Preview on IOLAVABE – the iSAT-ODE layer around VNODE-LP and bracketing enclosures – a nonlinear reachability library*

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Computing reachable sets for hybrid or continuous systems is an important step when one addresses verification, synthesis or monitoring tasks. A key issue lies in the calculation of the reachable set for continuous dynamics with nonlinear models when uncertainty is present in either parameters, control, or disturbance inputs. A nice way to solve this issue is to use guaranteed tools for numerical set integration of Ordinary Differential Equations (ODEs). VNODE-LP, Validated Numerical ODE through Literate Programming [1, 2], is a state-of-the-art software for computing bounds on the solution of an initial value problem (IVP) for ODEs. VNODE-LP is a fixed-order, variable-stepsize solver, based on interval Taylor series and the Hermite-Obreschkoff [3] methods. In general, it is suitable for computing bounds on the solution of an IVP ODE with point initial conditions or interval initial conditions with a sufficiently small width.

When the ODE system is nonlinear and the starting point (or the parameter vector) of the system under analysis is a wide interval vector, the enclosures returned by VNODE-LP may however diverge after a few computation steps. One way to address this shortcoming, while deriving guaranteed results, is to use the bracketing approach introduced in [4], which relies on the classical Müller's existence theorem [5] and a rule based on the sign of the elements of the system's Jacobian matrix [6]. Bracketing enclosures are computed as the solution of bracketing dynamical systems, whose enclosures are also obtained through VNODE-LP, but with now a point IVP ODE. A relaxed version of the bracketing approach [7] exists which is applicable only to monotone order-preserving systems. Experimental comparison of the bracketing enclosure method and VNODE-LP revealed that the two approaches have complementary strengths and should be combined [8]. However, there was no tool for obtaining these bracketing systems in an automatic and transparent way, these systems were to be derived manually.

In this talk, we give a preview on the IOLAVABE (iSAT-ODE Layer Around VNODE-LP and Bracketing Enclosures) library, which encapsulates these two approaches. While IOLAVABE has originally been developed as a part of the iSAT-ODE solver [8], we have only recently extracted it to form a stand-alone library that we hope will be useful in a wider range of contexts.

*This work has been supported by the German Research Council DFG within SFB/TR 14 "Automatic Verification and Analysis of Complex Systems" (`www.avacs.org`), by the French National Research Agency under contract ANR 2011 INS 006 04 "MAGIC-SPS" (`projects.laas.fr/ANR-MAGIC-SPS`), and by the Natural Sciences and Engineering Research Council of Canada.

In IOLAVABE, the bracketing systems are built automatically on the fly, through the FADBAD++ [9] automatic differentiation package. To accelerate and simplify re-computation of bracketing enclosures or solution bounds for the IVP ODE for arbitrary subranges of each integration time step without costly reinitialization of the solver, IOLAVABE also stores and reuses the Taylor coefficients computed by VNODE-LP.

This talk reports main features and illustrates the performance of the IOLAVABE library.

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