [\mathbf{p}]$

$$\dot{\mathbf{z}} = \begin{bmatrix} L_{\mathbf{f}} h(\mathbf{x}) \\ \vdots \\ L_{\mathbf{f}}^{M-1} h(\mathbf{x}) \\ L_{\mathbf{f}}^{M} h(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} z_2 \\ \vdots \\ z_M \\ \tilde{a}(\mathbf{z}, \mathbf{p}, d) \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \tilde{b}(\mathbf{z}, \mathbf{p}) \end{bmatrix} \cdot v$$

- Assumption of an additive disturbance d with $\tilde{a}(\mathbf{z}, \mathbf{p}, d) = a(\mathbf{z}, \mathbf{p}) + d$
- Sign condition $\tilde{b}(\mathbf{z}, \mathbf{p}) > 0$ holds due to physical parameter constraints
- Observer-based estimation of d with $[d] = [\underline{d} \, ; \, \overline{d}\,] = \hat{d} + [\underline{\Delta d} \, ; \, \overline{\Delta d}\,]$

Transformation into Nonlinear Controller Normal Form (4)

Goal: Accurate trajectory tracking and stabilization of the error dynamics despite the interval uncertainties $d \in [d]$ and $\mathbf{p} \in [\mathbf{p}]$

$$\dot{\mathbf{z}} = \begin{bmatrix} L_{\mathbf{f}}h(\mathbf{x}) \\ \vdots \\ L_{\mathbf{f}}^{M-1}h(\mathbf{x}) \\ L_{\mathbf{f}}^{M}h(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} z_2 \\ \vdots \\ z_M \\ \tilde{a}(\mathbf{z}, \mathbf{p}, d) \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \tilde{b}(\mathbf{z}, \mathbf{p}) \end{bmatrix} \cdot v$$

- Note: $\vartheta_{1,M,1}$ is the differentially flat system output

Transformation into Nonlinear Controller Normal Form (4)

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- Note: $\vartheta_{1,M,1}$ is the differentially flat system output

Generalization

- If the output definition $y = \vartheta_{1,j*,1}$, $j^* < M$, is used, y is no longer the flat system output
- The states $\vartheta_{1,j,1}$, $j > j^*$, act on the dynamics as disturbances

Interval-Based Sliding Mode Control (1)

Definition of tracking error signals

- Specification of a sufficiently smooth desired output trajectory $z_{1,d}^{(j)}$, $j = 0, ..., \delta - 1 = M - 1$
- Introduction of the error signals $\tilde{z}_1^{(j)} = z_1^{(j)} - z_{1,d}^{(j)}$
- Desired operating points are located on the *sliding surface*

$$s(\tilde{\mathbf{z}}) = \tilde{z}_1^{(M-1)} + \alpha_{M-2}\tilde{z}_1^{(M-2)} + \ldots + \alpha_0\tilde{z}_1^{(0)} = 0$$

- $\alpha_0, \ldots, \alpha_{M-2}$ are coefficients of a Hurwitz polynomial of order $M - 1$

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Guaranteed stabilizing control: Lyapunov function candidate

$$V = \frac{1}{2}s^2 > 0 \quad \text{with} \quad \dot{V} = s \cdot \dot{s} < 0 \quad \text{for} \quad s \neq 0$$

Interval-Based Sliding Mode Control (2)

Guaranteed stabilization despite uncertainties: Interval formulation of
a variable-structure control law

$$[v] := \left[\frac{-\tilde{a}\left(\mathbf{z}, \mathbf{p}, d\right) + z_{1,d}^{(M)} - \alpha_{M-2}\tilde{z}_1^{(M-1)} \ldots - \alpha_0 \tilde{z}_1^{(1)}}{\tilde{b}\left(\mathbf{z}, \mathbf{p}\right)} \right.$$

$$\left. - \frac{1}{\tilde{b}\left(\mathbf{z}, \mathbf{p}\right)} \underbrace{(\eta + \beta)}_{=: \tilde{\eta} > 0} \cdot \mathrm{sign}\{s\} \right] \Bigg|_{\substack{\mathbf{p} \in [\mathbf{p}] \\ d \in [d]}}$$

