# Image-based UAV cooperative localization using Interval Analysis

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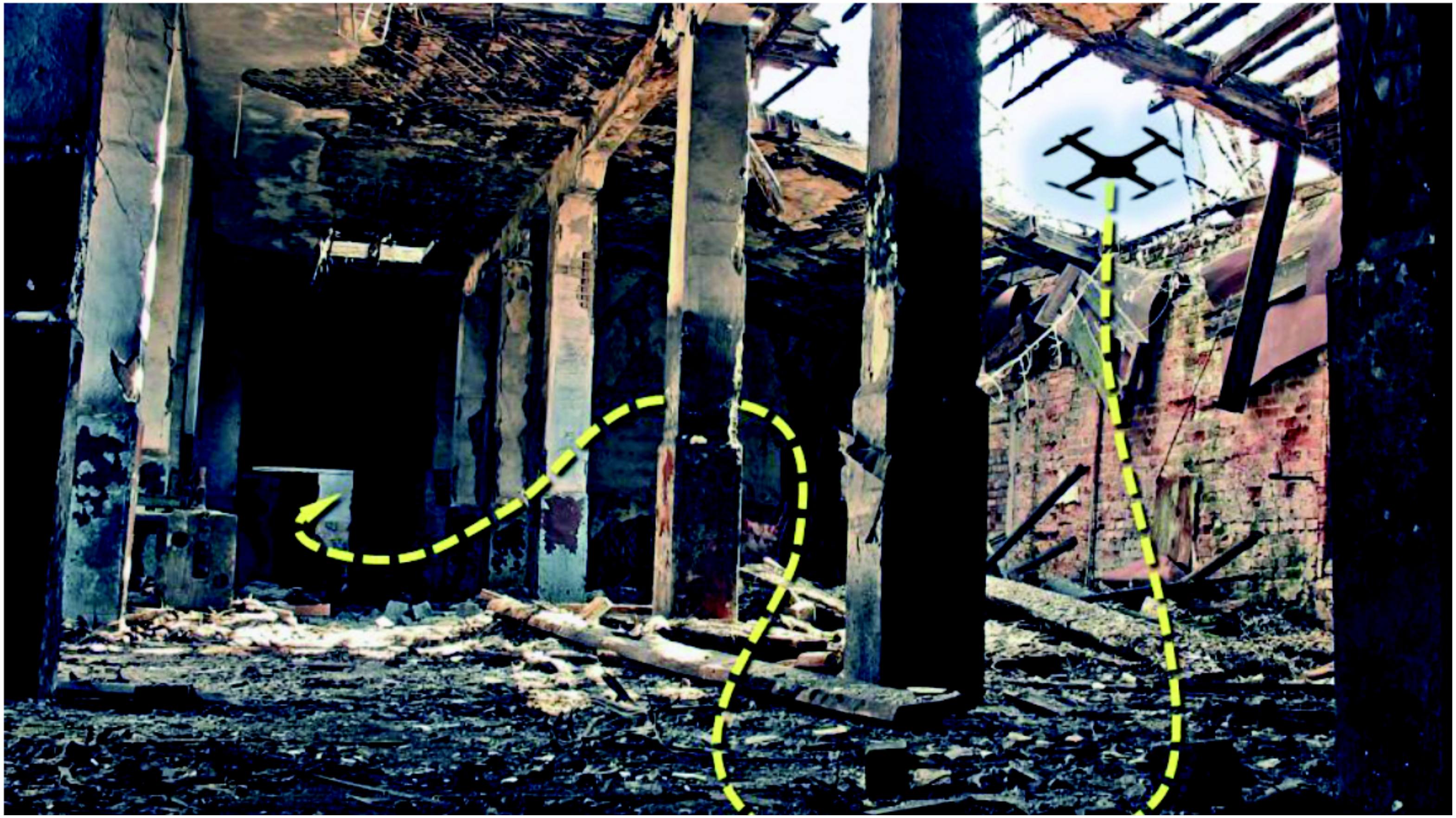
Journées STP-Nancy- 09/11/2017

# UAG navigation in GPS denied environements



# Problem statement

- Observation missions with UAVs in GPS denied environments

Use of other sensors


- Camera
- Inertial Measurement Unit

Uncertainty quantification

- For mapping acquired data
- For navigation safety

# Image-based UAV cooperative localization using Interval Analysis

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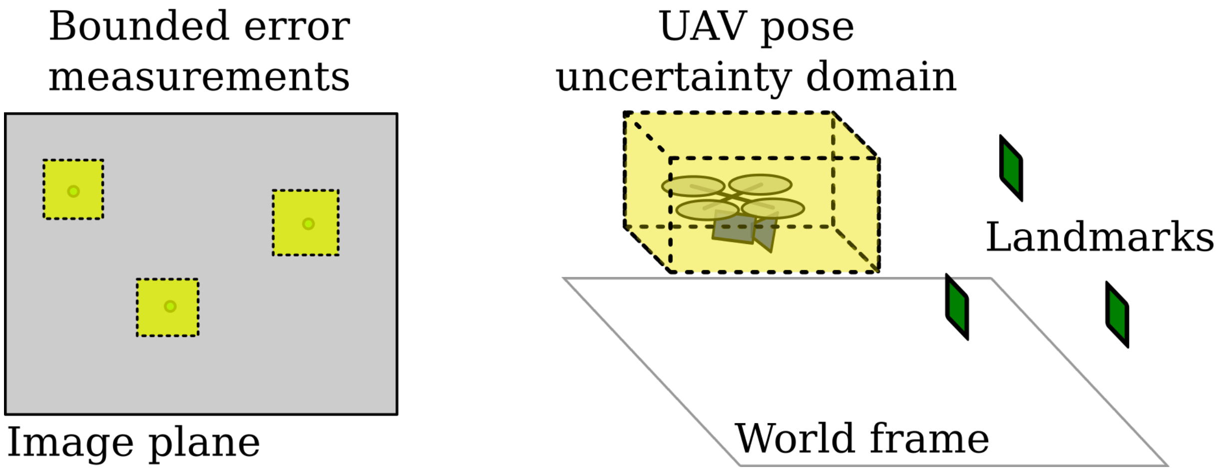
# Problem statement

Measurements are subject to errors → **uncertainties**

- **Bounded error** measurements
  - Image points
  - Landmarks positions

- Determine the set of all feasible pose compatible with the uncertain measurements: **the UAV pose uncertainty domain**



# Outline

Camera pose estimation

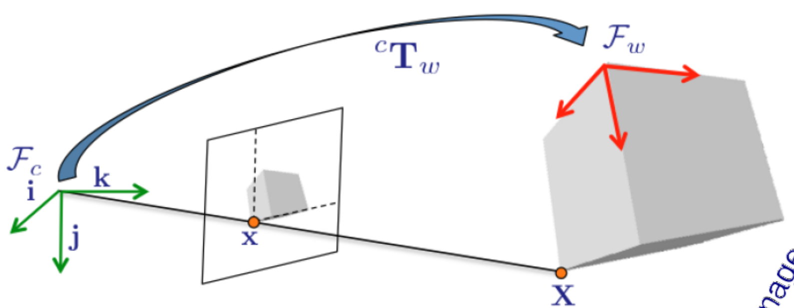
Image-based UAV pose domain characterization

