# Robust output/state feedback controllers design for uncertain systems described by interval state-space models

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- **Context and motivations**
- **Basic Concepts on intervals**
- **Problem Formulation**
- **Interval linear approaches**
- **Experimental validation**
- **Other propositions**
- **Conclusion and Perspectives**

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# 1- Context and Motivations

## Introduction

Magnetostrictive materials

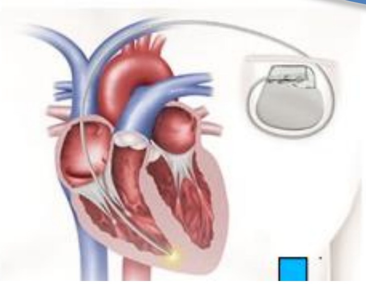
Thermoelectric materials

Piezoelectric

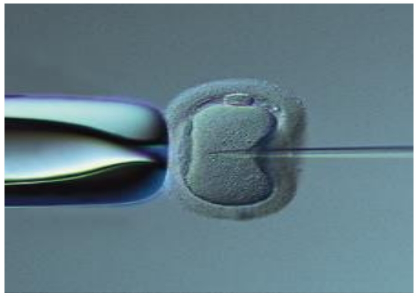
Others

**Smart materiels**

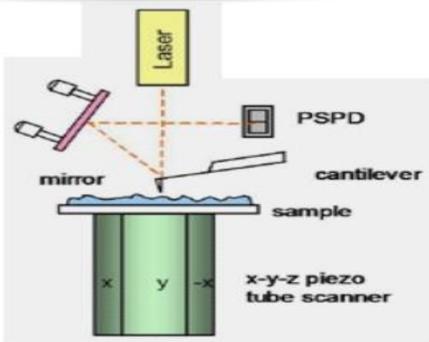
- -High resolution
- -High accuracy
- - High Bandwidth
- -Cost, Integration,..ect



Microrobot Surgery

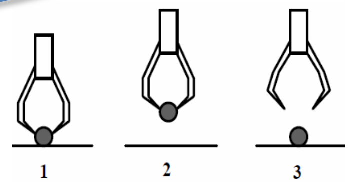


Microrobot manipulation

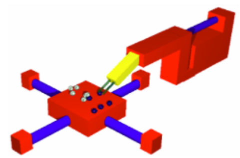



Scanning microscopy

Micro-pince




Micro-objets Manipulation



Micro assembly

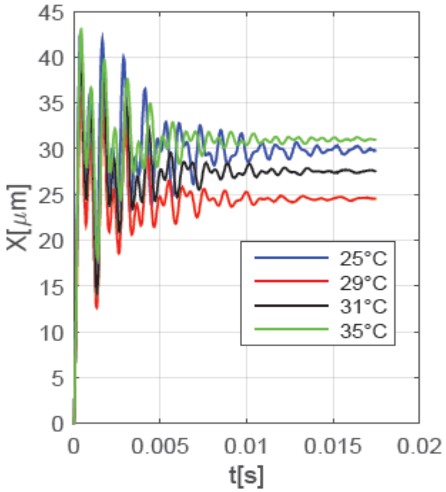
# 1- Context and Motivations

## Context

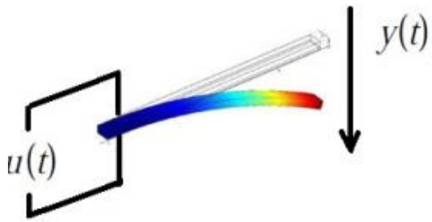
Smart materials are subjected to various uncertainties and contains serval nonlinearities.

**•Sensitive to the environment**

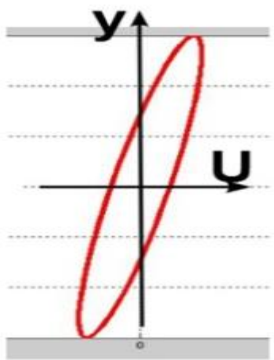
- Temperature ...etc.



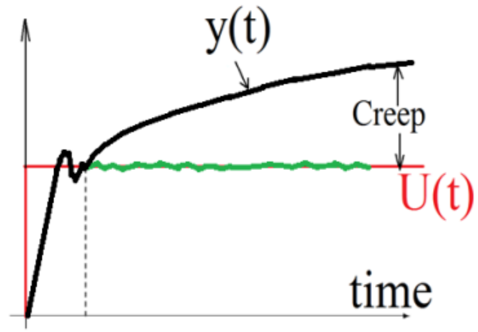

- vibrations ...etc.




**- Hysteresis**




**- creep (dérive)**




Modeling and control of such systems are difficult.

Robust control techniques are required to ensure the specified performances.

# 1- Context and Motivations

**Robust methods: H-2, H-inf, µ-synthesis**

- **Complex controllers**
- **Difficult to implement**

## ❖ Objectives

Seek for simple methods to control systems ensuring the stability and performances.

## ❖ Solution

**"Control Systems Using Intervals theory"**

- ❑ Simple methods to describe the parameters uncertainties just by **bounding the parameters.**
- ❑ Provide **low order controllers**.
- ❑ **Reliable computation**.

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## 2- Basic Concepts on intervals

### ❖ Operations on intervals

#### Interval operations

Given two intervals $[x] = [\underline{x}, \overline{x}]$ and $[y] = [\underline{y}, \overline{y}]$. The result of an operation $\diamondsuit \in \{+, -, \cdot, /\}$ between the two intervals is an interval that contains all possible solution:

$$[x]\diamondsuit[y] = \{x\diamondsuit y \mid x \in [x], y \in [y]\} \qquad (1)$$

#### Definition of interval matrix

An interval matrix is defined as a family of matrices:

$$A := [\underline{A}, \overline{A}] = \{A \in R^{n\times n}; \underline{A} \leq A \leq \overline{A}\} \qquad (2)$$

The midpoint and the radius of A are denoted respectively by:

$$A_c := \frac{1}{2}(\underline{A} + \overline{A}), \; A_\Delta := \frac{1}{2}(\underline{A} - \overline{A}) \qquad (3)$$

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# 2- Basic Concepts on intervals

## ❖ Interval systems

### Interval transfer function

An interval system denoted $[G](s,[p],[q])$is a system where $[p]$ and $[q]$ are two boxes of interval numbers:

$$[G](s,[p],[q]) = \frac{\sum_{j=0}^{m} [q_j]s^j}{\sum_{i=0}^{n} [p_j]s^j} \quad (4)$$

where $[q_j] \in [q_j^-, q_j^+]$ and $[p_i] \in [p_i^-, p_i^+]$

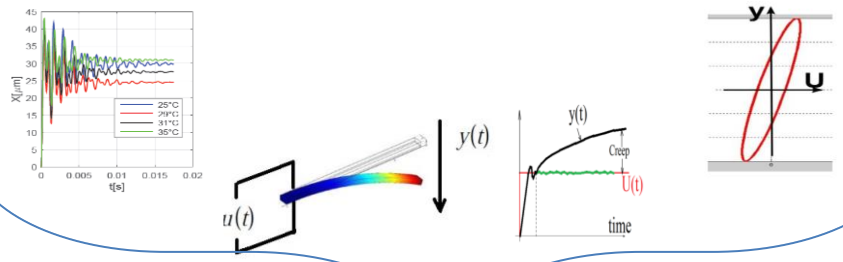
### Interval state-space

Interval uncertain system described by the state-space equation:

$$\begin{cases} \dot{x}(t) = \mathbf{[A]}x(t) + \mathbf{[B]}u(t) & ; \\ y(t) = \mathbf{[C]}x(t) + \mathbf{[D]}u(t) \end{cases} \quad (5)$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$, $\mathbf{[A]} \in IR^{n\times n}$, $\mathbf{[B]} \in IR^{n\times m}$, $\mathbf{[C]} \in IR^{p\times n}$, and $\mathbf{[D]} \in IR^{p\times m}$.