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$$\left. \left. - \frac{1}{\tilde{b}\left(\mathbf{z}, \mathbf{p}\right)} \underbrace{\left(\eta + \beta\right)}_{=:\tilde{\eta} > 0} \cdot \mathrm{sign}\{s\} \right] \right|_{\substack{\mathbf{p} \,\in\, [\mathbf{p}] \\ d \,\in\, [d]}}$$

Guaranteed stabilizing control: Extraction of suitable point values

$$v := \begin{cases} \overline{v} := \sup\{[v]\} & \text{for } s \geq 0 \\ \underline{v} := \inf\{[v]\} & \text{for } s < 0 \end{cases}$$

Interval-Based Sliding Mode Control (3)

Online optimization procedure for the computation of $\mathbf{u}(t_\nu)$

Real-time optimization of

$$\left[J_\nu^{<l>} \right] = \kappa_1 \cdot \left(\left[\Delta\vartheta_\nu^{<l>} \right] \right)^2 + \kappa_2 \cdot \left(\left[\dot{m}_{CG,\nu}^{<l>} \right] \right)^2 +$$
$$\kappa_3 \cdot \left(\left[\Delta\vartheta_\nu^{<l>} \right] - [\Delta\vartheta_{\nu-1}] \right)^2 + \kappa_4 \cdot \left(\left[\dot{m}_{CG,\nu}^{<l>} \right] - [\dot{m}_{CG,\nu-1}] \right)^2$$

on a suitable rapid control prototyping hardware with a fixed sampling period $t_\nu - t_{\nu-1}$: quantification of the control effort and the variation rates of the control signals

Software implementation

Interface between SIMULINK model containing C-XSC functions and LABVIEW by means of the NI SIMULATION INTERFACE TOOLKIT

Modeling
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System Structure
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Sliding Mode Control
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Predictive Control
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Interval-Based Sliding Mode Control (4)

Block diagram of the real-time control implementation
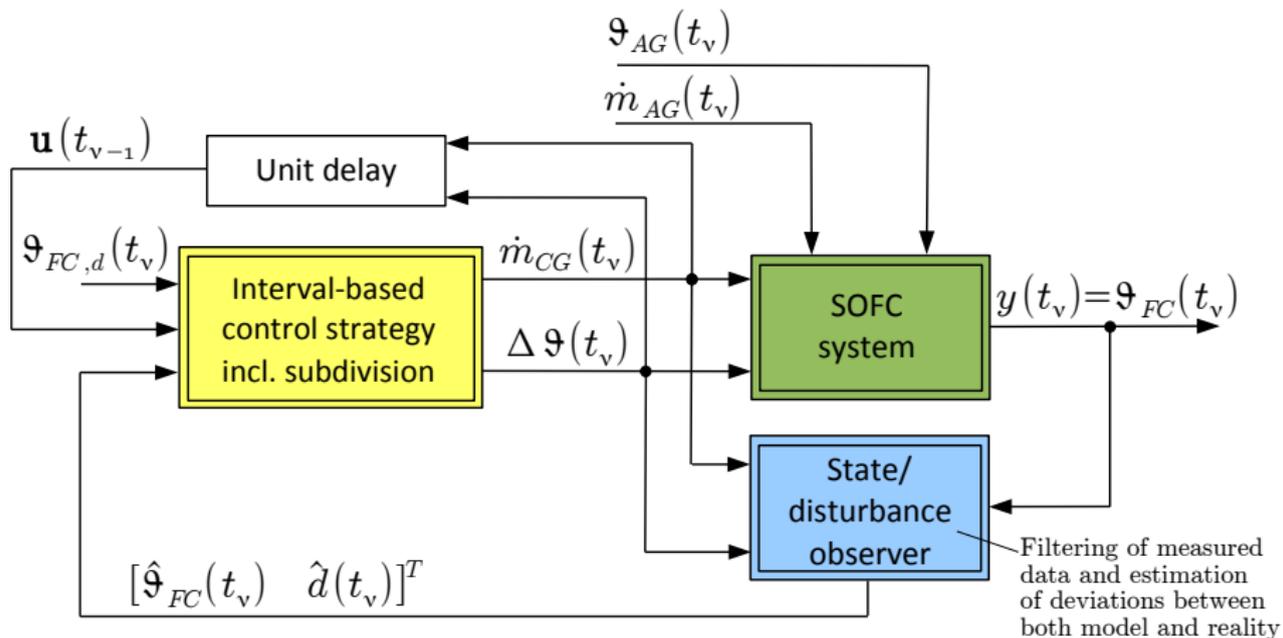
Experimental Results (1)

