Efficient Solution of a Class of Universally Quantified Constraints

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We present an algorithm for finding $x_1, \ldots, x_r$ such that

$$\begin{array}{c} \forall\, (y_1, \ldots, y_s) \in B_1 \,.\, \phi_1(x_1, \ldots, x_r, y_1, \ldots y_s) \\ \wedge \cdots \wedge \\ \forall\, (y_1, \ldots, y_s) \in B_n \,.\, \phi_n(x_1, \ldots, x_r, y_1, \ldots y_s) \end{array}$$

where each of the $B_1, \ldots, B_n$ is a box in $\mathbb{R}^s$ and each of the $\phi_1, \ldots, \phi_n$ is a Boolean combination of inequalities. Moreover, for each $\phi_1, \ldots, \phi_n$

- at most one of the inequalities contains a variable from $y_1, \ldots, y_s$ (all of them may contain $x_1, \ldots, x_r$), and
- this one inequality contains the variables $y_1, \ldots, y_s$ only linearly (but may contain $x_1, \ldots, x_r$ nonlinearly).

Such an algorithm can, for example, be used to

- compute Lyapunov-like functions for finding a basin of attraction of an given ODE wrt. to a target region [2], or to
- prove termination of loops in computer programs [1].

The algorithm uses interval methods in combination with linear programming relaxations. We will also report on some first results on speeding up the algorithm using splitting heuristics (joint work with Milan Hladík).

References

[1] Byron Cook, Andreas Podelski, and Andrey Rybalchenko. Proving program termination. *Commun. ACM*, 54(5):88–98, May 2011.

[2] Stefan Ratschan and Zhikun She. Providing a basin of attraction to a target region of polynomial systems by computation of Lyapunov-like functions. *SIAM Journal on Control and Optimization*, 48(7):4377–4394, 2010.