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An Algorithm Approach for Model Order Reduction of Discrete Time Interval Systems

By

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CONTENTS

AIM OF WORK:

- Extension and development of existing model order reduction techniques using interval systems
- Controller design based on interval systems reduced Model.
- To develop algorithms for controller reduction of interval systems.

INTRODUCTION:

- *Model Order Reduction* (MOR) is a branch of systems and control theory, which studies the properties of **dynamical systems** in application for **reducing their complexity, while preserving their input-output behavior.**
- *Interval systems* : many systems the coefficients are constant but uncertain within a finite range. Such systems are classified as interval systems.

PROBLEM STATEMENT

Let the transfer function of a higher order interval systems

$$G_n(z) = \frac{\left[c_{21}^-, c_{21}^+\right] + \left[c_{22}^-, c_{22}^+\right] z + \ldots. + \left[c_{2n}^-, c_{2n}^+\right] z^{n-1}}{\left[c_{11}^-, c_{11}^+\right] + \left[c_{12}^-, c_{12}^+\right] z + \ldots. + \left[c_{1,n+1}^-, c_{1,n+1}^+\right] z^{n}}$$

The reduced order model of a transfer function be

$$R_k(z) = \frac{N_k(z)}{D_k(z)}$$

ARITHMETIC RULES

The rules of interval arithmetic are

Let [e , f] and [g , h] be two intervals

Addition:

[e , f]+[g , h] = [e + g , f + h]

Subtraction

[e , f]-[g , h] = [e - h , f - g]

Multiplication

[e , f] x [g , h] = [Min (eg , eh, fg , fh), Max (eg , eh, fg , fh)]

Division

$$\frac{[e,f]}{[g,h]} = [e,f] \times \left[\frac{1}{h}, \frac{1}{g}\right]$$

PROPOSED EXTENTION OF AVAILABLE METHODS:

Proposed extensions are as follows

- Alpha truncation method and Pade approximation method.
- The stability of these methods had been verified by Kharitnov thereom.

From these methods it has been shown that the use of the *mixed methods* is superior to the use of simplified methods.

- D. Kranthi kumar, S. K. Nagar and J. P. Tiwari, "**Order Reduction of Interval Systems Using Alpha and Factor Divison Method**", 36th National Systems Conference (NSC-2012), Dec 6-8, 2012 at Annamali University, India. & Lecture Notes in Electrical Engineering (**springer**) Volume 188, 2013, pp 249-259

ALPHA TRUNCATION METHOD

Alpha truncation method: (Algorithm)

1) Replace $z = \frac{1+w}{1-w}$
2) Determine reciprocal transformation of denominator D(w).
3) Construct α table corresponding to $\hat{D}(w)$
4) Determine the reduced order denominator by Routh convergent $\hat{D}(w) = A_k(w)$
5) Reciprocate the above step.
6) Substitute $w = \frac{z-1}{z+1}$

ALPHA TRUNCATION METHOD (CONT….)

ALPHA TABLE

	$a_0^0 = \left[p_{11}^-, p_{11}^+ \right]$ $a_2^0 = \left[p_{13}^-, p_{13}^+ \right]$ ●●●●●●●●●● $a_0^1 = \left[p_{12}^-, p_{12}^+ \right]$ $a_2^1 = \left[p_{14}^-, p_{14}^+ \right]$ ●●●●●●●●●●
$\alpha_1 = \dfrac{a_0^0}{a_0^1}$ $\alpha_2 = \dfrac{a_0^1}{a_0^2}$ $\alpha_3 = \dfrac{a_0^2}{a_0^3}$ ●●●●●●●●●●	$a_0^2 = a_2^0 - \alpha_1 a_2^1$ $a_2^2 = a_4^0 - \alpha_1 a_4^1$ ●●●●●●●●●● $a_0^3 = a_2^1 - \alpha_2 a_2^2$ ●●●●●●●●●● $a_0^4 = a_2^2 - \alpha_3 a_2^3$ ●●●●●●●●●● ●●●●●●●●●●

ALPHA TRUNCATION METHOD (CONT….)

The general formula for reducing order is

$$A_1(w) = \alpha_1 w + 1$$

$$A_2(w) = \alpha_1 \alpha_2 w^2 + \alpha_2 w + 1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$A_k(w) = \alpha_1 A_{k-1} w + A_{k-2} w$$

PADE APPROXIMATION

Pade approximation:

Determination of the numerator polynomial of the reduced model by Pade approximation.

$$\frac{\left[c_{21}^{-},c_{21}^{+}\right]+\left[c_{22}^{-},c_{22}^{+}\right]z+....+\left[c_{2n}^{-},c_{2n}^{+}\right]z^{n-1}}{\left[c_{11}^{-},c_{11}^{+}\right]+\left[c_{12}^{-},c_{12}^{+}\right]z+....+\left[c_{1,n+1}^{-},c_{1,n+1}^{+}\right]z^{n}}=\frac{\left[d_{21}^{-},d_{21}^{+}\right]+\left[d_{22}^{-},d_{22}^{+}\right]z+....+\left[d_{2k}^{-},d_{2k}^{+}\right]z^{k-1}}{\left[d_{11}^{-},d_{11}^{+}\right]+\left[d_{12}^{-},d_{12}^{+}\right]z+....+\left[d_{1,k+1}^{-},d_{1,k+1}^{+}\right]z^{k}}$$