Preheater temperature
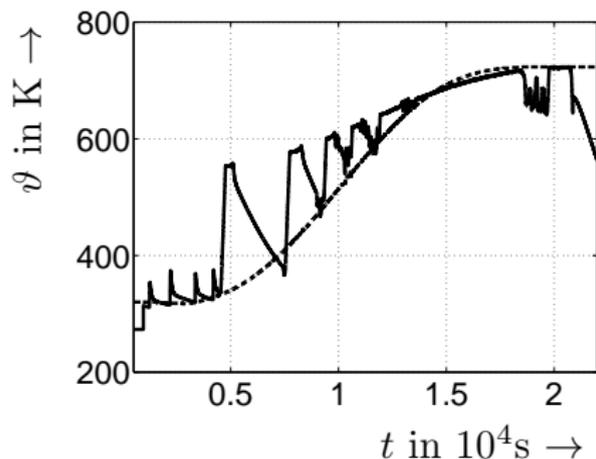
Mass flow

anode gas (ϑ_{AG}, dashed)
cathode gas (ϑ_{CG}, solid)
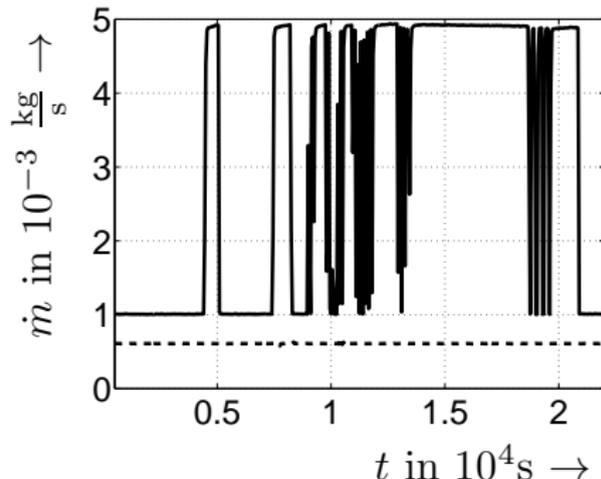
anode gas ($\dot{m}_{AG}$, dashed)
cathode gas ($\dot{m}_{CG}$, solid)

Enthalpy flow $v(t_\nu) = \dot{m}_{CG}(t_\nu) \cdot \Delta\vartheta(t_\nu)$

Modeling
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Experimental Results (2)

Stack temperatures

Control error $\vartheta_{FC,d} - \vartheta_{FC}$
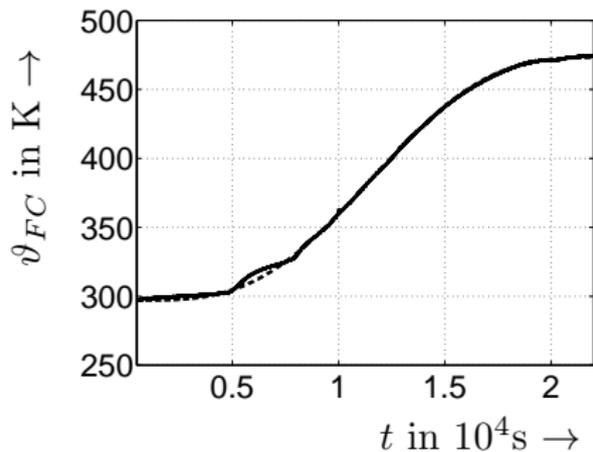
desired values ($\vartheta_{FC,d}$, dashed)
actual values (ϑ_{FC}, solid)

