Improvements on flow/guard intersection for nonlinear hybrid reachability

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SWIM, Brest, France June 5-7, 2013

Outline

ANR-Project : MAGIC-SPS

- Goal : To develop guaranteed methods and algorithms for integrity control and preventive monitoring of systems
- Different work package :
  - WP1 : Modelling and identification of systems with bounded uncertainties ;
  - WP2 : Identifiability and diagnosability of systems with bounded uncertainties ;
  - WP3 : Preventive monitoring of continuous systems with bounded uncertainties ;
  - WP4 : Preventive monitoring of hybrid systems with bounded uncertainties ;
  - WP5 : Dissemination
- Project duration = october 2012 to december 2014
- Partners

ANR-Project : MAGIC-SPS

Our work package : WP4

* Nonlinear hybrid reachability ;
* State estimation of HDS ;
* Feasibility of fault prognosis for HDS

Outline

Why reachability ?

- **State estimation of HDS ;**

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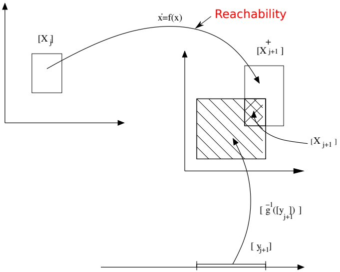
- **State estimation of HDS ;**

To estimate state, we use a classical predictor-corrector approach (Raissi, Ramdani,04).



-> **Prediction step** : computes the reachable set for the state vector.

-> **Correction step** : retains only those parts of the reachable set which are consistent with measurements and prior error bounds.

Hybrid system

Hybrid automaton (Alur, *et al.*, 95)

$$H = (\mathcal{Q}, \mathcal{D}, \mathcal{P}, \Sigma, \mathcal{A}, \mathrm{Inv}, \mathcal{F}),$$

$$\begin{aligned}
\mathrm{flow}(q): &\quad \dot{\mathbf{x}}(t) = f_q(\mathbf{x}, \mathbf{p}, t),\\
\mathrm{Inv}(q): &\quad \nu_q(\mathbf{x}(t), \mathbf{p}, t) < 0,\\
\\
e: &\quad (q \to q') = (q, \mathrm{guard}, \sigma, \rho, q'),\\
\mathrm{guard}(e): &\quad \gamma_e(\mathbf{x}(t), \mathbf{p}, t) = 0,
\end{aligned}$$

$$t_0 \leq t \leq t_N, \quad \mathbf{x}(t_0) \in \mathbb{X}_0 \subseteq \mathbb{R}^n, \quad \mathbf{p} \in \mathbb{P}$$

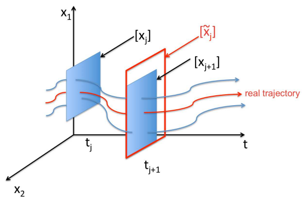
Outline

Guaranteed set integration with Taylor methods

(Moore,66) (Eijgenraam,81) (Lohner,88) (Rihm,94) (Berz,98) (Nedialkov,99)

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}, \mathbf{p}, t), \quad t_0 \leq t \leq t_N,\ \mathbf{x}(t_0) \in [\mathbf{x}_0]\,,\ \mathbf{p} \in [\mathbf{p}]$$

Time grid $\rightarrow \quad t_0 < t_1 < t_2 < \cdots < t_N$



- Analytical solution for $[\mathbf{x}](t),\ t \in [t_j, t_{j+1}]$

$$[\mathbf{x}](t) = [\mathbf{x}_j] + \sum_{i=1}^{k-1} (t - t_j)^i \mathbf{f}^{[i]}([\mathbf{x}_j], [\mathbf{p}]) + (t - t_j)^k \mathbf{f}^{[k]}([\tilde{\mathbf{x}}_j], [\mathbf{p}])$$