Rewriting the above equation

$$\left(\left[c_{21}^{-},c_{21}^{+}\right].\left[d_{11}^{-},d_{11}^{+}\right]\right)+\left(\left[c_{22}^{-},c_{22}^{+}\right].\left[d_{11}^{-},d_{11}^{+}\right]+\left[c_{21}^{-},c_{21}^{+}\right].\left[d_{12}^{-},d_{12}^{+}\right]\right)z+.....+\left(\left[c_{2n}^{-},c_{2n}^{+}\right].\left[d_{1,k+1}^{-},d_{1,k+1}^{+}\right]\right)z^{n-1+k}$$
$$=\left(\left[d_{21}^{-},d_{21}^{+}\right].\left[c_{11}^{-},c_{11}^{+}\right]\right)+\left(\left[d_{22}^{-},d_{22}^{+}\right].\left[c_{11}^{-},c_{11}^{+}\right]+\left[d_{21}^{-},d_{21}^{+}\right].\left[c_{12}^{-},c_{12}^{+}\right]\right)z+.....+\left(\left[d_{2k}^{-},d_{2k}^{+}\right].\left[c_{1,n+1}^{-},c_{1,n+1}^{+}\right]\right)z^{k-1+n}$$

PADE APPROXIMATION (CONT….)

Equating the coefficients of the above equation

$$\left(\left[c_{21}^{-},c_{21}^{+}\right].\left[d_{11}^{-},d_{11}^{+}\right]\right)=\left(\left[d_{21}^{-},d_{21}^{+}\right].\left[c_{11}^{-},c_{11}^{+}\right]\right)$$

$$\left(\left[c_{22}^{-},c_{22}^{+}\right].\left[d_{11}^{-},d_{11}^{+}\right]+\left[c_{21}^{-},c_{21}^{+}\right].\left[d_{12}^{-},d_{12}^{+}\right]\right)=\left(\left[d_{22}^{-},d_{22}^{+}\right].\left[c_{11}^{-},c_{11}^{+}\right]+\left[d_{21}^{-},d_{21}^{+}\right].\left[c_{12}^{-},c_{12}^{+}\right]\right)$$

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$$\left(\left[c_{2n}^{-},c_{2n}^{+}\right].\left[d_{1,k+1}^{-},d_{1,k+1}^{+}\right]\right)=\left(\left[d_{2k}^{-},d_{2k}^{+}\right].\left[c_{1,n+1}^{-},c_{1,n+1}^{+}\right]\right)$$

NUMERICAL EXAMPLE

Example: Consider a third order system described by the transfer function [13]

$$G_3(z) = \frac{[1,2]z^2 + [3,4]z + [8,10]}{[6,6]z^3 + [9,9.5]z^2 + [4.9,5]z + [0.8,0.85]}$$

Denominator is reduction by using Alpha truncation method

Step1: Substitute $z = \frac{1+w}{1-w}$

$$D(w) = [0.55,1.2]w^3 + [5.9,6.65]w^2 + [19.45,20.2]w + [20.7,21.35]$$

Step 2: Reciprocal of D(w) we get

NUMERICAL EXAMPLE (CONT….)

$$\hat{D}(w) = [20.7, 21.35]w^3 + [19.45, 20.2]w^2 + [5.9, 6.65]w + [0.55, 1.2]$$

Step 3: Construct α table

	$[20.7, 21.35]$	$[5.9, 6.65]$
	$[19.45, 20.2]$	$[0.55, 1.2]$
$\alpha_1 = [1.0247, 1.0977]$	$[4.5828, 6.0864]$	
$\alpha_2 = [3.1956, 4.4078]$	$[0.55, 1.2]$	
$\alpha_3 = [3.819, 11.0662]$		

Step 4: Denominator for second order

$$\hat{D}_2(w) = [3.2745, 4.8384]w^2 + [3.1956, 4.078]w + [1,1]$$

NUMERICAL EXAMPLE (CONT….)