# 3-Problem Formulation




**Microsystems sensitivities to the environment**

**+**

**Nonlinearities**

Approximated

- Linear Interval approximation
  - **Interval State-space representation**

$$\begin{cases} \dot{x}(t) = [A]x(t) + [B]u(t) & ; \\ y(t) = [C]x(t) + [D]u(t) \end{cases}$$

  - **Interval Transfer function representation**

$$[G](s,[p],[q]) = \frac{\sum_{j=0}^{m} [q_j] s^j}{\sum_{i=0}^{n} [p_j] s^j}$$

- Nonlinear interval approximation

$$\begin{cases} a_i \ddot{\delta}_i + b_i \dot{\delta}_i + \delta_i = d_{pi} U_i - h_i + \mathcal{C}_i(U_j, h_j) \\ \dot{h}_i = d_{pi} A_{bwi} \dot{U}_i - B_{bwi} \left|\dot{U}_i\right| h_i - C_{bwi} \dot{U}_i \left|h_i\right| \end{cases}$$

Theory of control → Interval controller design

Interval theory → Interval controller design

Interval controller design → Robust control

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## State-of-the-Art

# 1- Linear interval approaches

## a-Interval Transfer function representation

-PID controller,H∞ with interval techniques, RST-structured controller [E,Walter-1994, C.T.Chen-1997, Rakotondrabe-2009, Khadraoui-2012,-2014].

✓ Ensure the stability and the robustness

✗ Not well adapted to multivariable systems

## b- Interval State-space representation

✓ Well adapted to multivariable control

❖ *Interval State-feedback*

- Arithmetic intervals [Smagina-1997 , -2000,-2002], [Dugarova-1989], [Wei-1994].
- Non-fragile design [Marcia-2005].
- Analytical design [ Patre-2010 ]

✗ Address only the State-feedback

✗ No performances was discussed

❖ *Robust and Optimal control*

- Quadratic stability and LMIs [Mao- 2003,-2002], [Zhang-2006] and [Guang-2006].
- Optimal Guaranteed Cost Control [Min-2009, Li-1999 ].
- Optimal Guaranteed Cost Control with input constraints [Li-2005].
- Optimal Guaranteed Cost Control with Actuator Failures [Min-2008]

✗ Difficult to implement with a lot of parameters to set

# 2. Nonlinear interval approaches

- Interval-Based Sliding Mode Control [Rauh-2012,-2013,-2017].
- Nonlinear Model Predictive Control via interval arithmetic [Lydoire-2005].

✓ Good performances

**Our Focus**

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# 4- Interval linear Approaches

## ❖ Proposition

“Robust Output-feedback with regional pole assignment technique”

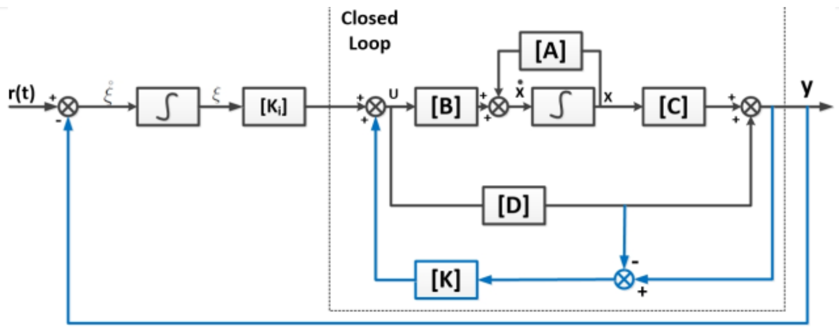
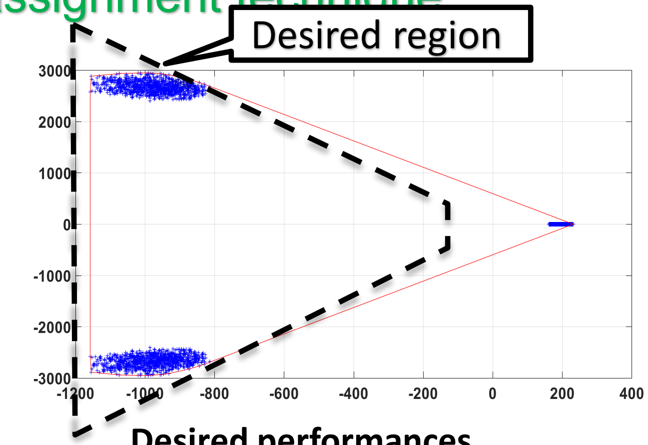



*Fig1. Output-Feedback with Integral Compensator.*

$$\begin{cases} \dot{x}(t) = [A]x(t) + [B]u(t) & ; \\ y(t) = [C]x(t) + [D]u(t) \end{cases}$$

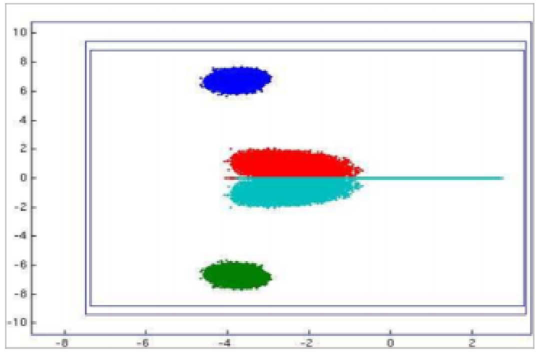



**Desired performances**

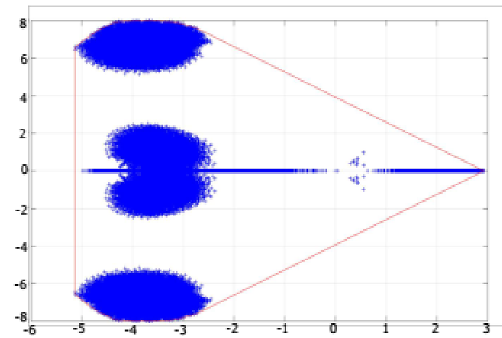
- **without overshoot**
- **settling time <2ms**

## ❖ Interval Eigenvalue Computation

- Symmetric matrix (Rohn,2005)
- Non-Symmetric matrix (Hladik, 2011)
- Vertex approach (Hussein,2011)
- ……



*Hladık formula (Hladık, 2011)*



*Vertex approach (Hussein,2011)*

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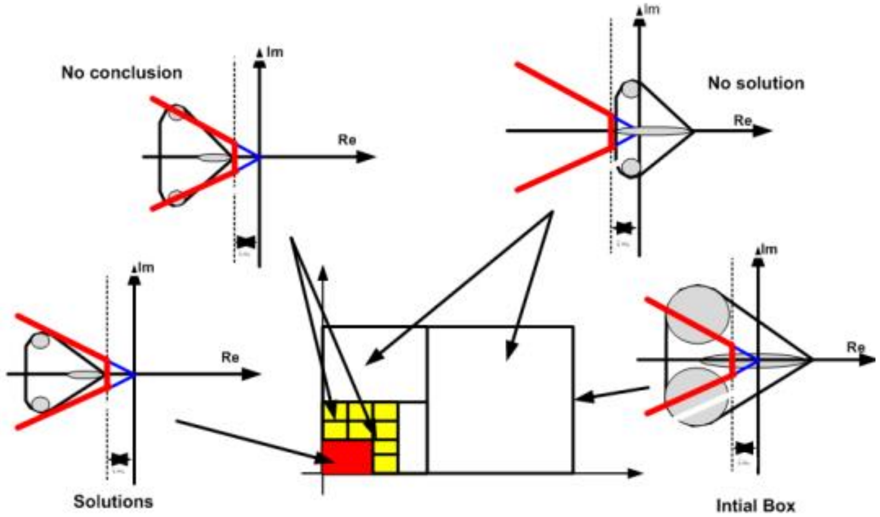
## Robust Output-feedback

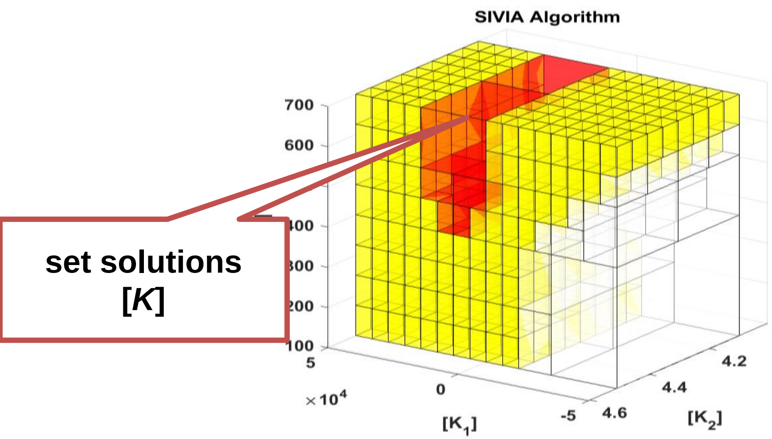
# The proposed recursive SIVIA-based algorithm