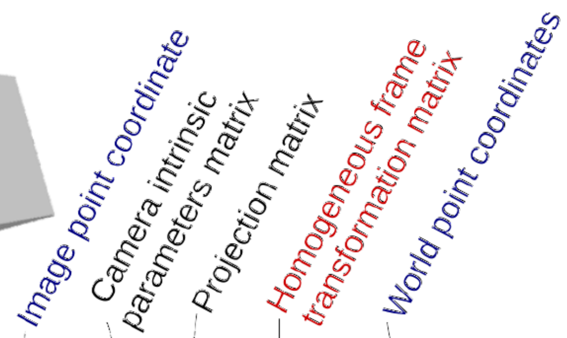
Cooperative UAV localization

Conclusion

# Camera pose estimation

- Given
  - ➢ A camera with known intrinsic parameters
  - ➢ 3D world points
  - ➢ 2D image point coordinates $\mathbf{x} = (u, v, 1)^T$
- Pose estimation: determine the pose of the camera in the world frame
- Using the projection equation of the world points in the image




$$x = K \, \Pi \, {}^cT_w \, {}^wX$$

With

$$^c\mathbf{T}_w = \begin{pmatrix} ^c\mathbf{R}_w & ^c\mathbf{t}_w \\ \mathbf{0}_{3\times1} & 1 \end{pmatrix}$$

the camera pose: orientation $^c\mathbf{R}_w$ , position $^c\mathbf{t}_w$

# Robot pose estimation

- Using Euler angle parametrization:

$$^{c}R_{w}(\psi, \theta, \phi) = R_z(\psi) R_y(\theta) R_x(\phi)$$

- We have $^{c}\mathbf{T}_{w} = {^{c}\mathbf{T}_{r}}\, {^{r}\mathbf{T}_{w}}(q)$
  - Where ***q*** is the robot pose $\mathbf{q} = (x, y, z, \phi, \theta, \psi)$

- Assuming the rigid transformation $^{c}\mathbf{T}_{r}$ is known

determining the robot pose <=> determining the camera pose

- Standard solution: minimizing the norm of reprojection error

$$\widehat{\mathbf{q}} = \underset{\mathbf{q}}{\operatorname{argmin}} \sum_{i=1}^{N} d\left(\mathbf{x}_i, \Pi\, {^{c}\mathbf{T}_{w}}\, {^{w}\mathbf{X}_i}\right)^2$$

with $d(\mathbf{x}, \mathbf{x}')$ the euclidean distance between two points

**N=3** → 2 ambigus solutions & **N >=4** for a single solution

# Image-based UAV pose domain characterization

- Determine the set of all feasible pose compatible with the uncertain measurements: **the UAV pose uncertainty domain**

- Amount on
  - Determine the 6-DOF $\mathbf{p}_k = (x_k, y_k, z_k)$ and $\mathbf{q}_k = (\phi_k, \theta_k, \psi_k)$
  - Altitude, roll and pitch directly measured
  - Focus : $x_k,\ y_k$ and $\psi_k$ for each robot

# Bounded errors measurements

- Having
  - Pixel measurements $\mathbf{x}_i$ subject to errors
  - Landmarks positions ${}^w\mathbf{X}$ known up to a given precision

- We assume
  - Bounded errors on Landmarks & Image-features

- Uncertainty representation: **Intervals** and **boxes**
  - Each quantity belongs to an interval generated from the measurement and its uncertainty

$\mathbf{x}_i$ and ${}^w\mathbf{X}$ measurements are included in *interval vectors* s.t.

$$\mathbf{x}_i \in [\mathbf{x}_i] \qquad {}^w\mathbf{X}_i \in [{}^w\mathbf{X}_i]$$

where the interval **width** represents the **uncertainty**

# Constraint satisfaction problem

- A **constraint satisfaction problem** (CSP) is defined by
  - A set of variables: vector **x**
  - With an initial domain: box [**x**]
  - And a set of constraints: g(**x**)=0

- The **solution set** of the CSP is S = $\{\mathbf{x} \in [\mathbf{x}] \mid g(\mathbf{x}) = 0\}$

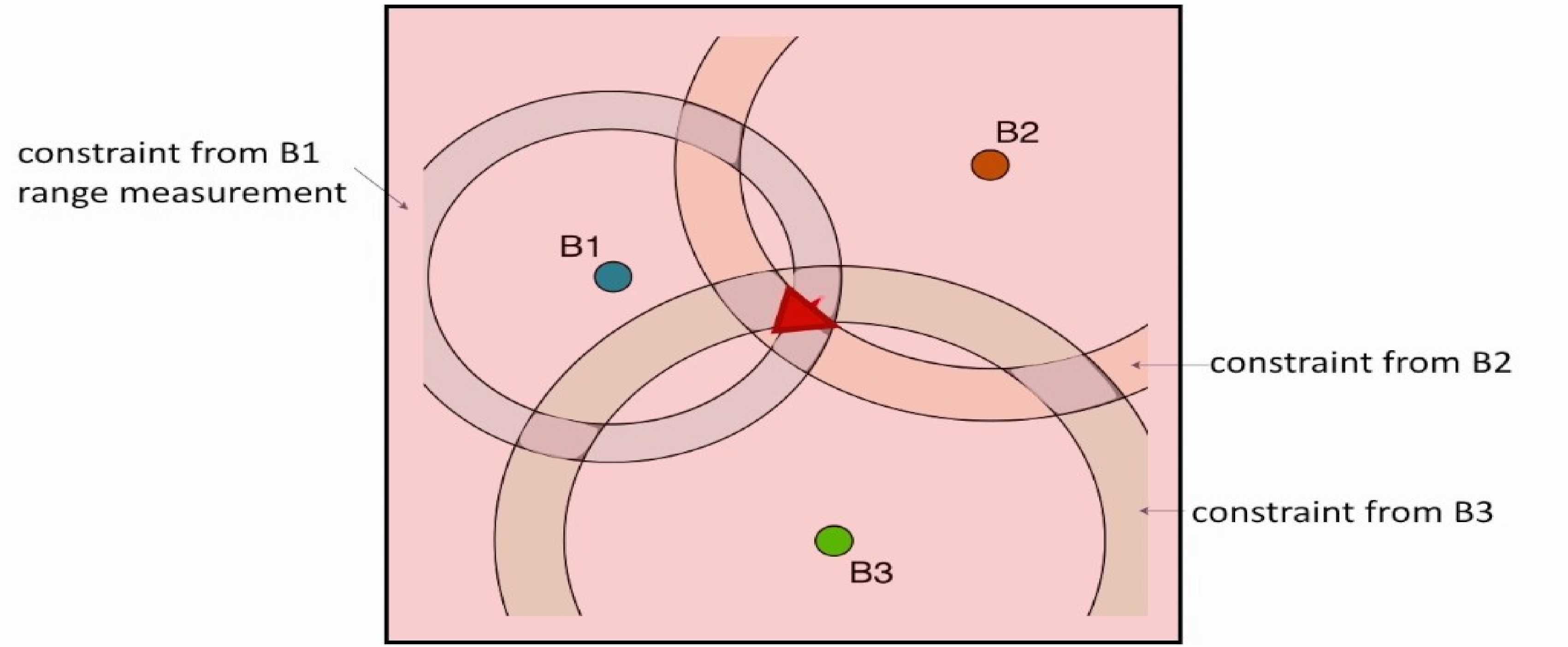
- A **contractor** for a CSP is an operator that reduces a box while keeping all the solutions.