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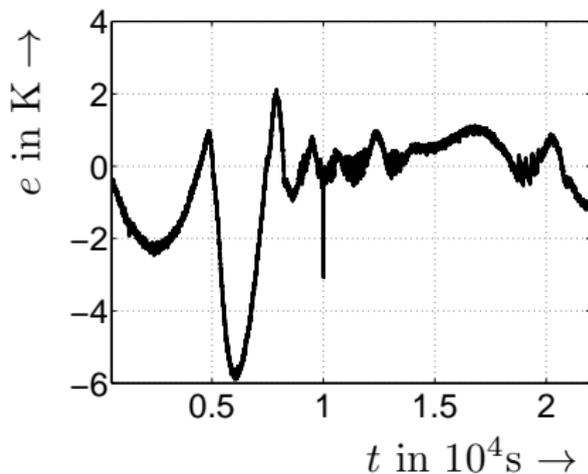
Influence of Actuator Constraints (1)
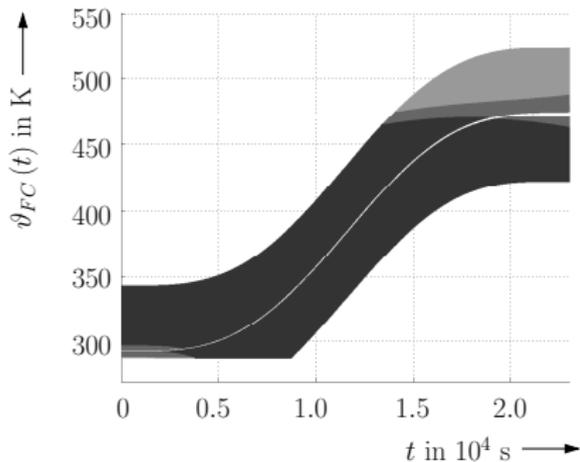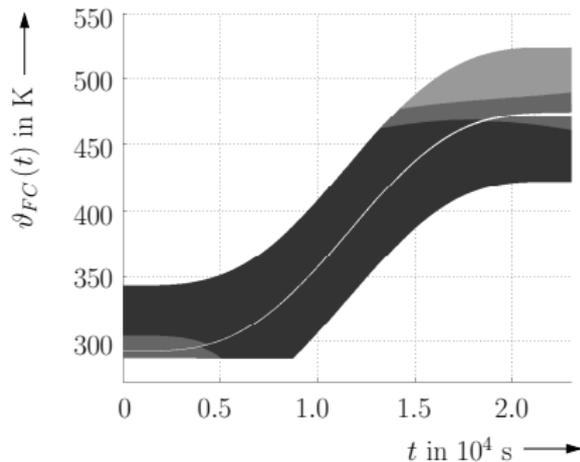
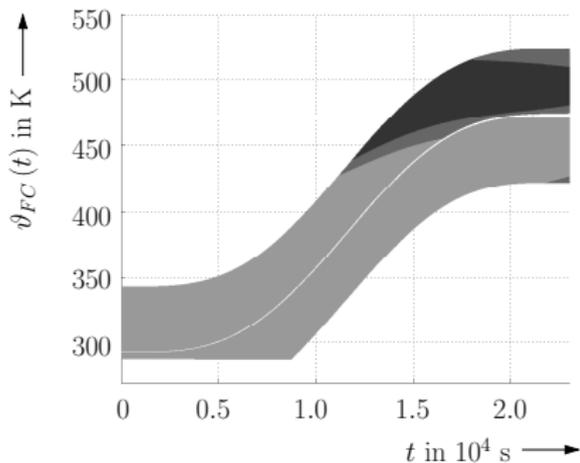$[d] = [-1 \; ; \; 1] \cdot 10^{-4} \frac{\text{K}}{\text{s}}, \; \tilde{\eta} = 0.0001$ $[d] = [-5 \; ; \; 5] \cdot 10^{-4} \frac{\text{K}}{\text{s}}, \; \tilde{\eta} = 0.0001$
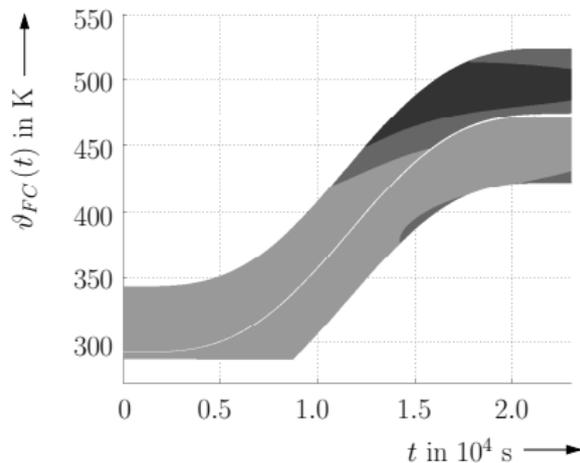
dark gray: guaranteed stabilizable
middle gray: undecided

light gray: violation of actuator constraints

Modeling
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System Structure
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Sliding Mode Control
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Predictive Control
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Conclusions
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Influence of Actuator Constraints (2)