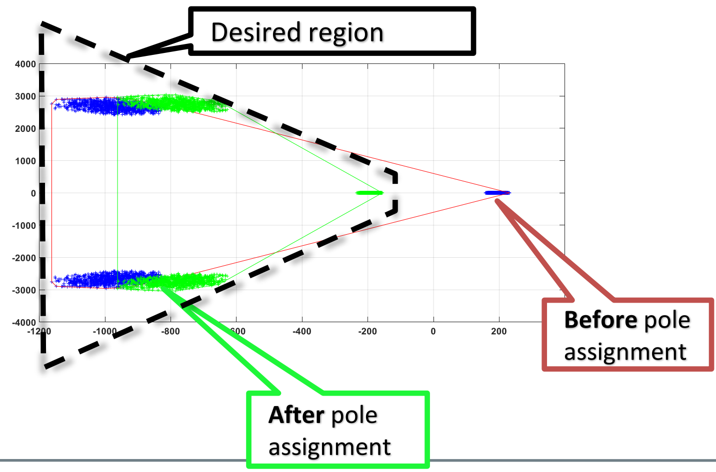
The proposed recursive SIVIA-based algorithm for solving a set-inversion problem:

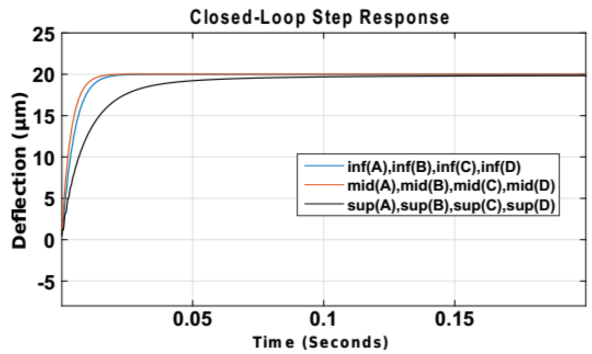
| | |
|---|---|
| Step 1 | Iteration i : - Calculate $eig([A_{aug\_cl}])$ |
| Step 2 | -If $eig([A_{aug\_cl}]) \subseteq \Omega_{Des}$ Then $[k_{in}] = [k_{in}] \cup [K]$ Go to step 6 |
| Step 3 | -If $eig([A_{aug\_cl}]) \cap \Omega_{Des} = \emptyset$ Then $[k_{Unf}] = [k_{Unf}] \cup [K]$ Go to step 6 |
| Step 4 | -If $[K] < \varepsilon$ Then $[k_{out}] = [k_{out}] \cup [K]$ Go to step 6 |
| Step 5 | -Else bisect $[K]$ and stack the two resulting boxes. |
| Step 6 | - If $stuck = \emptyset$ end; else go into $[K](i+1)$. |




-SIVIA: Set Inversion Via Interval Analysis [Jaulin 1993].



# 4- Interval linear Approaches

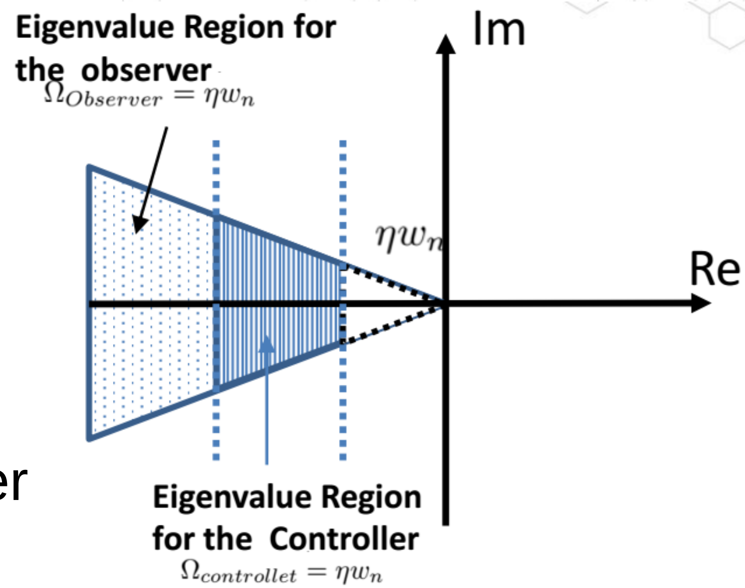
## a- Encountered problems and propositions

If the interval system can not be stabilized with output-feedback?

### ❖ Proposition

**"Robust state-feedback with interval observer"**

- ❑ Interval Luenberger observer.
- ❑ Regional eigenvalue assignment for both controller and observer.




$$\Omega_{controller} > 5 * \Omega_{observer}$$

### ❖ Advantages

Find easily the robust gains for the feedback controller also the robust gains for the observer in the presence of system uncertainties.

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# 4- Interval linear Approaches

## Robust observer-based state feedback

- **Controller**

$$\dot{x}(t) = \boldsymbol{A}x(t) - \boldsymbol{B}K\hat{x}(t) + \boldsymbol{B}N\xi(t)$$
$$u(t) = -K\hat{x} + N\xi(t)$$

- **Interval Observer**

$$\begin{cases} \dot{\hat{x}}(t) = \boldsymbol{A}\hat{x}(t) + \boldsymbol{B}u(t) + L(y - \hat{y}) \\ \quad = \boldsymbol{A}\hat{x}(t) + \boldsymbol{B}(-Ky + N\xi(t)) + L(y - \hat{y}) \\ \quad = (\boldsymbol{A} - L\boldsymbol{C} - \boldsymbol{B}K)\hat{x}(t) + \boldsymbol{B}N\xi(t) + Ly \\ \quad = (\boldsymbol{A} - L\boldsymbol{C} - \mathbf{B}K)\hat{x}(t) + \boldsymbol{B}N\xi(t) + L\boldsymbol{C}x \end{cases}$$

- **Interval augmented State-space model**

$$\begin{pmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \\ \dot{\xi}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} \boldsymbol{A} & -\boldsymbol{B}K & \boldsymbol{B}N \\ 0 & \boldsymbol{A} - L\boldsymbol{C} - \boldsymbol{B}K & \boldsymbol{B}N \\ -\boldsymbol{C} & 0 & 0 \end{pmatrix}}_{[A_{cl}]} \begin{pmatrix} x(t) \\ \hat{x}(t) \\ \xi(t) \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ I \end{pmatrix}}_{[B_{cl}]} r(t)$$

$$y(t) = \underbrace{\begin{pmatrix} \boldsymbol{C} & 0 & 0 \end{pmatrix}}_{[C_c]} \begin{pmatrix} x(t) \\ \hat{x}(t) \\ \xi(t) \end{pmatrix}$$