  - Ex: the *forward-backward* algorithm applies interval computations down then up the constraints syntactic tree

  - A fixed point algorithm applies contractors until no more reduction of the variables domain happens

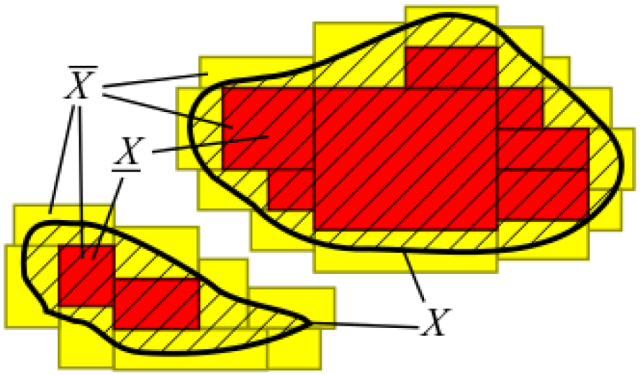
# Example: robot localization

- Goal: reduce the domain of the robot position (x, y)
- Constraints represent distances to three beacons B1, B2, B3
- Distance measurements are intervals
- Distance contractors are applied until a fixed point

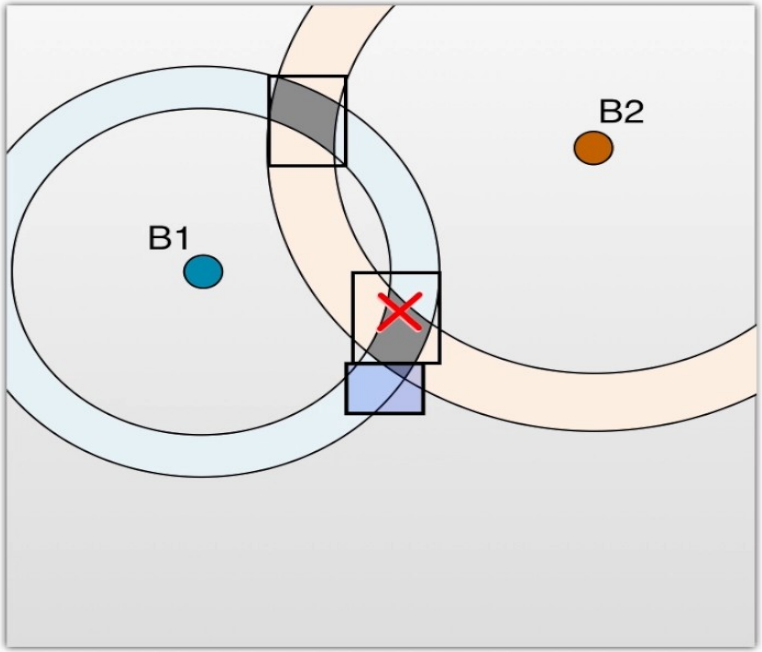


# Set Inversion via Interval Analysis

- A **subpaving** (set of non overlapping boxes) can approximate the solution set.



- The SIVIA algorithm is a branch and bound method that computes an outer approximation of the solution set, starting from an initial box,
  by recursively applying
  - **contraction**
    with contractors for the CSP
  - and **bisection**
    until boxes are too small



# Image-based pose domain characterization

- We define the robot pose estimation problem as a CSP
  - $\mathbf{r_k} = (x_k, y_k, z_k, \phi_k, \theta_k, \psi_k)$ is the robot pose
  - $(u_i, v_i)$ , $i$=1...N are pixel coordinates of landmarks in the image
  - $(X_i, Y_i, Z_i)$ , $i$=1...N are world coordinates of the landmarks

- The initial domains are intervals representing either measurements uncertainty or prior knowledge

- Projection constraints

$$({}^cX, {}^cY {}^cZ, 1)^T = {}^c\mathbf{T}_r {}^r\mathbf{T}_w(\mathbf{r})(X, Y, Z, 1)^T$$

$$C_{proj} = \{u = p_x x_i + u_0,\ v = p_y y_i + v_0\}$$

$$\text{with } x_i = \frac{{}^cX,}{{}^cZ,}, y_i = \frac{{}^cY,}{{}^cZ,}.$$

- Front looking constraints

$$C_{front} = \{{}^cZ, > 0\}$$

$$C_{cam,i} = \{C_{proj,i}, C_{front,i}\}$$

# Pose domain computation

- Real measurements does not always respect the specified error bounds → outliers.
- Inconsistencies in the CSP may lead to an empty solution set.

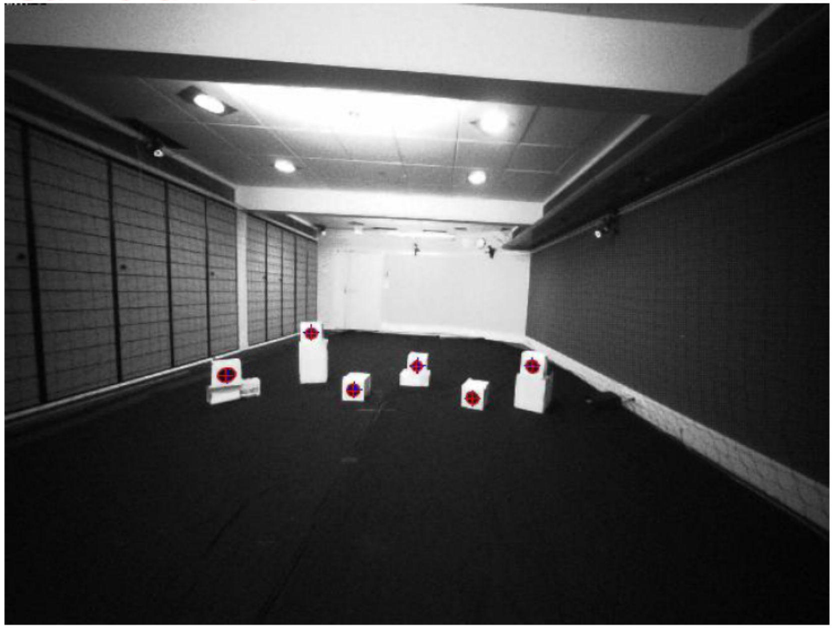
**Pose domain computation loop**

At each image acquisition:

- Compute the pose domain by solving the CSP with SIVIA.
  - pitch, roll, and altitude are measured to reduce the prior domain.
- If the solution subpaving is empty:
  - raise a « **fault detected** » flag
  - set the initial domain for the next epoch to « unknown »
- Otherwise (no fault detected) :
  - use velocity measurements to predict the pose domain until next image

# Experimental setup

Camera view



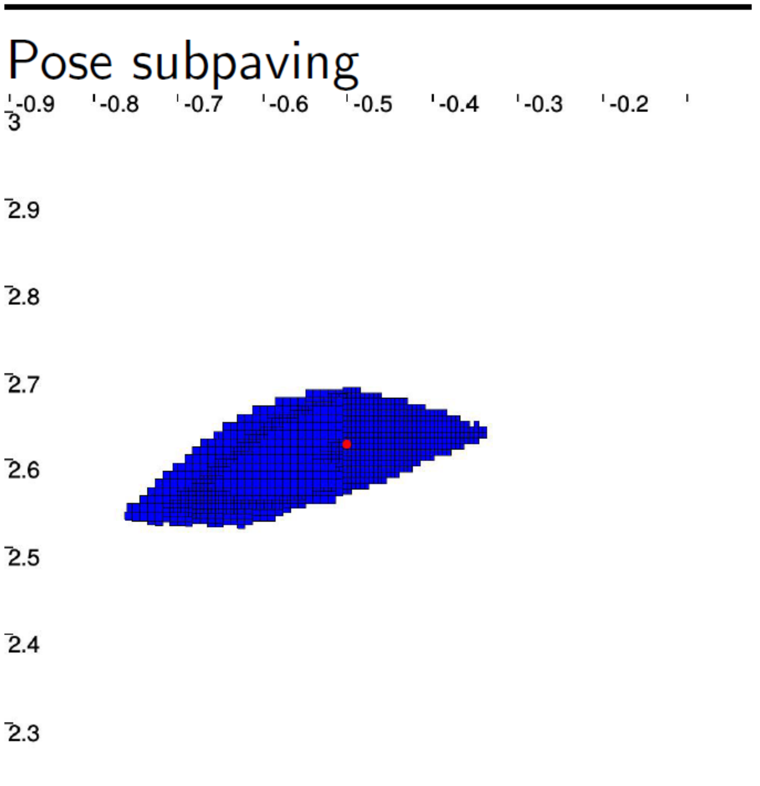
A Vicon motion capture system provides pose ground truth.