$[d] = [-1 \; ; \; 1] \cdot 10^{-4} \, \frac{\text{K}}{\text{s}}, \; \tilde{\eta} = 0.01$ $[d] = [-5 \; ; \; 5] \cdot 10^{-4} \, \frac{\text{K}}{\text{s}}, \; \tilde{\eta} = 0.01$

dark gray: guaranteed stabilizable
middle gray: undecided

light gray: violation of actuator constraints

Interval-Based Predictive Control of Uncertain Systems (1)

Online minimization of a performance criterion via sensitivity analysis

- Alternative for the *direct computation* of the control vector $\mathbf{u}(t)$
- Online evaluation and minimization of a cost function J over a finite time horizon (using predicted state and output trajectories)
- Definition of the error measure

$$J = \sum_{\mu=\nu}^{\nu+N_p} \mathcal{D}\left(\mathbf{y}\left(t_\mu\right) - \mathbf{y}_d\left(t_\mu\right)\right)$$

- Computation of the differential sensitivity

$$\frac{\partial J}{\partial \Delta \mathbf{u}_\nu} = \sum_{\mu=\nu}^{\nu+N_p} \left(\frac{\partial \mathcal{D}\left(\zeta\right)}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{x}\left(t_\mu\right)}{\partial \Delta \mathbf{u}_\nu} + \frac{\partial \mathcal{D}\left(\zeta\right)}{\partial \Delta \mathbf{u}_\nu} \right) , \; \zeta := \mathbf{h}\left(\mathbf{x}, \mathbf{u}\right) - \mathbf{y}_d\left(t_\mu\right)$$

Interval-Based Predictive Control of Uncertain Systems (2)

Definition of a piecewise constant control signal

Direct computation of preheater temperature and mass flow for the SOFC stack according to

$$\mathbf{u}\left(t_{\nu}\right) = \mathbf{u}\left(t_{\nu-1}\right) + \Delta\mathbf{u}_{\nu} \quad \text{with} \quad \Delta\mathbf{u}_{\nu} = -\alpha \left(\frac{\partial J}{\partial \Delta\mathbf{u}_{\nu}}\right)^{+} \cdot J$$

and the optional step size control parameter $0 < \alpha < 1$

Remarks

- All required partial derivatives are evaluated by means of algorithmic differentiation (FADBAD++)
- Account for worst case errors by interval evaluation of $J \in [J]$

Interval-Based Predictive Control of Uncertain Systems (3)

Typical performance criterion

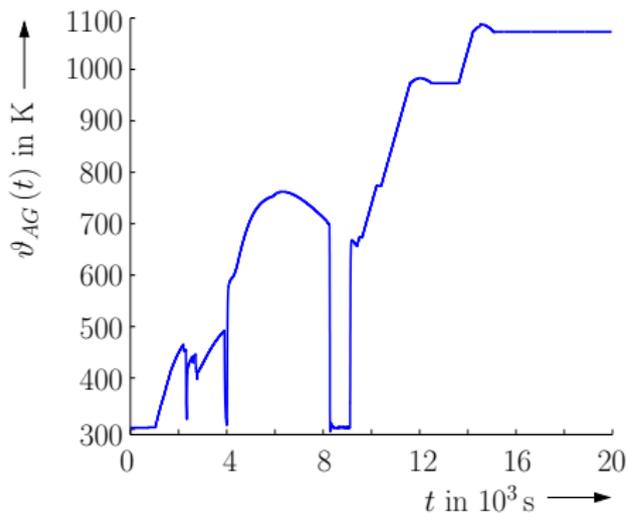- Minimization of the worst-case upper bound (using interval arithmetic)

$$\mathcal{D} = \kappa_1 \cdot (\theta_{FC} - \vartheta_{nom})^2 + \kappa_2 \cdot \frac{1}{M-1} \sum_{i=1}^{M} (\vartheta_{1,i,1} - \theta_{FC})^2$$
$$+ \kappa_3 \cdot (\dot{m}_{CG} - \dot{m}_{CG,nom})^2 + \kappa_4 \cdot (\vartheta_{CG} - \vartheta_{CG,nom})^2$$