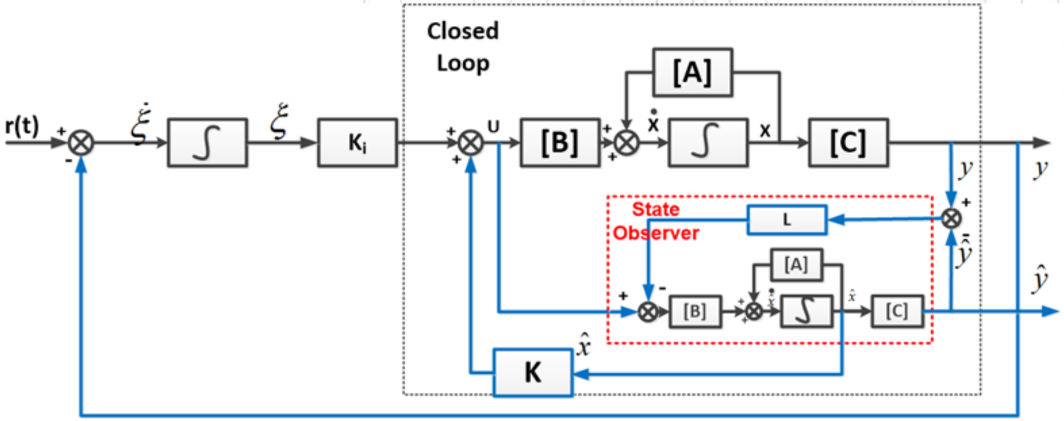
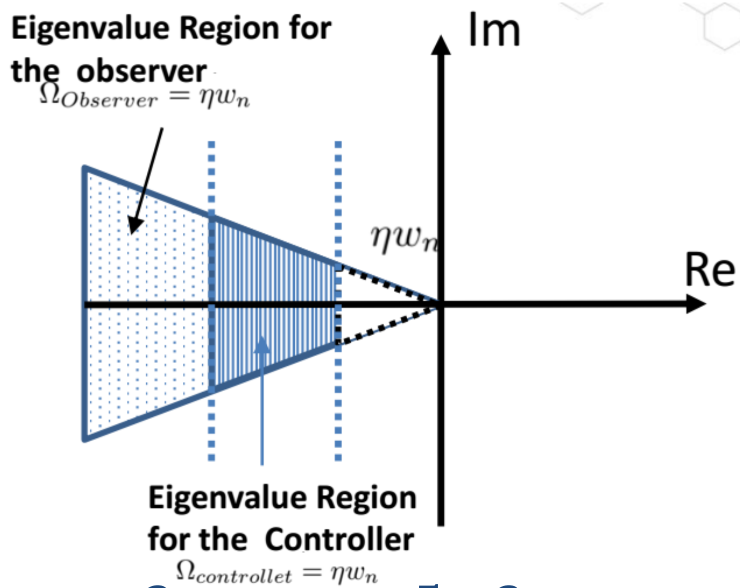



Fig.2. State-feedback with observer schema




$$\Omega_{controller} > 5 * \Omega_{observer}$$

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## 4- Interval linear Approaches

### Robust observer-based state feedback

- **Separation principal**

**Similarity transformation**

$$\overline{[A_{cl}]} = T[A_{cl}]T^{-1} \quad \Rightarrow \quad \lambda(\overline{[A_{cl}]}) = \lambda([A_{cl}])$$

If we consider that the system matrices that used to synthesis the controller and the observer are the same and belong to the interval system (***A; B; C***). We get,

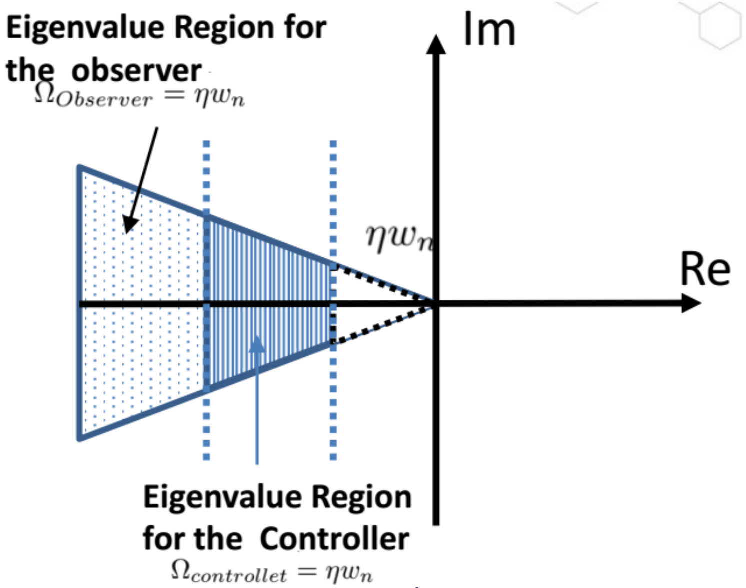
$$\overline{[A_{cl}]} = \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & I \\ I & -I & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{A} & -\boldsymbol{B}K & \boldsymbol{B}N \\ 0 & \boldsymbol{A}-L\boldsymbol{C}-\boldsymbol{B}K & \boldsymbol{B}N \\ -\boldsymbol{C} & 0 & 0 \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & I \\ I & -I & 0 \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{A}-\boldsymbol{B}K & \boldsymbol{B}N & -\boldsymbol{B}K \\ -\boldsymbol{C} & 0 & 0 \\ 0 & 0 & \boldsymbol{A}-L\boldsymbol{C} \end{pmatrix}$$

- **Problem Formulation**

$$\begin{cases} det(\overline{[A_{cl}]}) = \begin{pmatrix} SI-\boldsymbol{A}+BK & -\boldsymbol{B}N & BK \\ \boldsymbol{C} & SI & 0 \\ 0 & 0 & SI-\boldsymbol{A}+L\boldsymbol{C} \end{pmatrix} \\ = det\begin{pmatrix} SI-\boldsymbol{A}+BK & -\boldsymbol{B}N \\ \boldsymbol{C} & SI \end{pmatrix} det(SI-\boldsymbol{A}+L\boldsymbol{C}) \end{cases}$$

$$eig\left[\begin{pmatrix} SI-\boldsymbol{A}+\boldsymbol{B}K & -\boldsymbol{B}N \\ \boldsymbol{C} & SI \end{pmatrix}\right] \subseteq \Omega_{Desired\,region-controller} \quad (21)$$

$$eig\,[(SI-\boldsymbol{A}+L\boldsymbol{C})] \subseteq \Omega_{Desired\,region-oberever} \quad (22)$$




$$\Omega_{controller} > 5 * \Omega_{observer}$$

# 5-Experimental Validation

## 5DoF precise positioner

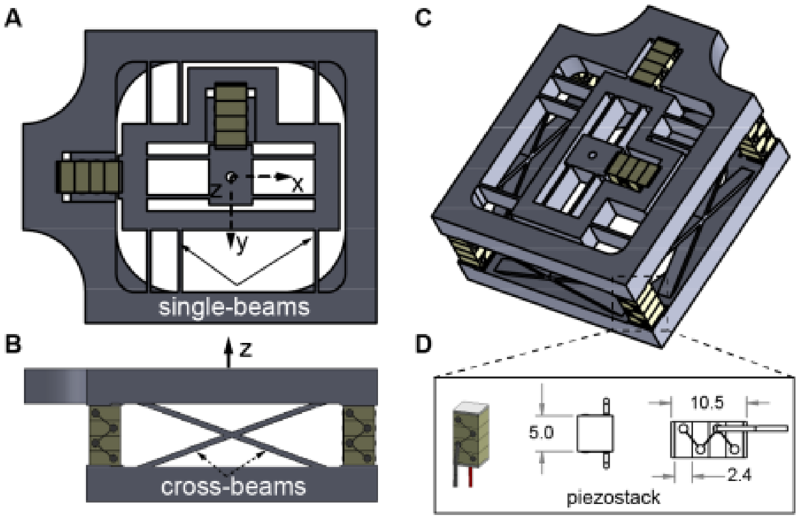



Fig.3. 3D CAD model of a 5DLL micro-positioner based on a monolithic passive structures

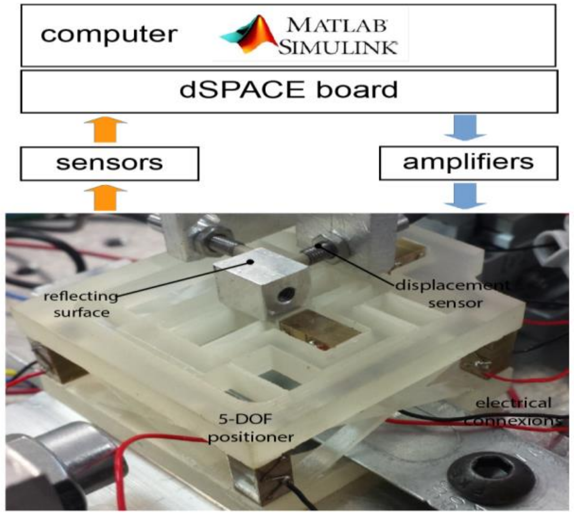



Fig.4. Experimental set-up

❖ *Proposed Model for a 3-DoF movement*

$$G(s) = \begin{pmatrix} G_{xx}(s) & 0 & G_{zx}(s) \\ 0 & G_{yy}(s) & G_{zy}(s) \\ 0 & 0 & G_{zz}(s) \end{pmatrix}$$