Initial domains

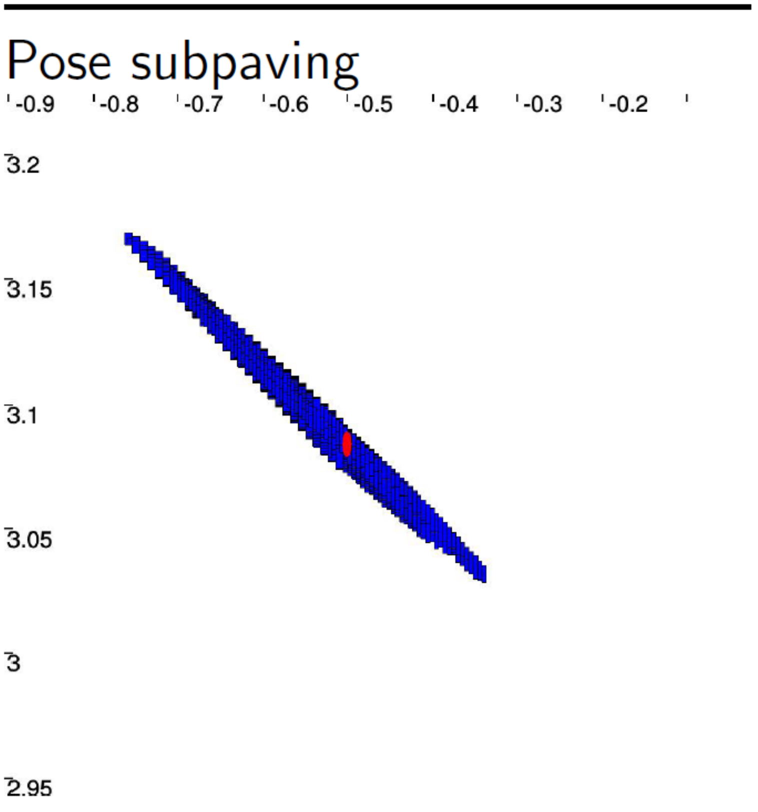
- pixel error $\pm 1px$
- 3D points error $\pm 1cm$
- initial pose domain

| | |
|---|---|
| x (m) | [-3,3] |
| y (m) | [-3.5,3.5] |
| z (deg) | $[z_{mes} \pm 1mm]$ |
| $\phi$ (deg) | $[\phi_{mes} \pm 0.05]^\circ$ |
| $\theta$ | $[\theta_{mes} \pm 0.05]^\circ$ |
| $\psi$ (deg) | $[\psi_{mes} \pm 20]^\circ$ |