  replaces the two-stage procedure of the sliding mode controller, where the enthalpy flow was firstly computed
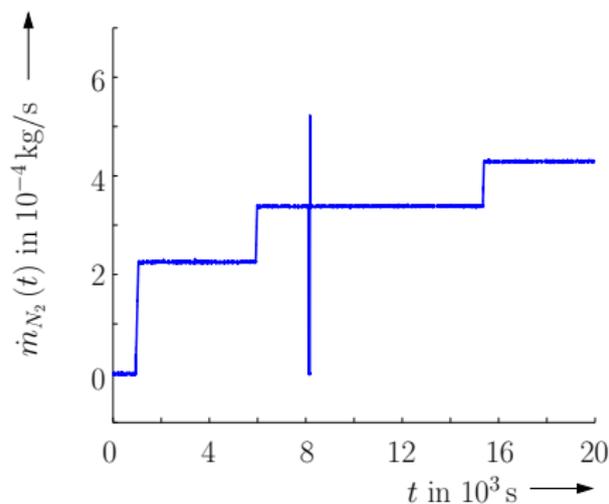
- Average cell temperature $\theta_{FC} = \frac{1}{M} \sum_{i=1}^{M} \vartheta_{1,i,1}$

- Penalization of overshoots over $\vartheta_{max} = 880\,\mathrm{K}$

Modeling
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System Structure
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Sliding Mode Control
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Predictive Control
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Conclusions
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Simulations and Experimental Results (1)
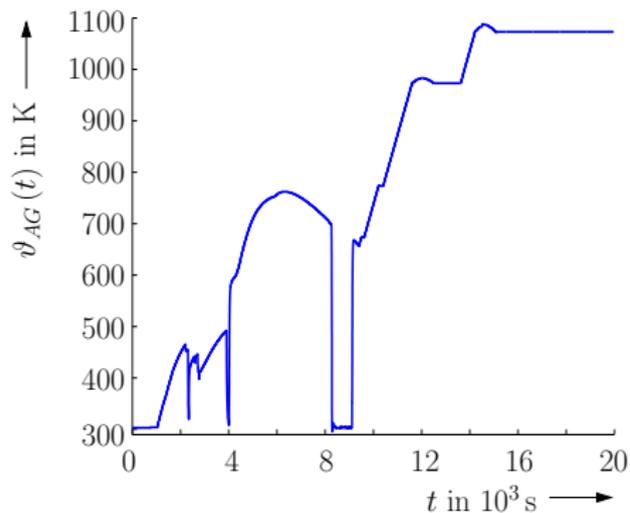
Measured anode gas temperature
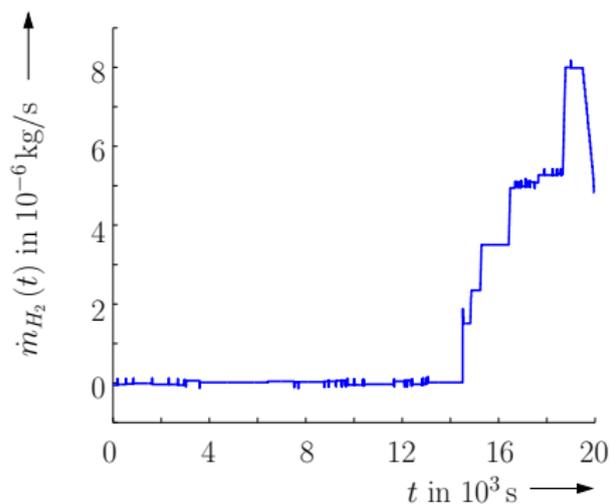
Measured mass flow of nitrogen at the anode

Simulations and Experimental Results (2)
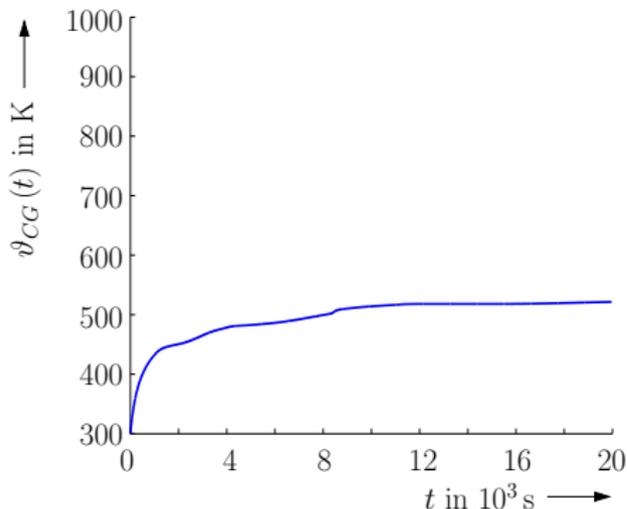
Measured anode gas temperature