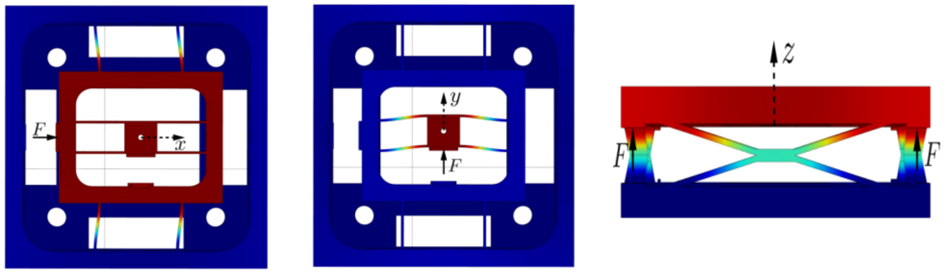



Fig.5. Simulation results showing the deformation of the structure in response to load force

# 6-Control synthesis framework

**Identification** → Interval model → Controller gains → Observer gains

$$G(s) = \begin{pmatrix} G_{xx}(s) & 0 & G_{zx}(s) \\ 0 & G_{yy}(s) & G_{zy}(s) \\ 0 & 0 & G_{zz}(s) \end{pmatrix}$$

**Box-Jenkins technique (System Identification Matlab Toolbox)**

$$G_{xx}(s) = \frac{25.84s+3.93e05}{s^2+2669s+3.471e06}$$

$$G_{yy}(s) = \frac{21.43s+1.806e05}{s^2+1666s+1.568e06}$$

$$G_{zz}(s) = \frac{-9.862s+1.548e04}{s^2+578.9s+1.29e05}$$

$$G_{zx}(s) = \frac{1.293s+969.7}{s^2+399.6s+4.153e05}$$

$$G_{zy}(s) = \frac{12.26s-5547}{s^2+5.587e-08s+8.696e05}$$

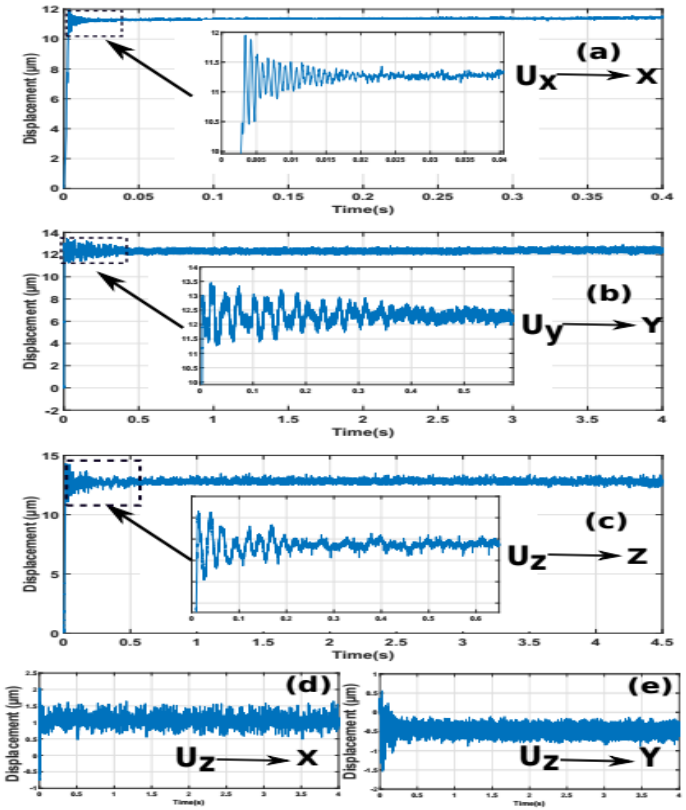



Fig. 5. Open-loop step response for $G_{xx}(s)$, $G_{yy}(s)$, $G_{zz}(s)$, $G_{zx}(s)$, and $G_{zy}(s)$.

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# 6-Control synthesis framework

Identification → **Interval model** → Controller gains → Observer gains

**we propose to consider each parameter as center and adding a radius of 10%. We therefore obtain:**

$$G_{xx}(s) = \frac{[b_{11}]s+[b_{12}]}{s^2+[a_{11}]s+[a_{12}]}$$

$$G_{yy}(s) = \frac{[b_{21}]s+[b_{22}]}{s^2+[a_{21}]s+[a_{22}]}$$

$$G_{zz}(s) = \frac{[b_{31}]s+[b_{32}]}{s^2+[a_{31}]s+[a_{32}]} \qquad (24)$$

$$G_{zx}(s) = \frac{[b_{41}]s+[b_{42}]}{s^2+[a_{41}]s+[a_{42}]}$$

$$G_{zy}(s) = \frac{[b_{51}]s+[b_{52}]}{s^2+[a_{51}]s+[a_{52}]}$$

where

$$\begin{array}{lll}
[b_{11}] = [23.2560, 28.4241] & ; & [a_{11}] = [2.4020, 2.9360] * e + 03; \\
[b_{12}] = [3.5369, 4.3231] * e + 05 & ; & [a_{12}] = [3.1238, 3.8182] * e + 06; \\
[b_{21}] = [19.2870, 23.5731] & ; & [a_{21}] = [1.4993, 1.8327] * e + 03; \\
[b_{22}] = [1.6253, 1.9867] * e + 05 & ; & [a_{22}] = [1.4111, 1.7249] * e + 06; \\
[b_{31}] = [-10.8483, -8.8757] & ; & [a_{31}] = [521.0099, 636.7901] \; ; \\
[b_{32}] = [1.3931, 1.7029] * e + 04 & ; & [a_{32}] = [1.1609, 1.4191] * e + 05; \\
[b_{41}] = [1.1636, 1.4224] & ; & [a_{41}] = [359.6399, 439.5601] \; ; \\
[b_{42}] = [0.8727, 1.0667] * e + 03 & ; & [a_{42}] = [3.7376, 4.5684] * e + 05; \\
[b_{51}] = [11.0340, 13.4861] & ; & [a_{51}] = [0.5028, 0.6146] * e - 07; \\
[b_{52}] = [-6.1018, -4.9922] * e + 03; & & [a_{52}] = [7.8263, 9.5657] * e + 05;
\end{array}$$

$$\begin{cases} \dot{x}(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t) \\ y(t) = \boldsymbol{C}x(t) \end{cases} \qquad (25)$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{xx} & 0 & \boldsymbol{A}_{zx} \\ 0 & \boldsymbol{A}_{yy} & \boldsymbol{A}_{zx} \\ 0 & 0 & \boldsymbol{A}_{zz} \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ -[a_{12}] & -[a_{11}] & 0 & 0 & -[a_{32}] & -[a_{31}] \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -[a_{22}] & -[a_{21}] & -[a_{42}] & -[a_{41}] \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -[a_{52}] & -[a_{51}] \end{bmatrix};$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}^t ; \boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{xx} & 0 & \boldsymbol{C}_{zx} \\ 0 & \boldsymbol{C}_{yy} & \boldsymbol{C}_{zy} \\ 0 & 0 & \boldsymbol{C}_{zz} \end{bmatrix}$$

$$\begin{array}{l}
C_{xx} = \begin{bmatrix} [b_{12}] - [a_{12}][b_{10}] & [b_{11}] - [a_{11}][b_{10}] \end{bmatrix}; \\
C_{yy} = \begin{bmatrix} [b_{12}] - [a_{12}][b_{10}] & [b_{11}] - [a_{11}][b_{10}] \end{bmatrix}; \\
C_{zx} = \begin{bmatrix} [b_{22}] - [a_{22}][b_{20}] & [b_{21}] - [a_{21}][b_{20}] \end{bmatrix}; \\
C_{zy} = \begin{bmatrix} [b_{32}] - [a_{32}][b_{30}] & [b_{31}] - [a_{31}][b_{30}] \end{bmatrix}; \\
C_{zz} = \begin{bmatrix} [b_{42}] - [a_{42}][b_{40}] & [b_{41}] - [a_{41}][b_{40}] \end{bmatrix};
\end{array}$$