Guaranteed set integration with Taylor methods

(Moore,66) (Eijgenraam,81) (Lohner,88) (Rihm,94) (Berz,98) (Nedialkov,99)

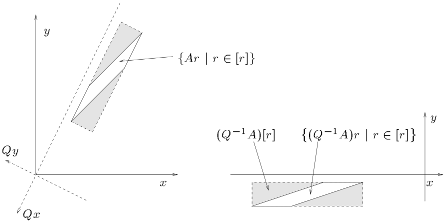
$$\dot{\mathbf{x}}(t) = f(\mathbf{x}, \mathbf{p}, t), \quad t_0 \leq t \leq t_N,\ \mathbf{x}(t_0) \in [\mathbf{x}_0]\,,\ \mathbf{p} \in [\mathbf{p}]$$

Mean-value approach

mean value forms + matrice preconditioning+ linear transforms

$$[\mathbf{x}](t) \in \left\{ \mathbf{v}(t) + \mathbf{A}^{a}(t)\mathbf{r}(t) \quad \middle| \quad \mathbf{v}(t) \in [\mathbf{v}](t),\ \mathbf{r}(t) \in [\mathbf{r}](t) \right\}.$$

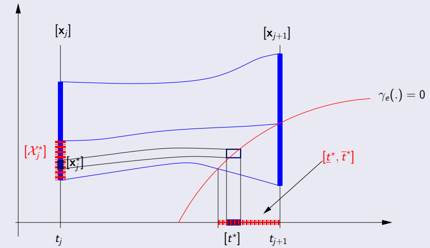
a. Two methods=QR, parralepipede



Outline

Computing flow/guards intersection

Time grid $\rightarrow \quad t_0 < t_1 < t_2 < \cdots < t_N$



Set membership guard crossing can be divided into three tasks :

- T1 : Detecting guard condition satisfaction ;
- T2 : Computing the state subset which intersects guard condition ;

  Compute $[\underline{t}^\star, \overline{t}^\star] \times [\mathcal{X}_j^\star] \Rightarrow$ Solve CSP $([t_j, t_{j+1}] \times [\mathbf{x}_j], \gamma_e \circ ([\mathbf{x}]) = 0)$
- T3 : Computing the image of the subset by the reset function.

Hybrid Reachability

Ramdani & Nedialkov, 2011

- **Interval Taylor methods**

  $\Rightarrow$ Analytical expressions for the boundaries of the continuous flows,

  $\Rightarrow$ Controlling wrapping effect

- **Interval constraint propagation techniques**

  $\Rightarrow$ Solve event detection/localization problems as an CSP

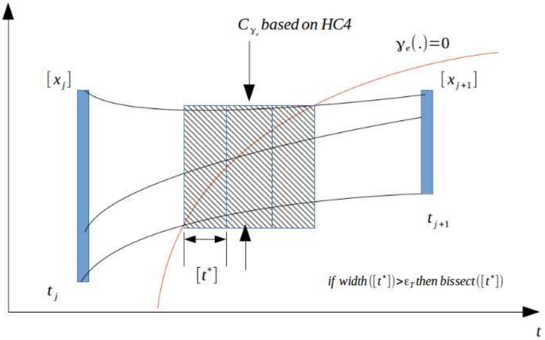
This talk

- Guaranteed relaxation of event detection/localization problems ;
- Flow/sets intersection with IBEX (G. Chabert) [a] CSP solver ;
- Test this new method on some example.

a. http ://www.emn.fr/z-info/ibex/

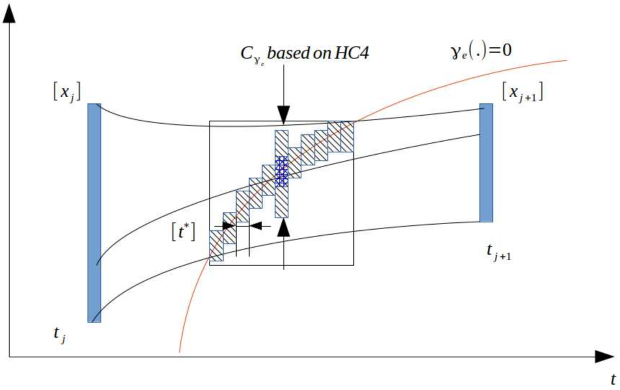
Outline

Our method



- Build for each guard condition a contractor based on HC4Revise ;
- Bisection is performed only in the direction of the time variable ;
- The domain of the state variables are contracted upon each subinterval of time obtained by the operation of bisection.

