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Model Order Reduction of Interval Systems for Mixed Methods

By

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CONTENTS

AIM OF WORK:

- Extension and development of existing model order reduction techniques using interval systems
- Controller design based on interval systems reduced model.
- To develop algorithms for controller reduction of interval systems.

INTRODUCTION:

- *Model Order Reduction* (MOR) is a branch of systems and control theory, which studies the properties of **dynamical systems** in application for **reducing their complexity, while preserving their input-output behavior.**
- *Interval systems* : many systems the coefficients are constant but uncertain within a finite range. Such systems are classified as interval systems.

- B. Bandyopadhyay, O. Ismail, and R. Gorez, "Routh Pade approximation for interval systems," *IEEE Trans. Autom. Control*, vol. 39, pp. 2454–2456, Dec1994.
- Dolgin, Y., and Zeheb, E., 'On Routh Pade model reduction of interval systems', *IEEE Trans. Autom. Control*, 48 (9), 1610–1612, 2003.
- Shih-Feng Yang,'Comments on the computation of interval Routh approximants', *IEEE Trans. Autom. Control*, 50 (2), 273-274, 2005.
- Dolgin, Y., 'Author's Reply," *IEEE Trans. Autom. Control*, 50 (2), 274-275, 2005.

PROBLEM STATEMENT

Let the transfer function of a higher order interval systems

$$G_n(s)=\frac{\left[c_{21}^-,c_{21}^+\right]+\left[c_{22}^-,c_{22}^+\right]s+....+\left[c_{2n}^-,c_{2n}^+\right]s^{n-1}}{\left[c_{11}^-,c_{11}^+\right]+\left[c_{12}^-,c_{12}^+\right]s+....+\left[c_{1,n+1}^-,c_{1,n+1}^+\right]s^{n}}$$

The reduced order model of a transfer function be

$$R_k(s)=\frac{N_k(s)}{D_k(s)}$$

ARITHMETIC RULES

The rules of interval arithmetic are

Let [e , f] and [g , h] be two intervals

Addition:

[e , f]+[g , h] = [e + g , f + h]

Subtraction

[e , f]-[g , h] = [e - h , f - g]

Multiplication

[e , f] x [g , h] = [Min (eg , eh, fg , fh), Max (eg , eh, fg , fh)]

Division

$$\frac{[e,f]}{[g,h]} = [e,f] \times \left[\frac{1}{h},\frac{1}{g}\right]$$

PROPOSED EXTENTION OF AVAILABLE METHODS:

Proposed extensions are as follows

- Differentiation method and Pade approximation, Factor division , Cauer second form.
- The stability of these methods had been verified by Kharitnov thereom.

From these methods it has been shown that the use of the *mixed methods* is superior to the use of simplified methods.

- D. Kranthi Kumar, S. K. Nagar and J.P. Tiwari, "New algorithm for model order reduction of interval systems", *Bonfring International Journal of Data Mining*, Vol. 3, No. 1, pp. 6-11, March 2013

DIFFERENTIATION METHOD

Differentiation method: (Algorithm)

1) High order transfer function.
2) Reciprocal transformation
3) Differentiate the transfer function
4) Second reciprocal transformation
5) Reduced order transfer function.

The iteration of differentiation depends upon (n-k)

PADE APPROXIMATION

Case2: Pade approximation

Determination of the numerator polynomial of the reduced model by Pade approximation.

$$\frac{\left[c_{21}^-,c_{21}^+\right]+\left[c_{22}^-,c_{22}^+\right]s+....+\left[c_{2n}^-,c_{2n}^+\right]s^{n-1}}{\left[c_{11}^-,c_{11}^+\right]+\left[c_{12}^-,c_{12}^+\right]s+....+\left[c_{1,n+1}^-,c_{1,n+1}^+\right]s^{n}}=\frac{\left[d_{21}^-,d_{21}^+\right]+\left[d_{22}^-,d_{22}^+\right]s+....+\left[d_{2k}^-,d_{2k}^+\right]s^{k-1}}{\left[d_{11}^-,d_{11}^+\right]+\left[d_{12}^-,d_{12}^+\right]s+....+\left[d_{1,k+1}^-,d_{1,k+1}^+\right]s^{k}}$$