# 6-Control synthesis framework

Identification → Interval model → **Controller gains** → Observer gains

$$det\begin{pmatrix} SI-\boldsymbol{A}+\boldsymbol{B}K & -\boldsymbol{B}N \\ \boldsymbol{C} & SI \end{pmatrix} = det\begin{pmatrix} SI-\boldsymbol{A}_{xx}-\boldsymbol{B}_{xx}K_{xx} & 0 & \boldsymbol{A}_{zx}-\boldsymbol{B}_{xx}K_{zx}-\boldsymbol{B}_{zx}K_{zz} & \boldsymbol{B}_{xx}N_x & 0 & 0 \\ 0 & SI-\boldsymbol{A}_{yy}-\boldsymbol{B}_{yy}K_{yy} & \boldsymbol{A}_{zy}-\boldsymbol{B}_{yy}K_{zy}-\boldsymbol{B}_{zy}K_{zz} & 0 & \boldsymbol{B}_{yy}N_y & 0 \\ 0 & 0 & SI-\boldsymbol{A}_{zz}-\boldsymbol{B}_{zz}K_{xx} & 0 & 0 & \boldsymbol{B}_{zz}N_z \\ -\boldsymbol{C}_{xx} & 0 & -\boldsymbol{C}_{zx} & SI & 0 & 0 \\ 0 & -\boldsymbol{C}_{yy} & -\boldsymbol{C}_{zz} & 0 & SI & 0 \\ 0 & 0 & -\boldsymbol{C}_{zz} & 0 & 0 & SI \end{pmatrix}$$

$$det\begin{pmatrix} SI-\boldsymbol{A}+\boldsymbol{B}K & -\boldsymbol{B}N \\ \boldsymbol{C} & SI \end{pmatrix} = det\begin{pmatrix} SI-\boldsymbol{A}_{xx}-\boldsymbol{B}_{xx}K_{xx} & \boldsymbol{B}_{xx}N_x \\ -\boldsymbol{C}_{xx} & SI \end{pmatrix} det\begin{pmatrix} SI-\boldsymbol{A}_{yy}-\boldsymbol{B}_{yy}K_{yy} & \boldsymbol{B}_{yy}N_y \\ -\boldsymbol{C}_{yy} & SI \end{pmatrix} det\begin{pmatrix} SI-\boldsymbol{A}_{zz}-\boldsymbol{B}_{zz}K_{zz} & \boldsymbol{B}_{zz}N_z \\ -\boldsymbol{C}_{zz} & SI \end{pmatrix}$$

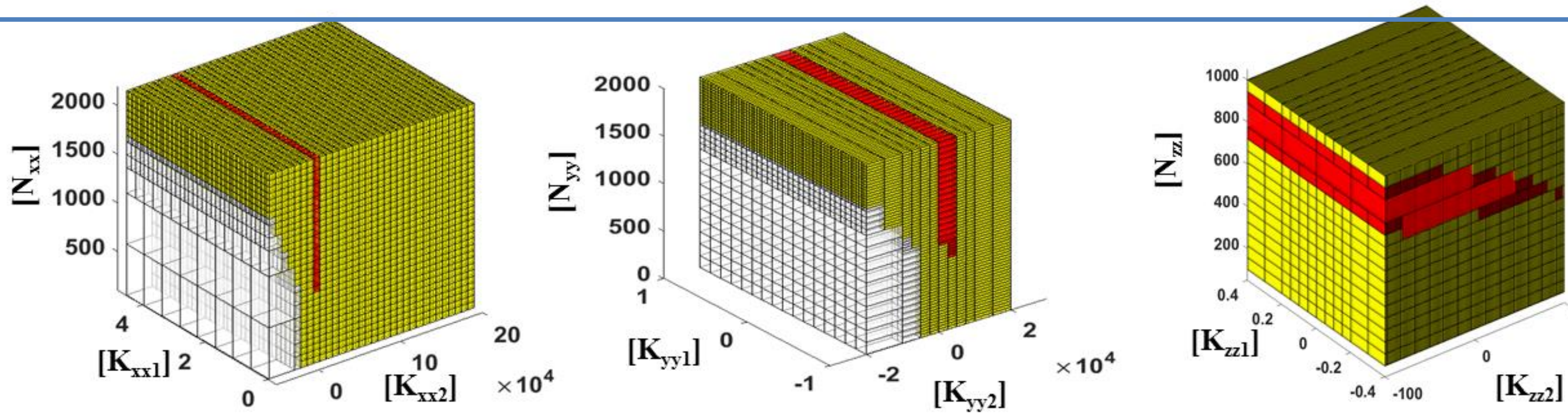


**Fig.5. Resulting solution gains for the controller.**

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# 6-Control synthesis framework

Identification → Interval model → Controller gains → **Observer gains**

$$det(SI - A + LC) = \begin{pmatrix} SI - \boldsymbol{A}_{xx} - L_{xx}\boldsymbol{C}_{xx} & 0 & \boldsymbol{A}_{zx} - L_{zx}\boldsymbol{C}_{xx} - L_{zz}\boldsymbol{C}_{zx} \\ 0 & SI - \boldsymbol{A}_{yy} - L_{yy}\boldsymbol{C}_{yy} & \boldsymbol{A}_{zy} - L_{zy}\boldsymbol{C}_{yy} - L_{zz}\boldsymbol{C}_{zy} \\ 0 & 0 & SI - \boldsymbol{A}_{zz} - L_{zz}\boldsymbol{C}_{zz} \end{pmatrix}$$

$$det(SI - \boldsymbol{A} + L\boldsymbol{C}) = det\left( SI - \boldsymbol{A}_{xx} - L_{xx}\boldsymbol{C}_{xx} \right) det\left( SI - \boldsymbol{A}_{yy} - L_{yy}\boldsymbol{C}_{yy} \right) det\left(SI - \boldsymbol{A}_{zz} - L_{zz}\boldsymbol{C}_{zz}\right)$$

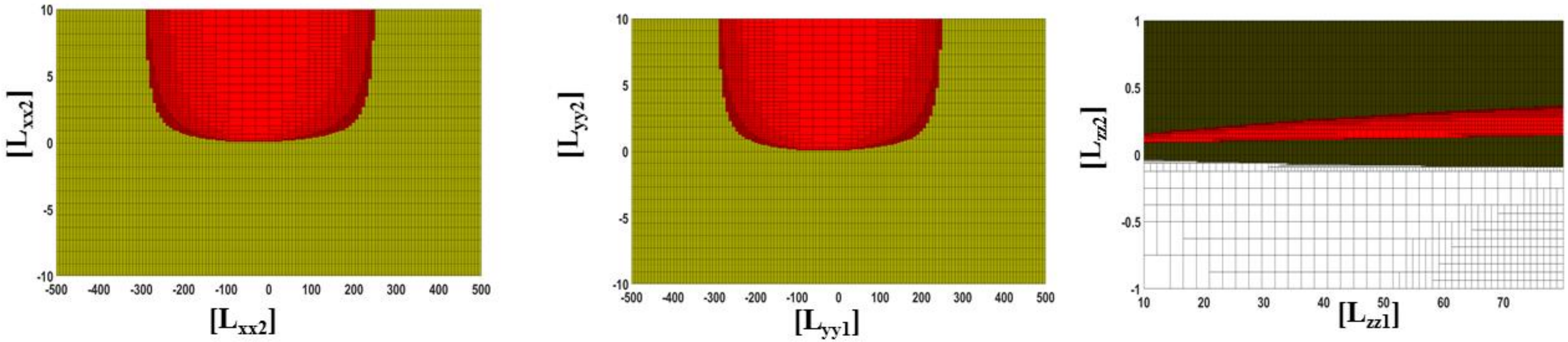



**Fig.7. Resulting solution gains for the Observer.**

# 6-Control synthesis framework

## Simulation  Validation using Monte-Carlo technique

We Select **Randomly** from the **solution boxes** he gains of the controller and the observer as:
**Controller:** $[K_{xx}; N_{xx}] = [1; 0{:}05; 1200]$, $[K_{yy}; N_{yy}] = [1; 0{:}05; 1200]$, $[K_{zz}; N_{zz}] = [0{:}2; -10; 800]$,
**Observer:** $L_{xx} = [50; 5]$ , $L_{yy} = [50; 5]$, $L_{zz} = [0{:}2; 100]$