# Pose uncertainty domain subpaving

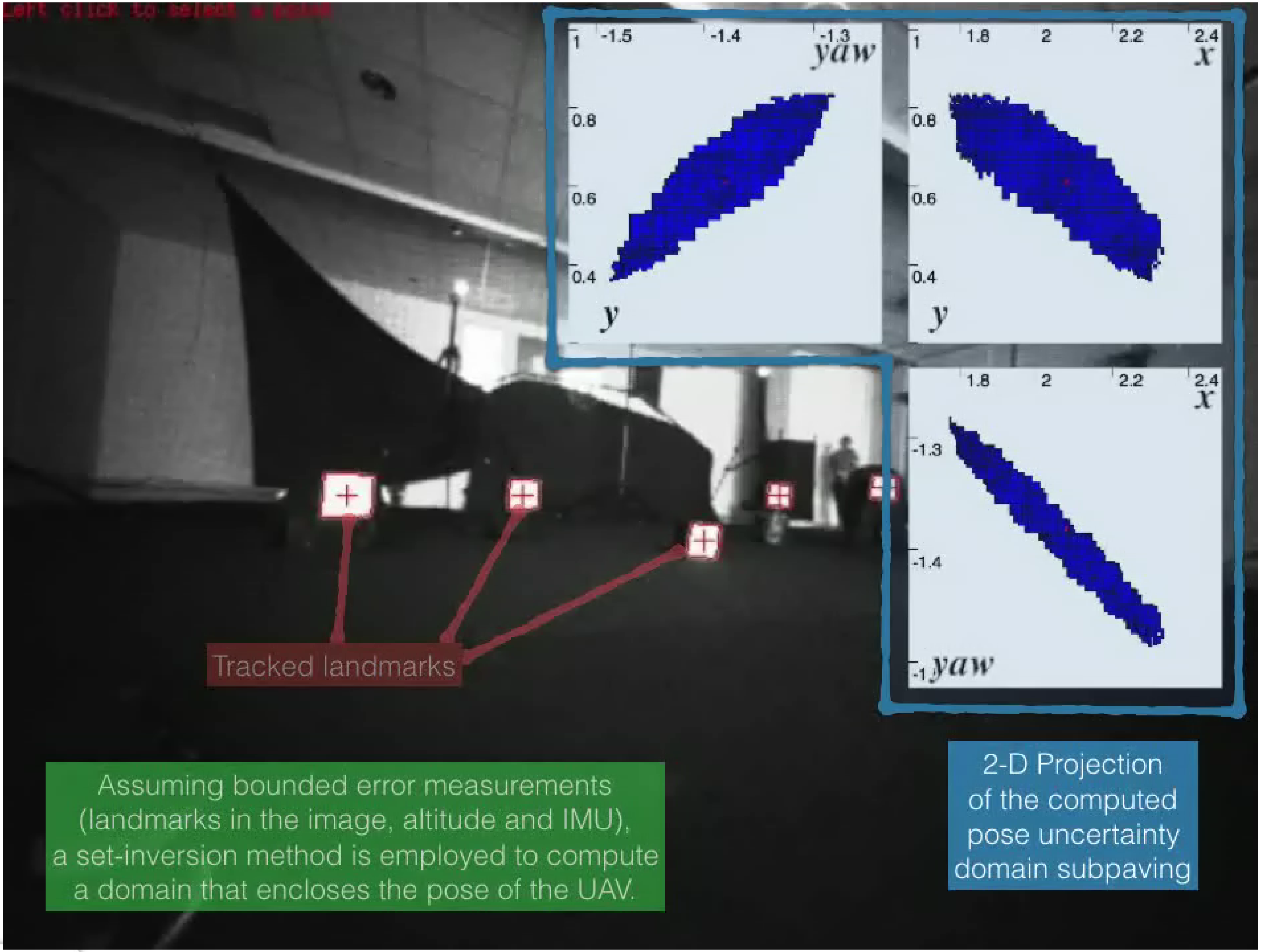



X,Y plane

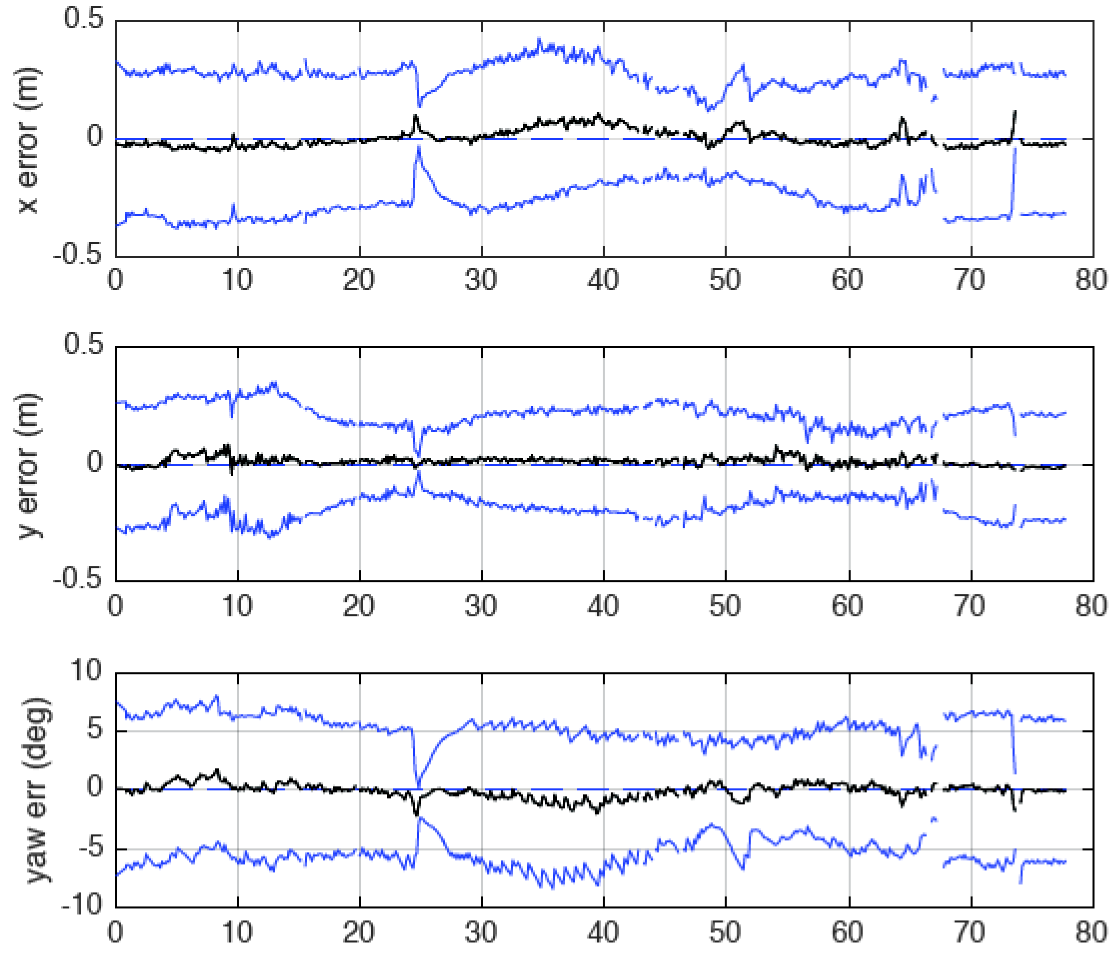



X, Yaw plane

# Experiment video



# Pose domain bounds




- Domain mean radius:
  - 25 cm horizontal
  - +/-5 deg. Heading
- Domain center can be used as a point estimate.

Mean errors:
  - 3.7 cm horizontal (5.1 cm with EKF)
  - 0.45 deg. Heading

(0.64° with EKF)

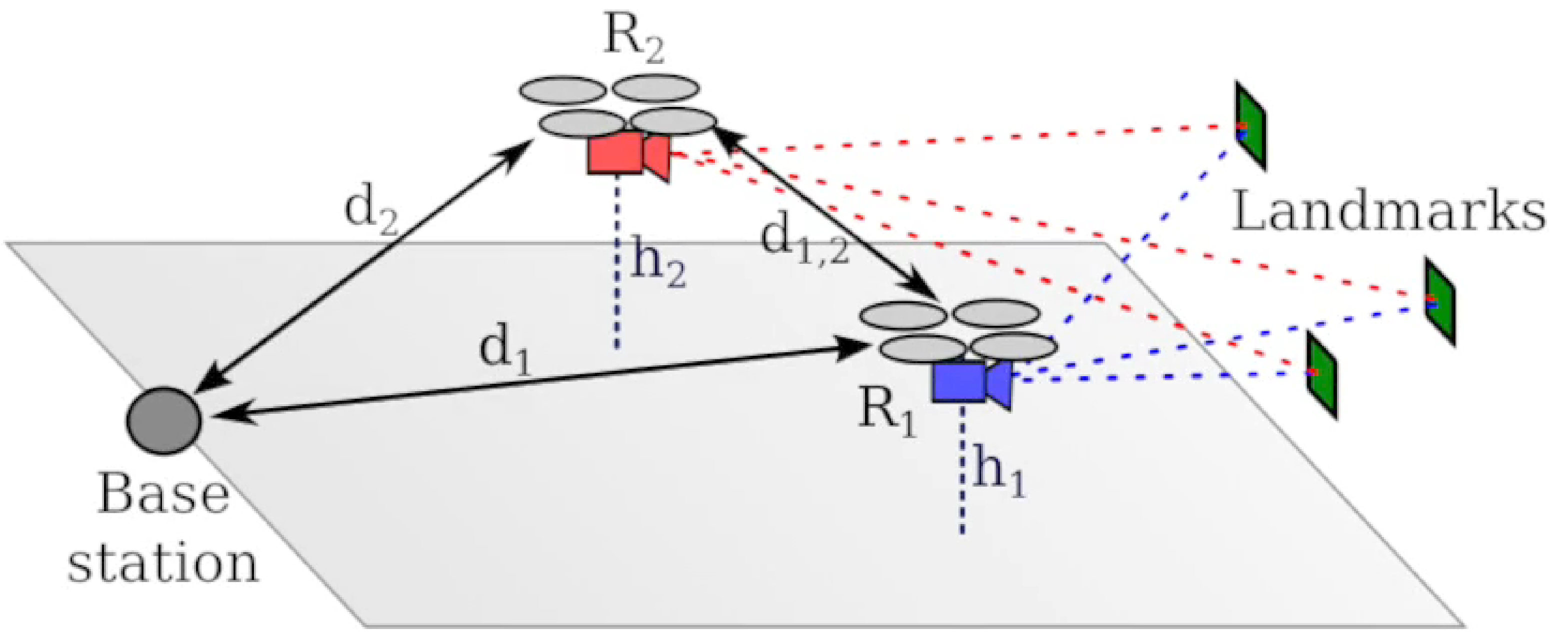
# Outline

Camera pose estimation

Image-based UAV pose domain characterization

Cooperative UAV localization

# UAV cooperative localization




- 2 Robots $R_k,\ k \in \{1,2\}$ equipped with
  - Altimeter (altitude $h_k$ )
  - IMU (roll $\psi_k$ , pitch $\theta_k$ )
  - Camera (image points $\{\mathbf{x}_{i,k}, \ldots, \mathbf{x}_{N,k}\}$ , $\{{}^w\mathbf{X}_i, \ldots, {}^w\mathbf{X}_N\}$ )
  - Communication link (range $d_{k,j}$ to robot $R_j$ and $d_k$ to the base station)

# Cooperative pose-domain characterization

- Principle
  - Combine ranging & camera measurements

  → Use IMU & Altitude & distance to the base measurement & vision to compute a robot pose

  → Cooperative positionning (enables position refining)
    - inter-UAV ranging
    - Data exchanges

# Range measurements

- Information provided $\mathbf{p}_k = (x_k, y_k, z_k)$
- Range of $R_k$ to base $\mathbf{b} = (x_B, y_B, z_B)$

$$d_k = \|\mathbf{p}_k - \mathbf{b}\| \quad \text{with } k \in \{1, 2\}$$

  - Constraints the position of $R_k$ to be on the **sphere** $S^2(\mathbf{b}, d_k)$

- Inter-robot range (cooperation)

$$d_{k,j} = \|\mathbf{p}_k - \mathbf{p}_j\| \quad \text{with } (k, j) \in \{1, 2\}^2 \text{ and } k \neq j$$

  - Admissible positions for $R_2$ given $R_1$ is $S^2(\mathbf{b}, d_2)$ $S^2(\mathbf{p}_1, d_{1,2})$
  - Ambiguity (2 possible position solution) & 1-DOF unknown

# Cooperative pose-domain characterization

- Principle
  - Combine ranging & camera measurements

→ CSP using IMU & Altitude & distance to the base measurement & vision to compute a robot pose