Rewriting the above equation

$$\left(\left[c_{21}^-,c_{21}^+\right].\left[d_{11}^-,d_{11}^+\right]\right)+\left(\left[c_{22}^-,c_{22}^+\right].\left[d_{11}^-,d_{11}^+\right]+\left[c_{21}^-,c_{21}^+\right].\left[d_{12}^-,d_{12}^+\right]\right)s+.....+\left(\left[c_{2n}^-,c_{2n}^+\right].\left[d_{1,k+1}^-,d_{1,k+1}^+\right]\right)s^{n-1+k}$$

$$=\left(\left[d_{21}^-,d_{21}^+\right].\left[c_{11}^-,c_{11}^+\right]\right)+\left(\left[d_{22}^-,d_{22}^+\right].\left[c_{11}^-,c_{11}^+\right]+\left[d_{21}^-,d_{21}^+\right].\left[c_{12}^-,c_{12}^+\right]\right)s+.....+\left(\left[d_{2k}^-,d_{2k}^+\right].\left[c_{1,n+1}^-,c_{1,n+1}^+\right]\right)s^{k-1+n}$$

PADE APPROXIMATION (CONT….)

Equating the coefficients of the above equation

$$\left(\left[c_{21}^{-},c_{21}^{+}\right].\left[d_{11}^{-},d_{11}^{+}\right]\right)=\left(\left[d_{21}^{-},d_{21}^{+}\right].\left[c_{11}^{-},c_{11}^{+}\right]\right)$$

$$\left(\left[c_{22}^{-},c_{22}^{+}\right].\left[d_{11}^{-},d_{11}^{+}\right]+\left[c_{21}^{-},c_{21}^{+}\right].\left[d_{12}^{-},d_{12}^{+}\right]\right)=\left(\left[d_{22}^{-},d_{22}^{+}\right].\left[c_{11}^{-},c_{11}^{+}\right]+\left[d_{21}^{-},d_{21}^{+}\right].\left[c_{12}^{-},c_{12}^{+}\right]\right)$$

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$$\left(\left[c_{2n}^{-},c_{2n}^{+}\right].\left[d_{1,k+1}^{-},d_{1,k+1}^{+}\right]\right)=\left(\left[d_{2k}^{-},d_{2k}^{+}\right].\left[c_{1,n+1}^{-},c_{1,n+1}^{+}\right]\right)$$

FACTOR DIVISION METHOD

CASE 3:

Determination of the numerator coefficients of the order reduced model by using factor division method:

Any method of reduction which relies upon calculating the reduced denominator first and then the numerator, where has already been calculated

$$G_n(s) = \frac{N(s)D_k(s) \big/ D(s)}{D(s)}$$

$$N(s)D_k(s) = \left[u_{11}^-, u_{11}^+\right] + \left[u_{12}^-, u_{12}^+\right]s + ...$$

$$\frac{N(s)D_k(s)}{D(s)} = \frac{\left[u_{11}^-, u_{11}^+\right] + \left[u_{12}^-, u_{12}^+\right]s + ...}{\left[c_{11}^-, c_{11}^+\right] + \left[c_{12}^-, c_{12}^+\right]s + ...}$$

FACTOR DIVISION METHOD (CONT......)

$$\left[\alpha_{11}^{-},\alpha_{11}^{+}\right]=\frac{\left[u_{11}^{-},u_{11}^{+}\right]}{\left[c_{11}^{-},c_{11}^{+}\right]}\left\{\begin{array}{l}\left[u_{11}^{-},u_{11}^{+}\right]\left[u_{12}^{-},u_{12}^{+}\right].....\\\left[c_{11}^{-},c_{11}^{+}\right]\left[c_{12}^{-},c_{12}^{+}\right].....\end{array}\right\}$$

$$\left[\alpha_{12}^{-},\alpha_{12}^{+}\right]=\frac{\left[r_{11}^{-},r_{11}^{+}\right]}{\left[c_{11}^{-},c_{11}^{+}\right]}\left\{\begin{array}{l}\left[r_{11}^{-},r_{11}^{+}\right]\left[r_{12}^{-},r_{12}^{+}\right].....\\\left[c_{11}^{-},c_{11}^{+}\right]\left[c_{12}^{-},c_{12}^{+}\right].....\end{array}\right\}$$