❖ **Results for the observer**

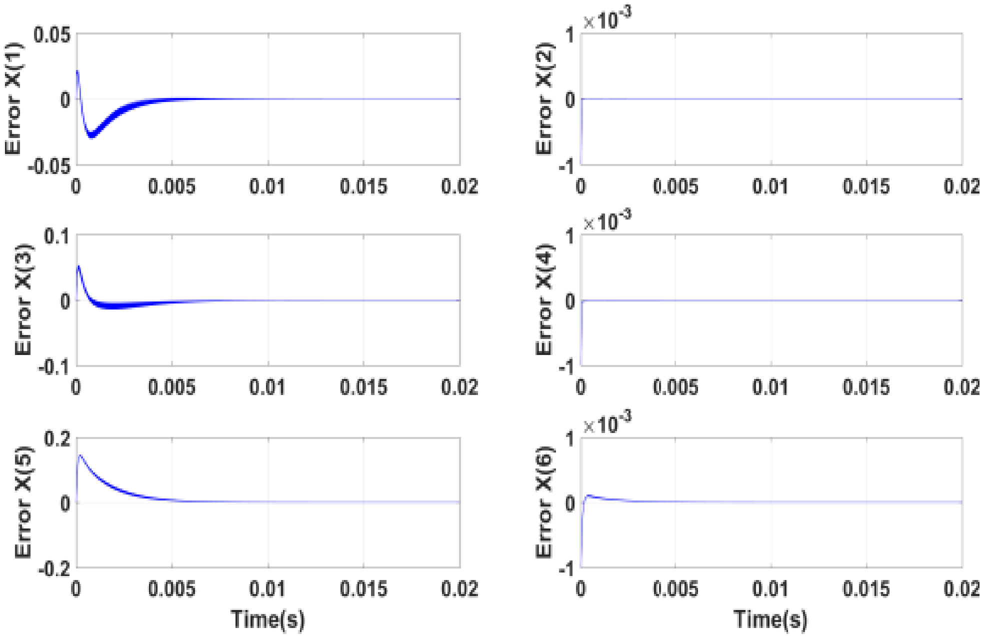



**Fig.8. The error between the real states and the estimated ones (Simulation).**

❖ **Results for the Observer-based controller**

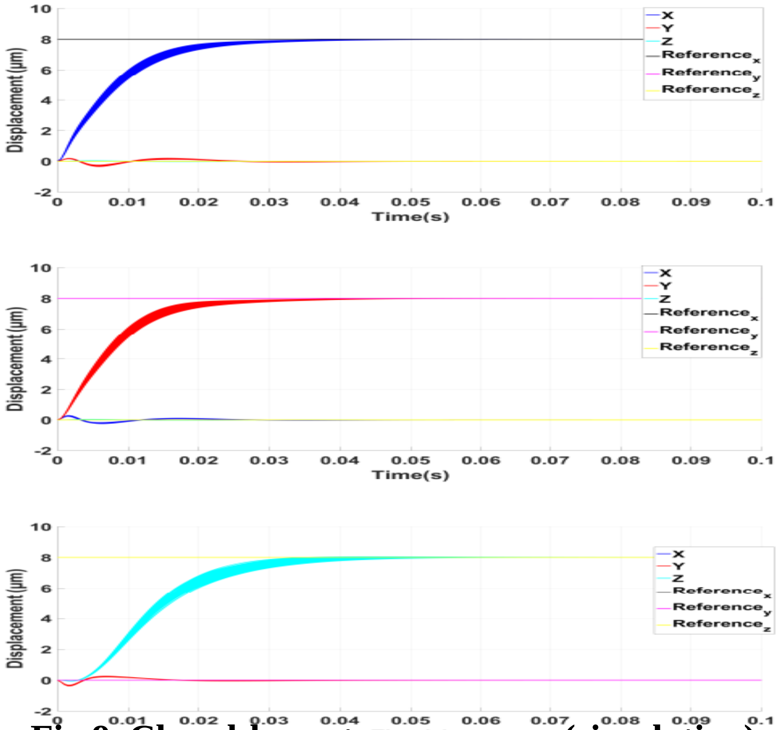



**Fig.9. Closed-loop step responses (simulation)**

# 6-Control synthesis framework

## Exprimental Validation

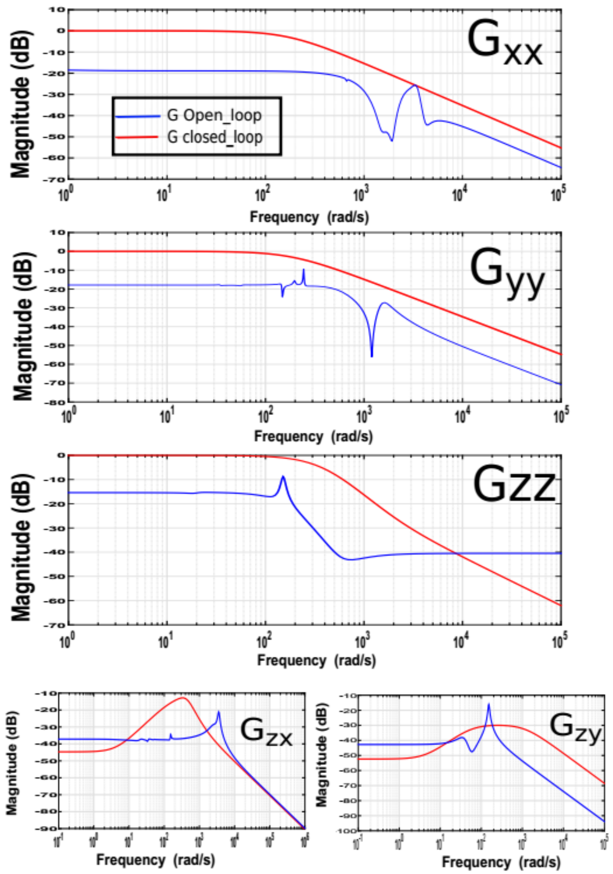


Fig.10. Frequency responses (experimentation)

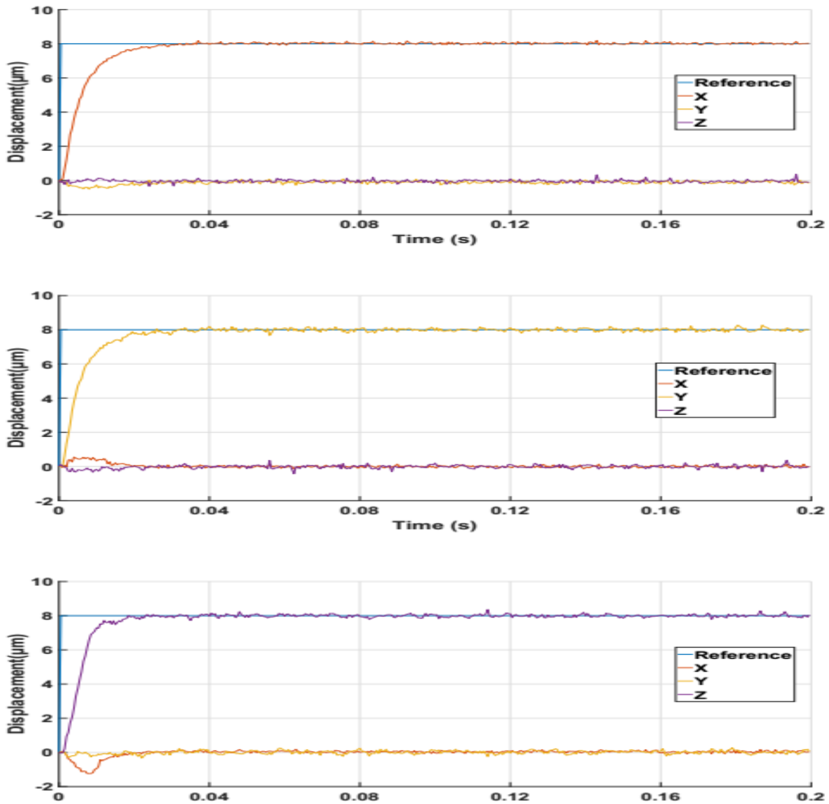


Fig.11. Closed_loop step response (experimentation)