Our method



Event $e = q \to q'$ occurs if :

- width($[t^*]$$\leq \varepsilon_T$;
- width($[X^*] \leq \varepsilon_Z$).

Outline

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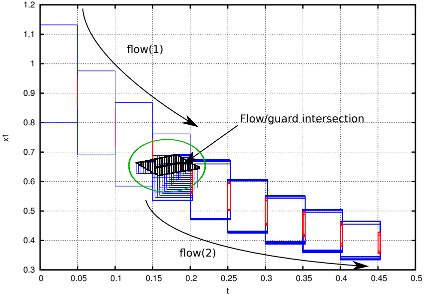
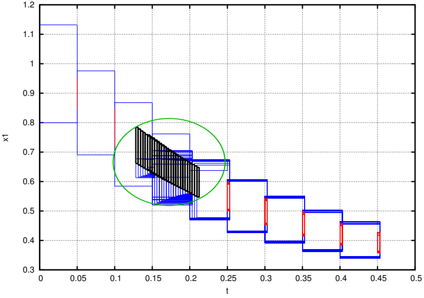
Consider a hybrid dynamical system (Brusselator), $q = 1, 2$ and one jump $e = 1 \to 2$ given by :

$$\begin{cases} \text{flow}(1) : f_1 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 - (b_1 + 1)x_1 + a_1 x_1^2 x_2 \\ b_1 x_1 - a_1 x_1^2 x_2 \end{pmatrix} \\ \text{inv}(1) : \nu_1(x_1, x_2) = -4x_1 + x_2 + 2 \\ \text{flow}(2) : f_2 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 - (b_2 + 1)x_1 + a_2 x_1^2 x_2 \\ b_2 x_1 - a_2 x_1^2 x_2 \end{pmatrix} \\ \text{inv}(2) : \nu_2(x_1, x_2) = -\nu_1(x_1, x_2) \\ \text{guard}(1) : \gamma_1(x_1, x_2) = \nu_1(x_1, x_2) \\ \text{reset}(1) : \rho_1(x_1, x_2) = (\alpha_1 x_1, \alpha_2 x_2) \end{cases} \tag{1}$$

with $\alpha_1 = \alpha_2 = (1; 1)$, $a_1 = 1.5$, $a_2 = 3.5$, $b_1 = 1$, $b_2 = 3.5$ and $x_0 \in [1, 1.1] \times [0.1, 0.3]$. We took for this simulation a constant integration time step $h = 0.05$,$\varepsilon_Z = 0.2$ and $\varepsilon_T = 0.005$.