....

$$\left[\alpha_{1,k-2}^{-},\alpha_{1,k-2}^{+}\right]=\frac{\left[x_{11}^{-},x_{11}^{+}\right]}{\left[c_{11}^{-},c_{11}^{+}\right]}\left\{\begin{array}{l}\left[x_{11}^{-},x_{11}^{+}\right]\left[x_{12}^{-},x_{12}^{+}\right]\\\left[c_{11}^{-},c_{11}^{+}\right]\left[c_{12}^{-},c_{12}^{+}\right]\end{array}\right\}$$

$$\left[\alpha_{1,k-1}^{-},\alpha_{1,k-1}^{+}\right]=\frac{\left[y_{11}^{-},y_{11}^{+}\right]}{\left[c_{11}^{-},c_{11}^{+}\right]}\left\{\begin{array}{l}\left[y_{11}^{-},y_{11}^{+}\right]\\\left[c_{11}^{-},c_{11}^{+}\right]\end{array}\right\}$$

FRACTION DIVISION METHOD (CONT….)

$$\left[r_{1,i}^{-}, r_{1,i}^{+}\right] = \left[u_{1,i+1}^{-} u_{1,i+1}^{+}\right] - \left[\alpha_{11}^{-}, \alpha_{11}^{+}\right]\left[c_{1,i+1}^{-}, c_{1,i+1}^{+}\right]; \text{Where i} = 0,\ 1,\ 2,\ \ldots \text{k} - 2$$

$$\left[s_{1,i}^{-}, s_{1,i}^{+}\right] = \left[r_{1,i+1}^{-} r_{1,i+1}^{+}\right] - \left[\alpha_{12}^{-}, \alpha_{12}^{+}\right]\left[c_{1,i+1}^{-}, c_{1,i+1}^{+}\right]; \text{Where i} = 0,\ 1,\ 2,\ \ldots \text{k} - 3$$

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$$\left[y_{1,i}^{-}, y_{1,i}^{+}\right] = \left[x_{11}^{-}, x_{11}^{+}\right] - \left[\alpha_{1,k-2}^{-}, \alpha_{1,k-2}^{+}\right]\left[c_{11}^{-}, c_{11}^{+}\right]$$

The reduced transfer function is

$$R_k(s) = \frac{\left[\alpha_{11}^{-}, \alpha_{11}^{+}\right] + \left[\alpha_{12}^{-}, \alpha_{12}^{+}\right]s + \ldots.}{D_k(s)}$$

CAUER SECOND FORM

CASE 4: Determination of numerator coefficients of reduced order model by using Cauer second form

Coefficient values from Cauer second form $\left[h_i^-, h_i^+\right]$

(i = 1 , 2 , ... k) are evaluated by forming Routh array as

$$\left[h_1^-, h_1^+\right] = \frac{\left[c_{11}^-, c_{11}^+\right]}{\left[c_{21}^-, c_{21}^+\right]} \begin{Bmatrix} \left[c_{11}^-, c_{11}^+\right]\left[c_{12}^-, c_{12}^+\right]\ldots.. \\ \left[c_{21}^-, c_{21}^+\right]\left[c_{22}^-, c_{22}^+\right]\ldots.. \end{Bmatrix}$$

$$\left[h_2^-, h_2^+\right] = \frac{\left[c_{21}^-, c_{21}^+\right]}{\left[c_{31}^-, c_{31}^+\right]} \begin{Bmatrix} \left[c_{21}^-, c_{21}^+\right]\left[c_{22}^-, c_{22}^+\right]\ldots.. \\ \left[c_{31}^-, c_{31}^+\right]\left[c_{32}^-, c_{32}^+\right]\ldots.. \end{Bmatrix}$$

....

CAUER SECOND FORM (CONT ….)