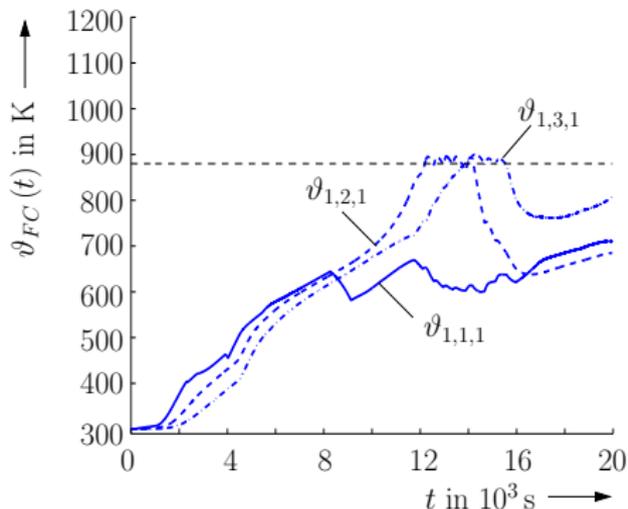Measured mass flow of hydrogen at the anode

Simulations and Experimental Results (3)

Preheater temperature at the cathode (simulated)

Temperatures in the stack module (finite volume model, simulated)

$\implies$ Necessity for an energy-based measure for the reliable detection and reduction of overestimation in the predicted state enclosures

Conclusions and Outlook on Future Work

- Control-oriented modeling of a complex thermodynamic application
- Verified parameter identification as the basis for control design
- Guaranteed stabilization of the error dynamics
- Online optimization of performance criteria (energy efficiency and lifetime)
- Real-time use of interval arithmetic and algorithmic differentiation for control purposes

Conclusions and Outlook on Future Work

- Control-oriented modeling of a complex thermodynamic application
- Verified parameter identification as the basis for control design
- Guaranteed stabilization of the error dynamics
- Online optimization of performance criteria (energy efficiency and lifetime)
- Real-time use of interval arithmetic and algorithmic differentiation for control purposes
- Extension of the control strategies by a sensitivity-based estimation of non-measurable states
- Extension of the system models by a description of the preheater dynamics
- Extension to scenarios with switchings of the output segment
- Design of interval-based extensions of backstepping controllers

Merci beaucoup pour votre attention!
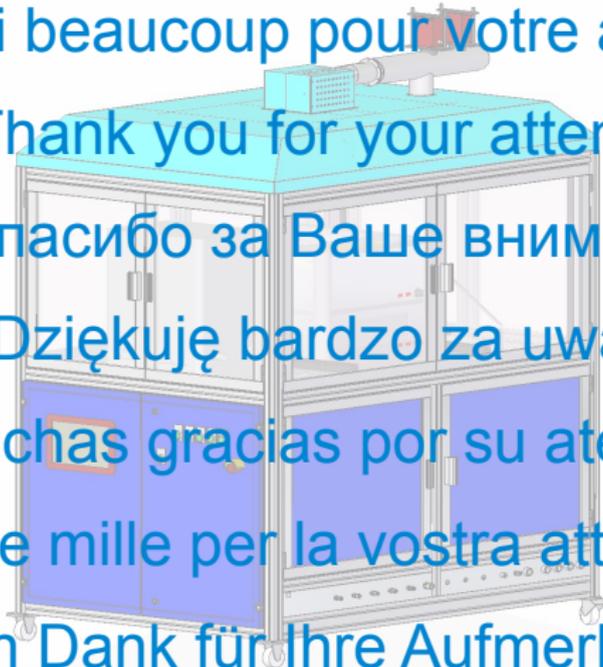
Thank you for your attention!

Спасибо за Ваше внимание!

Dziękuję bardzo za uwagę!

¡Muchas gracias por su atención!

Grazie mille per la vostra attenzione!

Vielen Dank für Ihre Aufmerksamkeit!

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