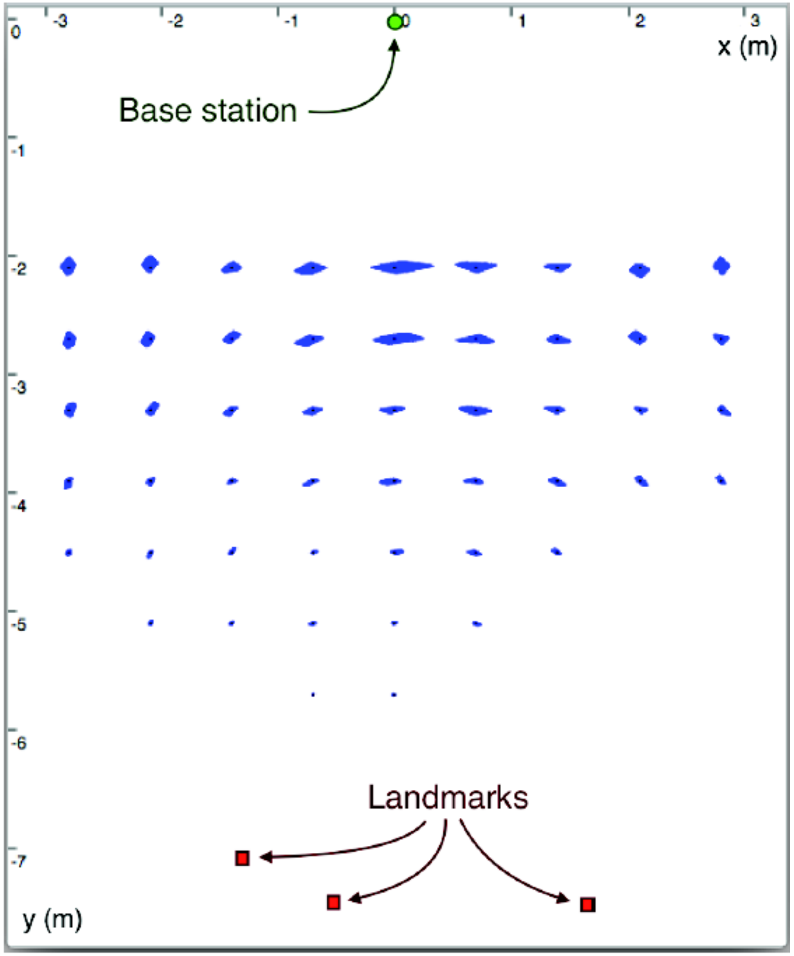
- Image constraints $C_{cam,i} = \{C_{proj,i}, C_{front,i}\}$
- Add distance to the base constraint

$$C_{base,r_k} : \{d_k^2 = (x_B - x_k)^2 + (y_B - y_k)^2 + (z_B - z_k)^2\} \quad k \in 1,2$$

→ Cooperative positionning (enables position refining)

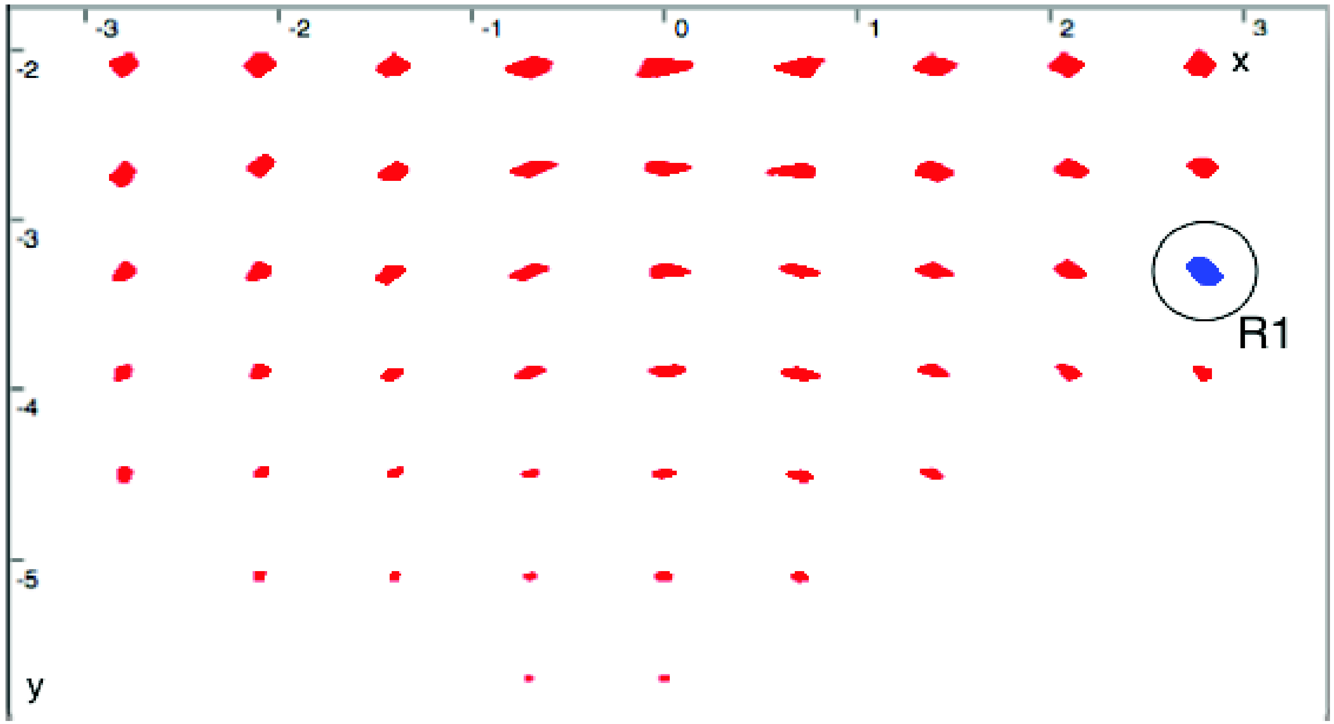
- inter-UAV ranging $C_{r_1,r_2} = \{d_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}$
- Data exchanges

# Single robot: camera & range



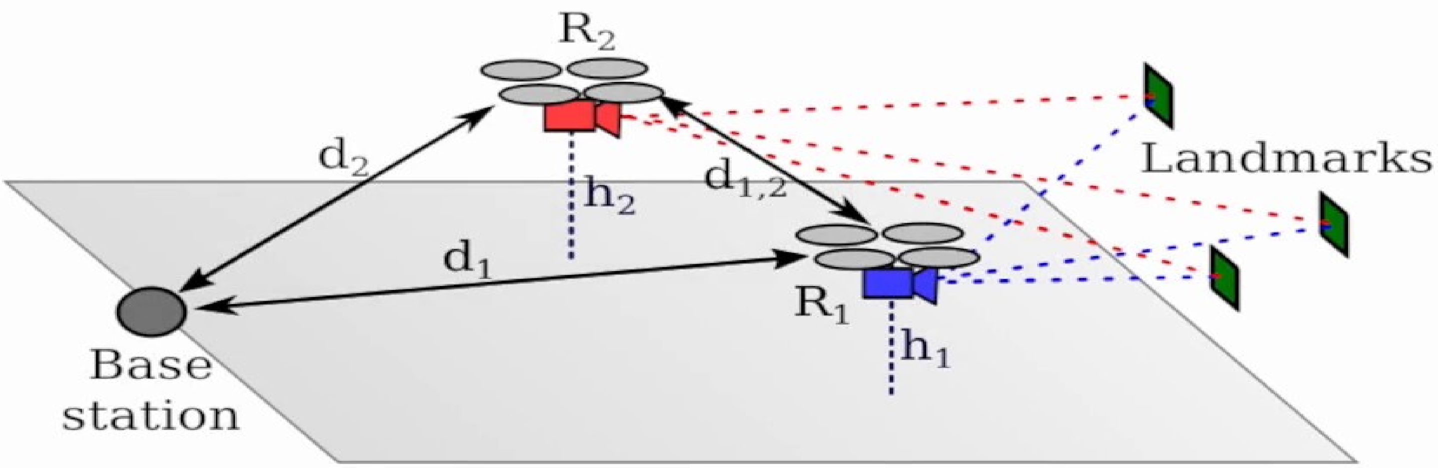

Position domains for a single robot, at 50 different positions, with range measurement to the base station

# Cooperative localization




Position domains for cooperative localization. R1 is fixed (in blue, circled), R2 at 49 different positions (in red)