The first two rows are copied from the original system numerator and denominator coefficients and rest of the elements are calculated by using well known Routh approximation.

$$\left[c_{i,j}^{-}, c_{i,j}^{+}\right] = \left[c_{i-2,j+1}^{-}, c_{i-2}^{+}\right] - \left[h_{i-2}^{-}, h_{i-2}^{+}\right]\left[c_{i-1,j+1}^{-}, c_{i-1,j+1}^{+}\right]$$

Where i = 3,4 , ... and j = 1,2, ….

$$\left[h_i^{-}, h_i^{+}\right] = \frac{\left[c_{i,1}^{-}, c_{i,1}^{+}\right]}{\left[c_{i+1}^{-}, c_{i+1}^{+}\right]}$$

The inverse Routh array is constructed as

$$\left[d_{i+1,1}^{-}, d_{i+1,1}^{+}\right] = \frac{\left[d_{i,1}^{-}, d_{i,1}^{+}\right]}{\left[h_i^{-}, h_i^{+}\right]}$$

CAUER SECOND FORM (CONT ….)

Where i= 1, 2 , ..., k and k ≤ n.

Also

$$\left[d^{-}_{i+1,\,j+1}, d^{+}_{i+1,\,j+1}\right] = \frac{\left[d^{-}_{i,\,j+1}, d^{+}_{i,\,j+1}\right] - \left[d^{-}_{i+2,\,j}, d^{+}_{i+2,\,j}\right]}{\left[h^{-}_{i}, h^{+}_{i}\right]}$$

INTEGRAL SQUARE ERROR

The integral square error (ISE) between the transient response of higher order system (HOS) and lower order system (LOS) is determined to compare different approaches of model reduction.

$$ISE = \int_0^\infty \left[y(t) - y_r(t) \right]^2 dt$$

Where, y(t) and $y_r(t)$

are the unit step responses of original system and reduced order system.

NUMERICAL EXAMPLE

Example: Consider a third order system described by the transfer function [13]

$$G_3(s)=\frac{[2,3]s^2+[17.5,18.5]s+[15,16]}{[2,3]s^3+[17,18]s^2+[35,36]s+[20.5,21.5]}$$

Method 1: Reduction by using Differentiation method

For getting second order model, number of times to be differentiated is n-k= 3-2=1

$$D_2(s)=[4.25,4.5]s^2+[17,18]s+[15.375,16.125]$$

$$N_2(s)=[5.8333,6.1667]s+[10.0005,10.6676]$$

NUMERICAL EXAMPLE (CONT…)

The reduced second order model is

$$R_2(s) = \frac{[5.8333, 6.1667]s + [10.0005, 10.6676]}{[4.25, 4.5]s^2 + [17, 18]s + [15.375, 16.125]}$$