(a) x_1 flow with HC4

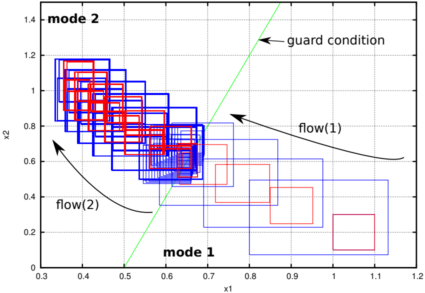


(b) x_1 flow without HC4

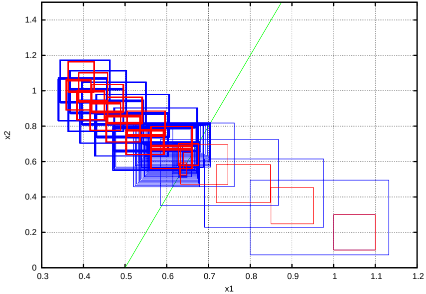
TCG[1] with contractor=0.148s

TCG without contractor=0.192s

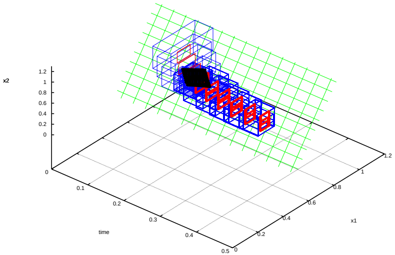
1. TOTAL TIME CROSSING GUARD CONDITION



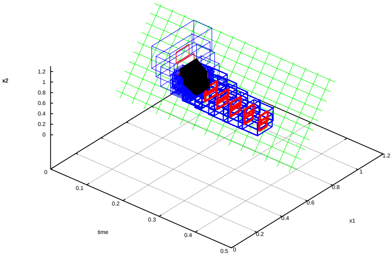
(c) phase plan $x_2 \times x_1$ with HC4



(d) phase plan $x_2 \times x_1$ without HC4



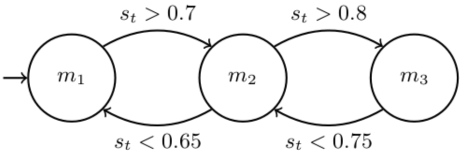
(e) 3D with HC4



(f) 3D without HC4

Outline

Evaluation on Benchmarks : Vehicle Model [2]



$$\frac{dx}{dt} = vc_t; \frac{dy}{dt} = vs_t; \frac{dv}{dt} = u_1$$

$$\frac{dc_t}{dt} = \sigma v^2 s_t; \frac{ds_t}{dt} = -\sigma v^2 c_t; \frac{d\sigma}{dt} = u_2$$

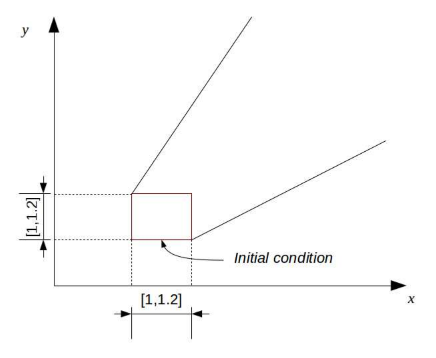
$$x \in [1, 1.2] \quad y \in [1, 1.2] \quad v \in [0.8, 0.81]$$

$$s_t \in [0.7, 0.71] \quad c_t \in [0.7, 0.71] \quad \sigma = [0, 0.05]$$

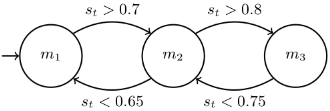
2. Bench proposed by Sriram Sankaranarayanan and Xin Chen

Goal of this benchmark ?

Find all positions reached by vehicule over t$\in [0, 10]$

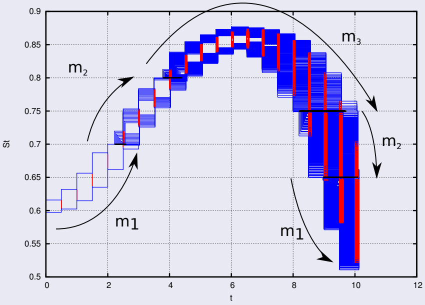


$x \in [1, 1.2]$ $y \in [1, 1.2]$ $v \in [0.8, 0.81]$
$s_t \in [0.7, 0.71]$ $c_t \in [0.7, 0.71]$ $\sigma = [0, 0.05]$