# Cooperative localization

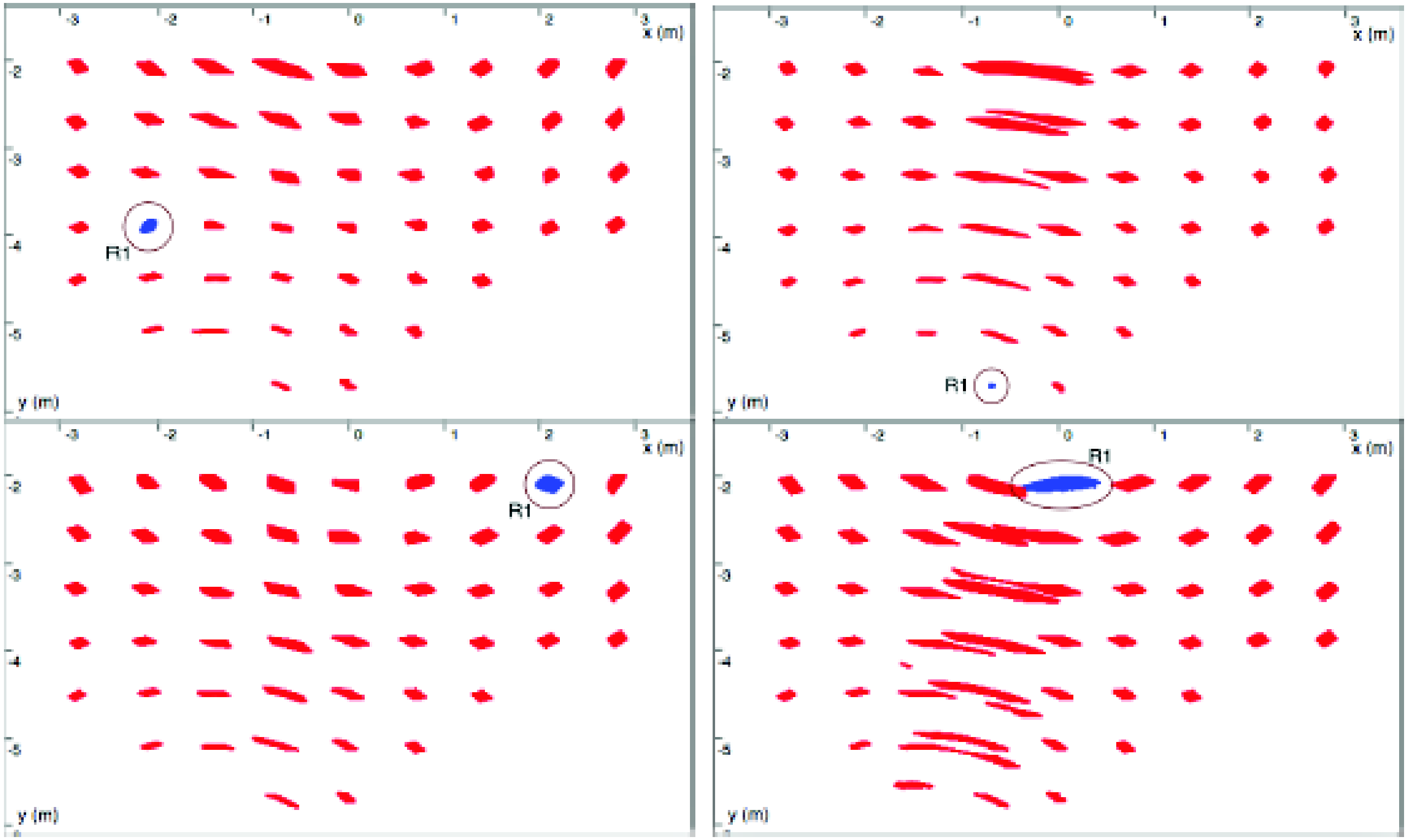



**1. Simulation results**

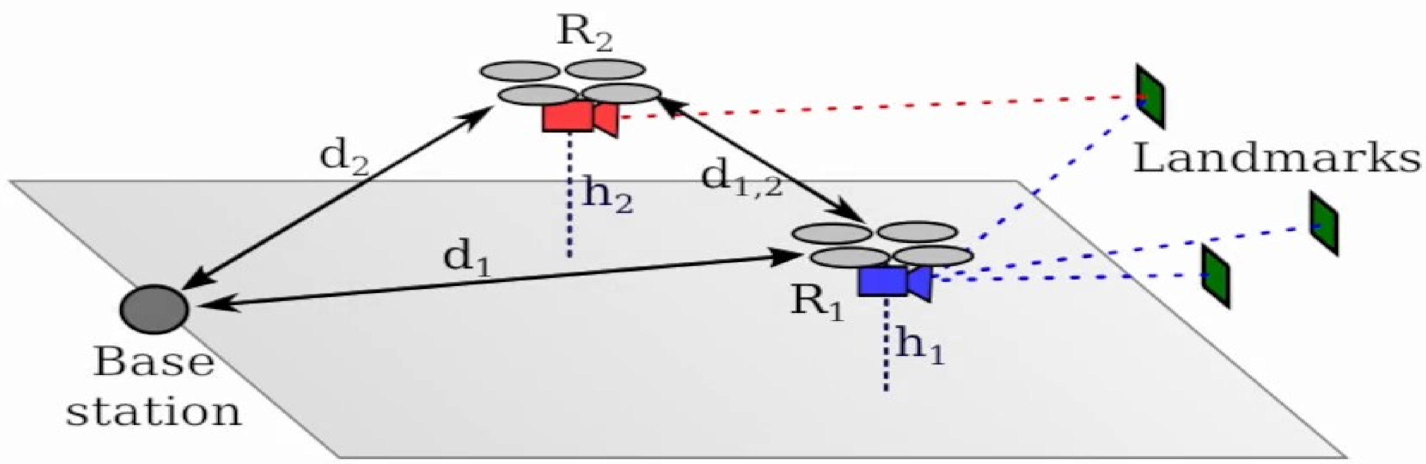
UAVs R1 (blue) and R2 (red) track 3 landmarks with their cameras. Pose uncertainty domains for 49 positions of R2 are computed for each position of R1. Horizontal position uncertainty domains are plotted.

# Cooperative localization

- Results when R1 sees 3 landmarks and R2 sees only one (for different fixed position of R1)