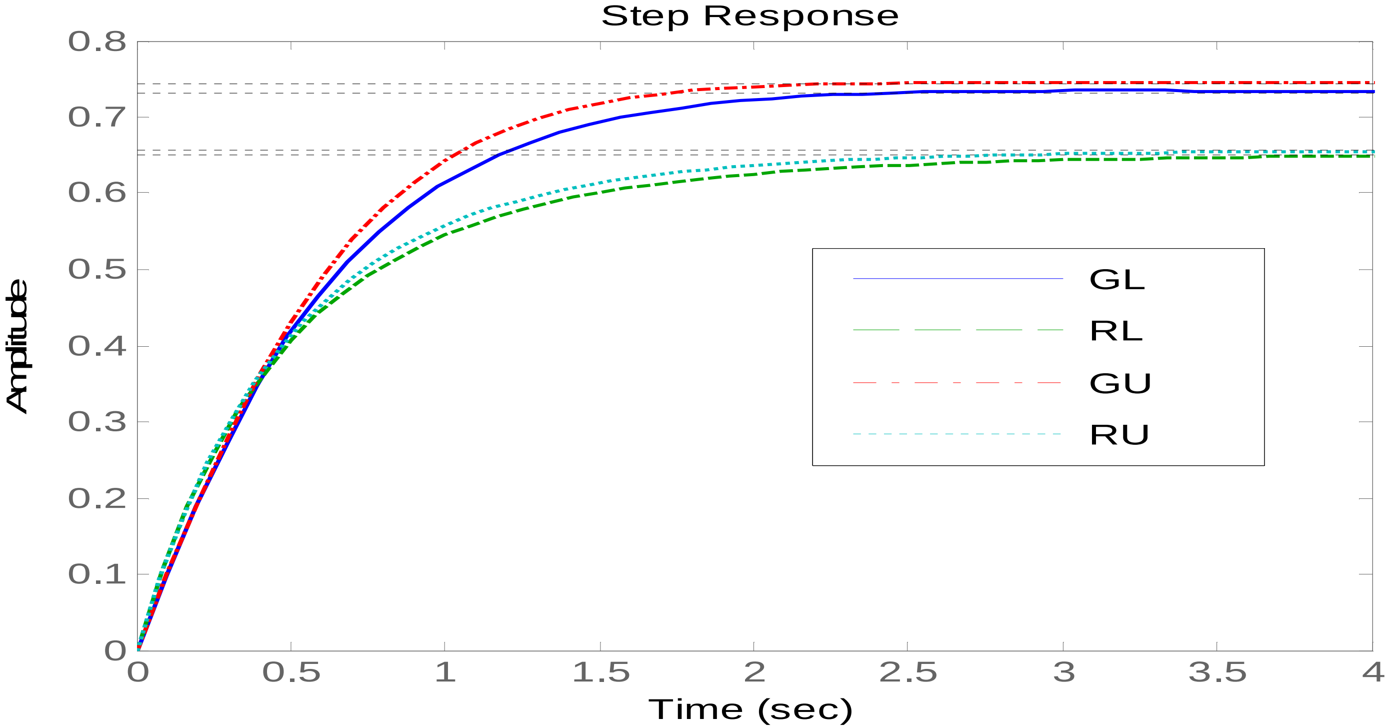

Fig. 1: Step Response of original model and reduced model using Differentiation method

NUMERICAL EXAMPLE (CONT…)

Method 2:

Reduction by using Differentiation and Pade approximation

$$D_2(s) = [4.25, 4.5]s^2 + [17,18]s + [15.375,16.125]$$

Numerator is reduced by Pade approximation

$$\left[d_{21}^-, d_{21}^+\right] = [10.7267, 12.5854]$$

$$\left[d_{22}^-, d_{22}^+\right] = [3.3018, 10.2867]$$

The reduced transfer function is

$$R_2(s) = \frac{[3.3018, 10.2867]s + [10.7267, 12.5854]}{[4.25, 4.5]s^2 + [17,18]s + [15.375, 16.125]}$$

Method 3:

Reduction by using Differentiation and Factor division method

$$D_2(s) = [4.25, 4.5]s^2 + [17,18]s + [15.375,16.125]$$

Numerator is reduced by Factor division method

$$\frac{N(s)D_2(s)}{D(s)} = \frac{[230.625, 258] + [531.0562, 586.3125]s + ...}{[20.5, 21.5] + [35, 36]s + ...}$$

$$N_2(s) = [3.4809, 10.7884]s + [10.7267, 12.5854]$$

The reduced transfer function is

$$R_2(s) = \frac{[3.4809, 10.7884]s + [10.7267, 12.5854]}{[4.25, 4.5]s^2 + [17,18]s + [15.375,16.125]}$$