# 6-Control synthesis framework

## Exprimental Validation

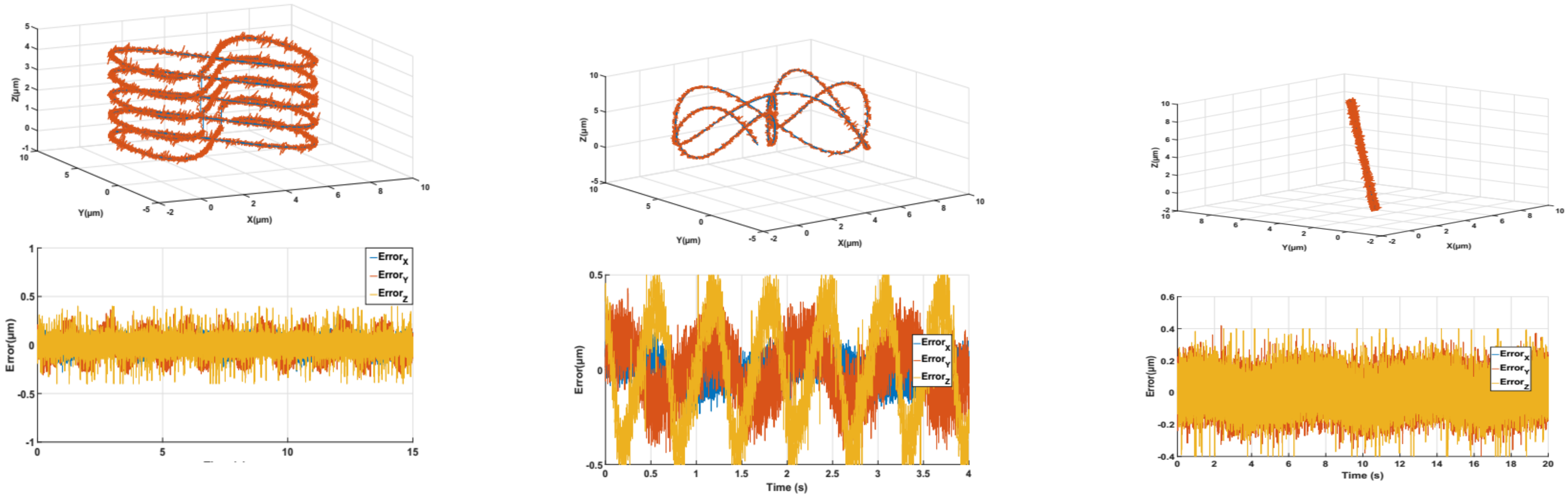
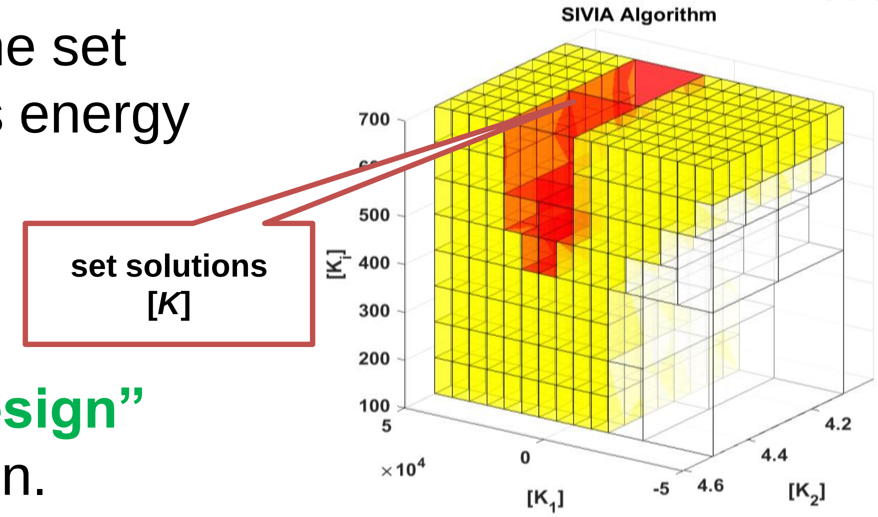



**Fig.12. Complex trajectory tests (experimentation)**

# 7- Other propositions

## a- Encountered problems and propositions

How to choose **the optimal solution** from the set solutions [*K*] that minimize the inputs/outputs energy and ensures the best behaviors ?




**State-of-the-Art** ❖ **Proposition**

**"Robust and Optimal output-feedback design"**

- ❑ The Linear Quadratic (LQ) tracker design.
- ❑ Particle Swarm Optimization (PSO) technique.

$$J = \int_0^\infty (\tilde{X}^T Q \tilde{X} + u^T R u) + \frac{1}{2} e^T V e \;\; = \frac{1}{2} tr(\boldsymbol{PS}) \;\; \Rightarrow \;\; \begin{cases} 0 = [A_c^T][P] + [P][A_c] + [C]^T K^T R K [C] + Q \\ S = X_{ss} X_{ss}^T = A_c^{-1} B_c r_0 r_0^T B_c^T A_c^{-T}. \end{cases}$$

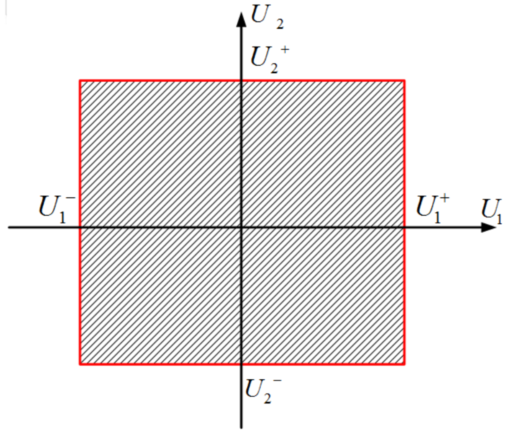
## ❖ Advantages

Robustly ensure the desired performances and, in addition to that, minimize the inputs/outputs energy described by a linear quadratic cost function.

# 7- Other propositions

## a- Encountered problems and propositions

All actuators are usually subjected to input constraints. How to find the range of robust gains that satisfy the **input constrains**?




State-of-the-Art ❖ Proposition

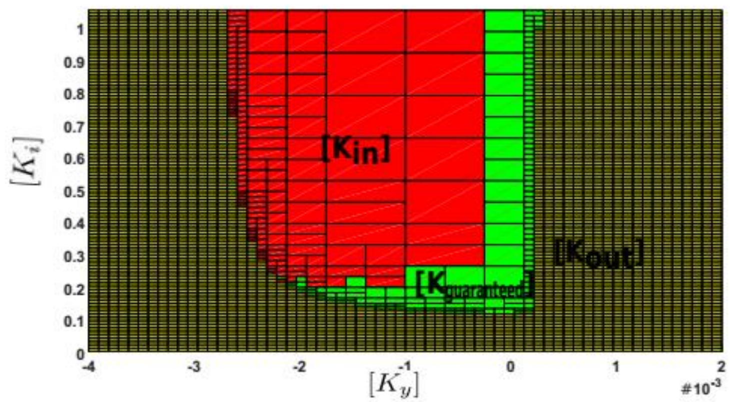
## Robust and guaranteed output-feedback control

❑ Interval computation of input control.

$$\begin{cases} u^* = (I + K^* D^*)^{-1} K^* \begin{pmatrix} C_c^t & B_c \end{pmatrix}^t (-A_c^{-1} B_c r) \\ u^*([A],[B],[C],[D],[K]) \equiv [\underline{u}, \overline{u}] \subseteq [\underline{U}, \overline{U}] \end{cases}$$

❖ Advantages

Provides the robust set solutions **[*K*]** that ensures that the magnitude of the applied control inputs are included inside the physical limitations of the actuator.



# 8-Conclusion and perspectives

## ❖ Conclusion

- ❖ State-of-the-art for linear and nonlinear control design using **interval analysis** were presented.

- ❖ The algorithm based on Set Inversion Via Interval Analysis (SIVIA) was adapted to synthesis a robust output-feedback and an observer based state-feedback controllers.

- ❖ **Interval LQ tracker** and **input constraints** syntheses were combined with the proposed SIVIA-Based algorithm to find the **optimal** and **guaranteed** gains.

- ❖ A simulation (using Monte-Carlo methods) and experiment tests were carried out to validate the proposed algorithm.

## ❖ Perspectives

- ❖ The use of the **nonlinear interval approaches** to enhance the control of Micro/nano systems Under high-speed conditions

Thank You For your Attention