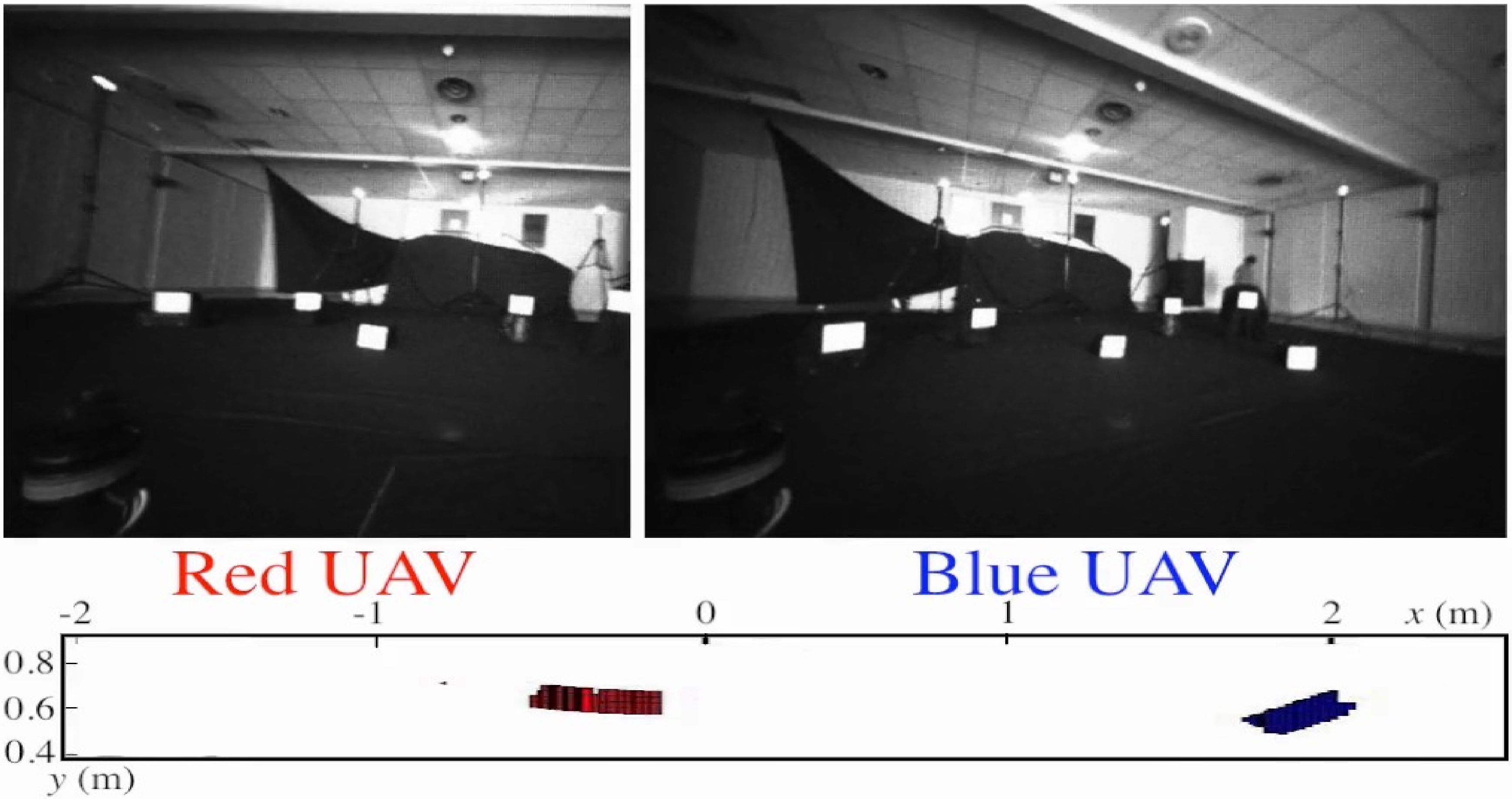
# Cooperative localization




## 2. Simulation results with reduced visibility

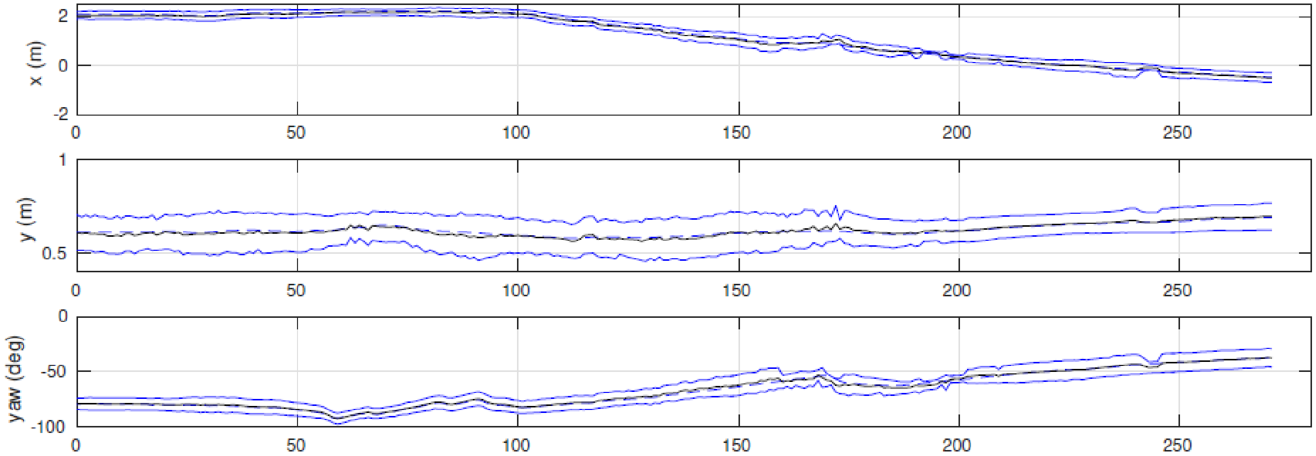
UAV R1 (in blue) tracks 3 landmarks, but **R2 (in red) only sees one**. Pose uncertainty domains for 49 positions of R2 are computed for each position of R1. Horizontal position uncertainty domains are plotted.

# Cooperative localization: experimental validation



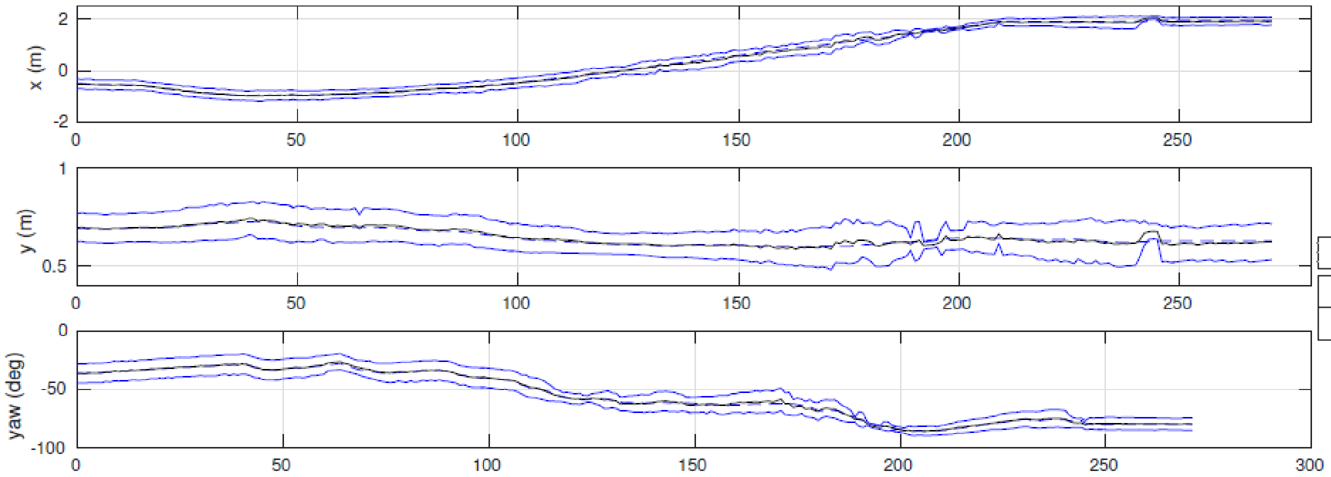

Landmarks are white boxes, tracked in the images from embedded camera. Pose domains for two UAVs (red and blue) are computed from landmarks and distances measurements. Horizontal position domains are plotted.

# Cooperative localization: experimental results




## Pose domain width

| | min | mean | max |
|---|---|---|---|
| x (cm) | 5.4 | 36.39 | 58.35 |
| y (cm) | 3.61 | 16.57 | 24.6 |
| yaw (deg) | 2.94 | 13.78 | 20.47 |




## Absolute error

| Error | min | mean | max |
|---|---|---|---|
| yaw (deg) | 0.001 | 0.7 | 6 |
| 2D horizontal (cm) | 0.19 | 3.49 | 23.61 |

# Conclusion

- Set-membership approach to vision-based robot pose estimation
- Compute a domain that contains all the feasible robot poses, given bounded error camera measurements. and onboard sensor data (attitude, altitude, and speed).
- Inconsistency detection (empty set computation) enables fault detection.
- Extension to cooperative localization is straightforward
  - Although reduced visibility, pose domain computation still possible for second robot