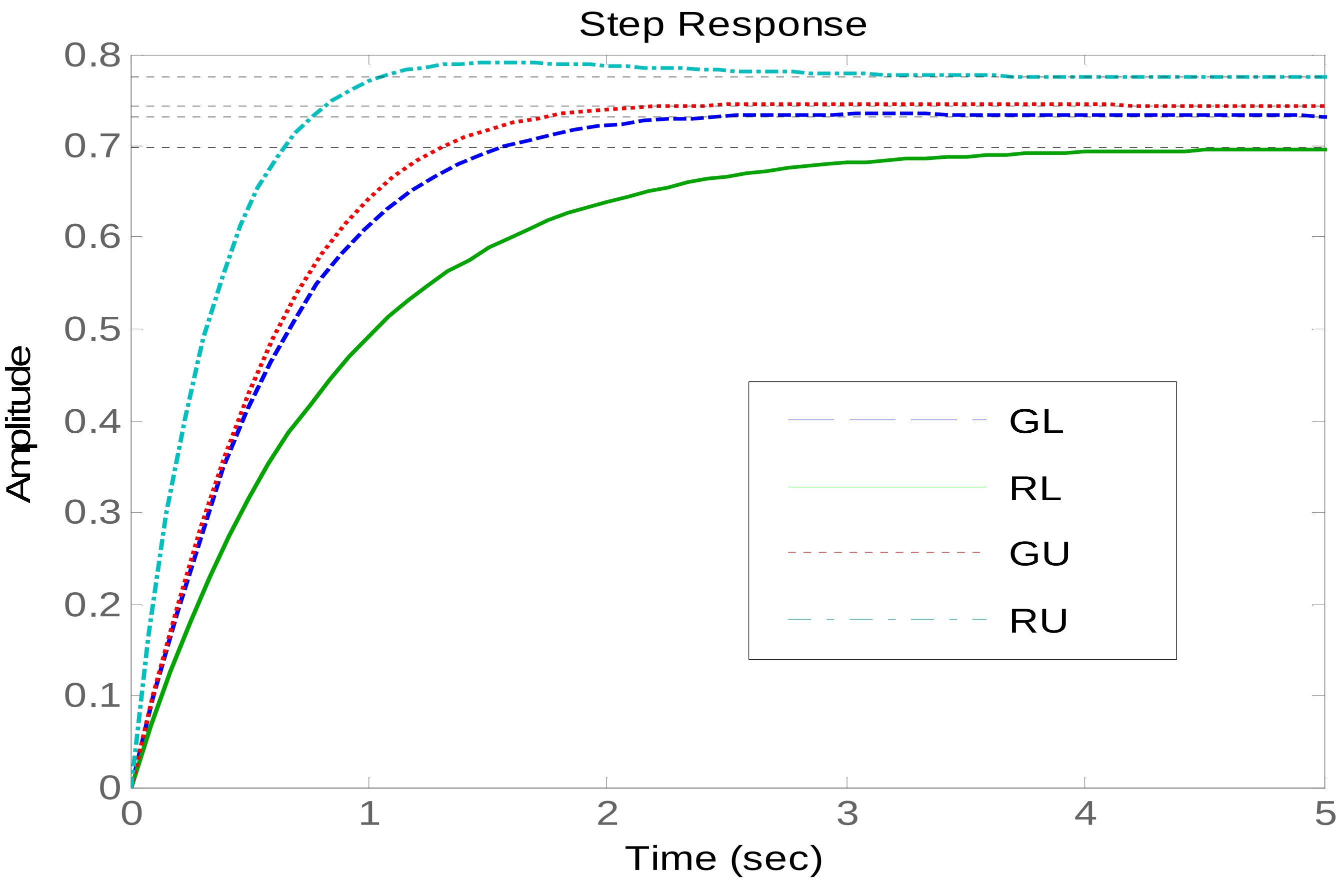

Fig. 2: Step Response of original model and reduced model using Differentiation method and Factor division method

Method 4: Reduction by using differentiation method and Cauer second form

Numerator is reduced by Cauer second form

$$\left[h_1^-, h_1^+\right] = [1.2812, 1.4333]$$

$$\left[h_2^-, h_2^+\right] = [1.1046, 1.8859]$$

$$\left[d_{21}^-, d_{21}^+\right] = [10.7271, 12.5856]$$

$$\left[d_{22}^-, d_{22}^+\right] = [4.2604, 9.6099]$$

$$N_2(s) = \left[4.2604, 9.6099\right]s + \left[10.7271, 12.5856\right]$$

Denominator is reduced by differentiation method

$$D_2(s) = \left[4.25, 4.5\right]s^2 + \left[17, 18\right]s + \left[15.375, 16.125\right]$$

The reduced order transfer function

$$R_2(s) = \frac{[4.2604, 9.6099]s + [10.7271, 12.5856]}{[4.25, 4.5]s^2 + [17, 18]s + [15.375, 16.125]}$$