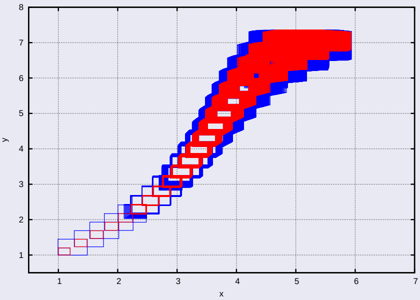


Results : Vehicle Model

$\sigma = [0, 0.01]$ and h=0.5



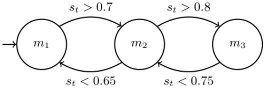
(g) St×t



(h) Y×X space

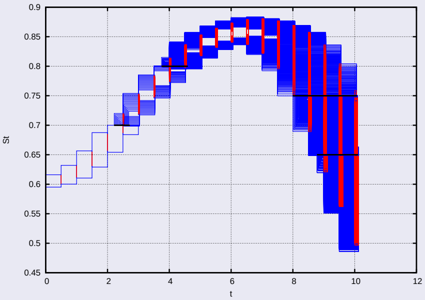
CPU times=1m7.504s

$$x \in [1, 1.2] \quad y \in [1, 1.2] \quad v \in [0.8, 0.81]$$
$$s_t \in [0.6, 0.61] \quad c_t \in [0.7, 0.71] \quad \sigma = [0, 0.01]$$

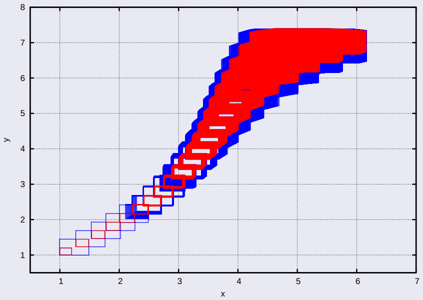


Results : Vehicle Model

$\sigma = [0, 0.02]$ and h=0.5



(i) St×t

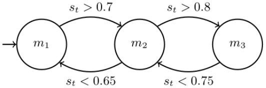


(j) Y×X space

CPU times=3m50.944s with $h = 0.5$

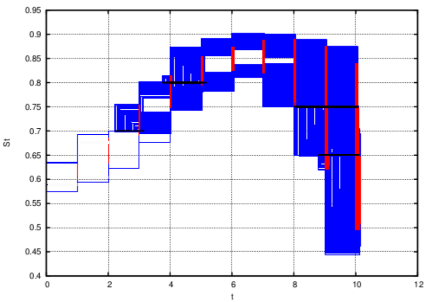
$x \in [1, 1.2]$ $\quad y \in [1, 1.2]$ $\quad v \in [0.8, 0.81]$

$s_t \in [0.6, 0.61]$ $\quad c_t \in [0.7, 0.71]$ $\quad \sigma = [0, 0.02]$

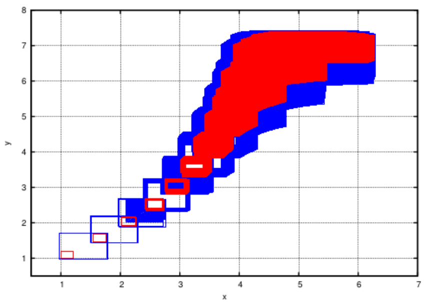


Results : Vehicle Model

$\sigma = [0, 0.05]$ and h=1



(k) St×t

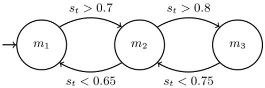


(l) Y×X space

CPU times=13m10.604s with $h = 1$

$x \in [1, 1.2] \quad y \in [1, 1.2] \quad v \in [0.8, 0.81]$

$s_t \in [0.6, 0.61] \quad c_t \in [0.7, 0.71] \quad \sigma = [0, 0.05]$



Outline

Concluding remarks

Conclusion

- Minimize intersection of flow/guard ;
- Positive impact on computation times.

Future works : to develop scalable methods

$\rightarrow$ Investigation on hybrid reachability methods without using bisection ;

$\rightarrow$ Merging boxes without over-approximation