Step 4: Reciprocal of $\hat{D}_2(w)$

$$D_2(w) = [1,1]w^2 + [3.1956, 4.078]w + [3.2745, 4.8384]$$

Step 5: substitute $w = \dfrac{z-1}{z+1}$

$$D_2(z) = [7.4701, 10.2462]z^2 + [4.549, 7.6768]z + [-0.1333, 2.6428]$$

Numerator is reduced by Pade approximation

$$\frac{[8,10] + [3,4]z + [1,2]z^2}{[0.8, 0.85] + [4.9, 5]z + [9, 9.5]z^2 + [6,6]z^3}$$

$$= \frac{\left[d_0^-, d_0^+\right] + \left[d_1^-, d_1^+\right]z}{[-0.1333, 2.6428] + [4.549, 7.6768]z + [7.4701, 10.2462]z^2}$$

NUMERICAL EXAMPLE (CONT….)

$$\left[d_0^-, d_0^+\right] = \left[-1.6662, 33.035\right];$$

$$\left[d_1^-, d_1^+\right] = \left[-161.642, 119.5877\right]$$

Step 6: The reduced transfer function

$$R_2(z) = \frac{\left[-161.6452, 119.5877\right]z + \left[-1.6662, 33.035\right]}{\left[7.4701, 10.2462\right]z^2 + \left[4.549, 7.6768\right]z + \left[-0.1332, 2.6428\right]}$$

NUMERICAL EXAMPLE (CONT…)

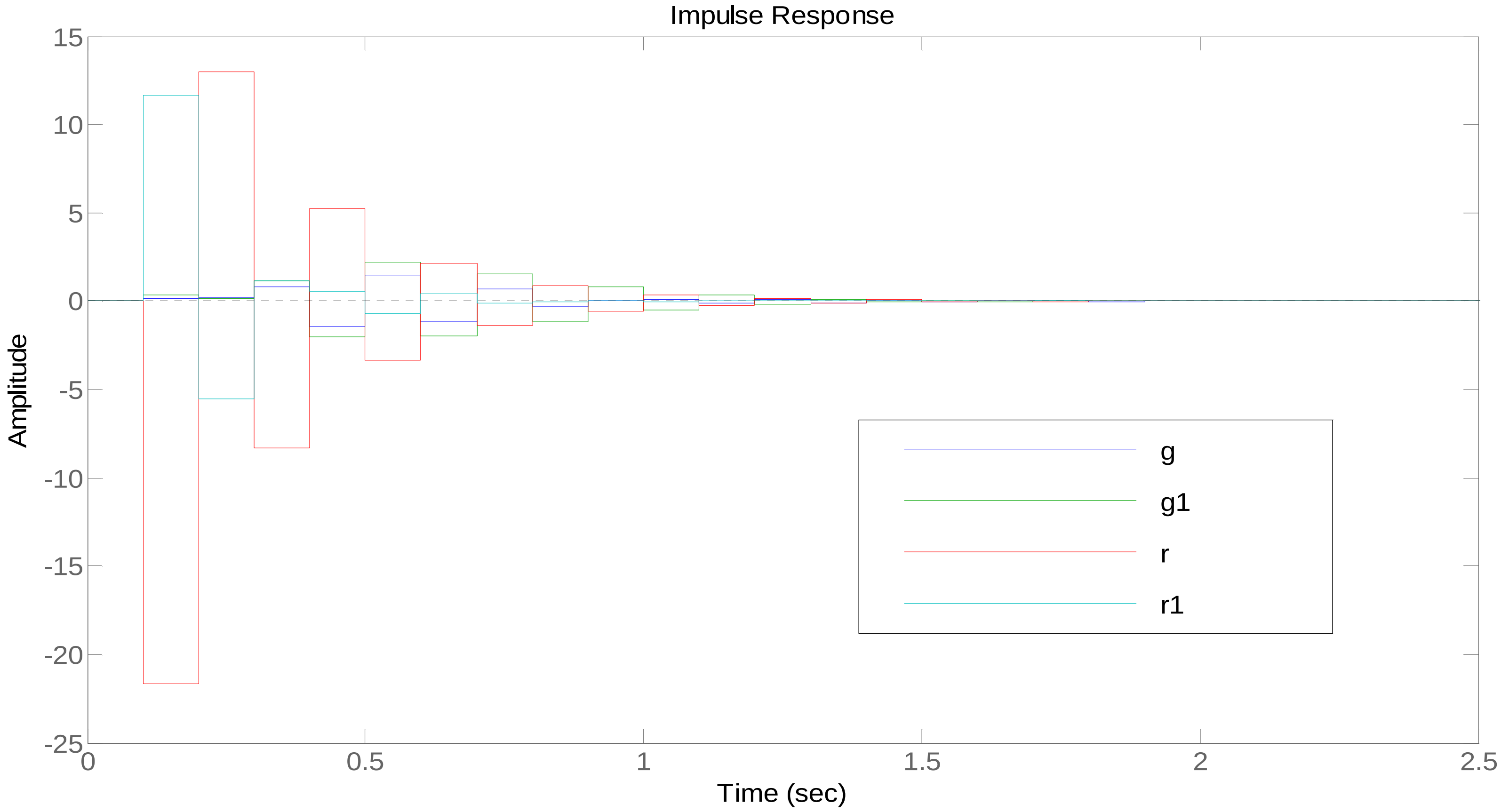

Fig. 1: Impulse Response of original model and reduced model

CONCLUSIONS :

- The proposed method guarantees the stability of reduced model if the original system is stable. This proposed method is conceptually simple and comparable with other available methods.

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THANK YOU