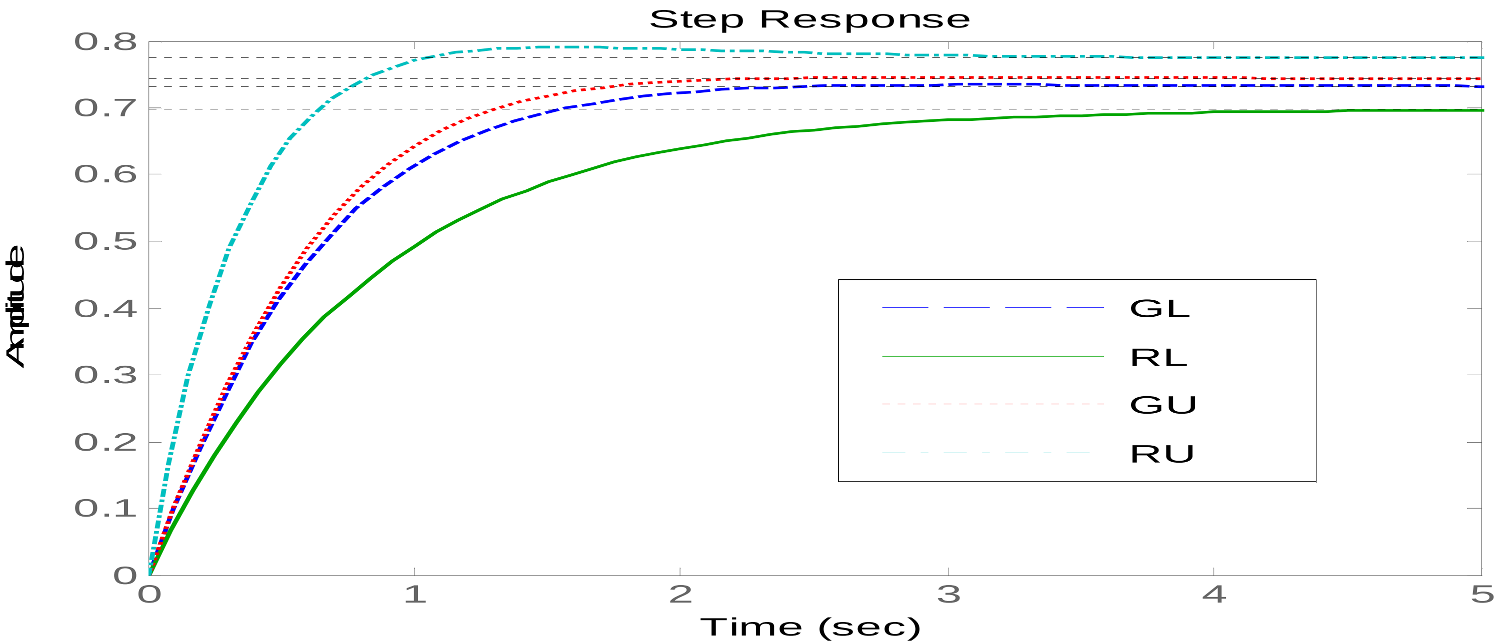

Fig. 3: Step Response of original model and reduced model using Differentiation method and Cauer Second form

Comparison of Reduced Order Models

Method of Order reduction	**ISE for lower limit errL**	ISE for upper limit errU
Differentiation	0.0531	0.0617
Differentiation and Pade approximation	0.0094	0.0105
Differentiation and factor division	0.0094	0.0074
Differentiation and Cauer second form	0.0094	0.0073
G.V.K. Sastry [14]	0.2256	0.0095

CONCLUSIONS :

- In this report differentiation method is mixed with Pade approximation, Factor division method and Cauer second form are employed for order reduction.
- The proposed method guarantees the stability of reduced model if the original system is stable. This proposed method is conceptually simple and comparable with other available methods.

REFERENCES:

1) Aoki, M., 'Control of Large-Scale Dynamic Systems by Aggregation', *IEEE Trans. Autom. Control*, 13, 246-253, 1986.

2) Shamash, Y., 'Stable Reduced Order Models Using Pade Type Approximation', *IEEE Trans. Autom. Control*, 19, 615-616, 1974.

3) Hutton, M.F., and Friedland, B., 'Routh Approximation for Reducing Order of Linear Time Invariant System', *IEEE Trans. Autom. Control*, 20, 329-337, 1975.

4) Shamash, Y., 'Model Reduction Using Routh Stability Criterion and The Pade Approximation', *International Journal of Control*, 21, 475-484, 1975.

5) Krishnamurthy, V., and Seshadri, V., 'Model Reduction Using Routh Stability Criterion', *IEEE Trans. Autom. Control*, 23, 729-730, 1978.

6) Glover, K., 'All Optimal Hankel-norm Approximations of Linear Multivariable Systems and their Error Bounds', *International Journal of Control*, 39 (6), 1115-1193, 1984.

7) Sinha, N.K., and Kuszta, B., 'Modelling and Identification of Dynamic Systems', *New York Van Nostrand Reinhold*, Ch.8, 133- 163, 1983.

8) G. Parmar, " A Mixed method for large-scale systems modelling using eigen spectrum analysis and Cauer second form," *IETE Journal of Research*, vol. 53, no 2, pp. 93, 2007.

9) Wan Bai-Wu, "Linear model reduction is using Mihailov Criterion and Pade approximation technique," *International Journal of Control*, vol 33, no. 6, pp. 1073, 1981.

10) V. L. Kharitonov, "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations," *Differentsial'nye Uravneniya*, vol. 14, pp. 2086–2088, 1978.

11) S. P. Bhattacharyya, "Robust stabilization against structured perturbations," (Lecture Notes In Control and Information Sciences). New York: Springer-Verlag. 1987.

12) B. Bandyopadhyay, O. Ismail, and R. Gorez, "Routh Pade approximation for interval systems," *IEEE Trans. Autom. Control*, vol. 39, pp. 2454–2456, Dec1994.

13) B. Bandyopadhyay, "γ-δ Routh approximations for interval systems," *IEEE Trans. Autom. Control*, vol. 42, pp. 1127-1130, 1997.

14) G V K Sastry, G R Raja Rao and P M Rao, "Large scale interval system modelling using Routh approximants," *Electronics Letters*, vol 36, no 8, pp. 768. April 2000.

15) Dolgin, Y., and Zeheb, E., ‘On Routh Pade model reduction of interval systems’, *IEEE Trans. Autom. Control,* 48 (9), 1610–1612, 2003.

16) Hwang, C., and Yang, S.F., ‘Comments on the computation of interval Routh approximants’, *IEEE Trans. Autom. Control*, 44 (9), 1782–1787, 1999.

17) Shih-Feng Yang,‘Comments on the computation of interval Routh approximants’, *IEEE Trans. Autom. Control*, 50 (2), 273-274, 2005.

18) Dolgin, Y., ‘Author’s Reply,” *IEEE Trans. Autom. Control*, 50 (2), 274-275, 2007.

19) Ismail, O., and Bandyopadhyay, B., ‘Model order reduction of linear interval systems using Pade approximation’, *IEEE International symposium on circuit and systems*, 1995.

20) Saraswathi, G., ‘A mixed method for order reduction of interval systems’, *International Conference on Intelligent and Advanced Systems*, 1042-1046, 2007.

21) Singh, V. P., and Chandra, D., ‘Routh approximation based model reduction using series expansion of interval systems’, *IEEE International conference on power, control & embedded systems (ICPCES),* 1, 1-4, 2010.

22) Singh, V. P., and Chandra, D., ‘Model reduction of discrete interval system using dominant poles retention and direct series expansion method’, *IEEE 5th International power engineering and optimization conference (PEOCO),* 1, 27-30, 2011.

23)Younseok Choo., ‘A Note on discrete interval system reduction via retention of Dominant Poles’, *International Journal of Control, Automation, and System*, 5 (2), 208-211, 2007.

24)D. Kranthi Kumar, S. K. Nagar and J.P. Tiwari, “Model order reduction of interval systems using modified Routh approximation and factor division method,” *35th National System Conference (NSC)*, IIT Bhubaneswar, India, 2011.

25)Hansen, E., ‘Interval arithmetic in matrix computations’, Part I, *SIAM J. Numerical Anal*., 308-320, 1965.

26)D. Kranthi Kumar, S. K. Nagar and J.P. Tiwari, “New algorithm for model order reduction of interval systems”, *Bonfring International Journal of Data Mining*, Vol. 3, No. 1, pp. 6-11, March 2013

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