A simple controller for the line following of sailboats

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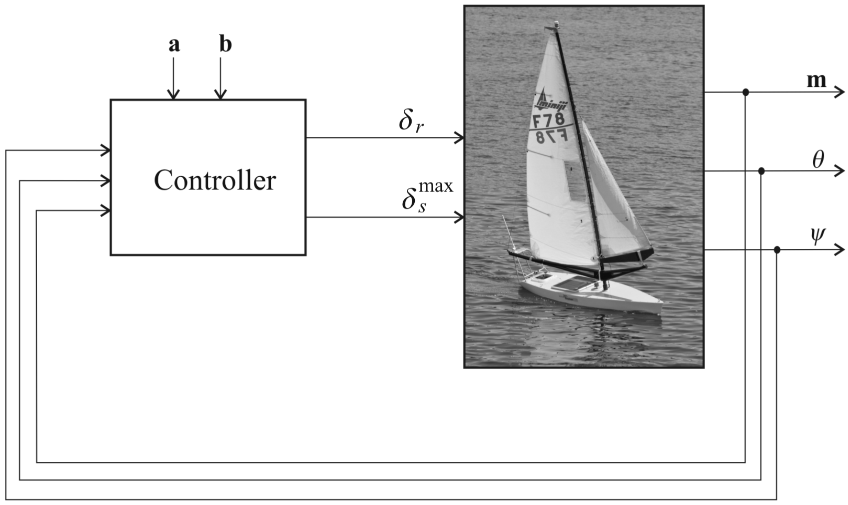
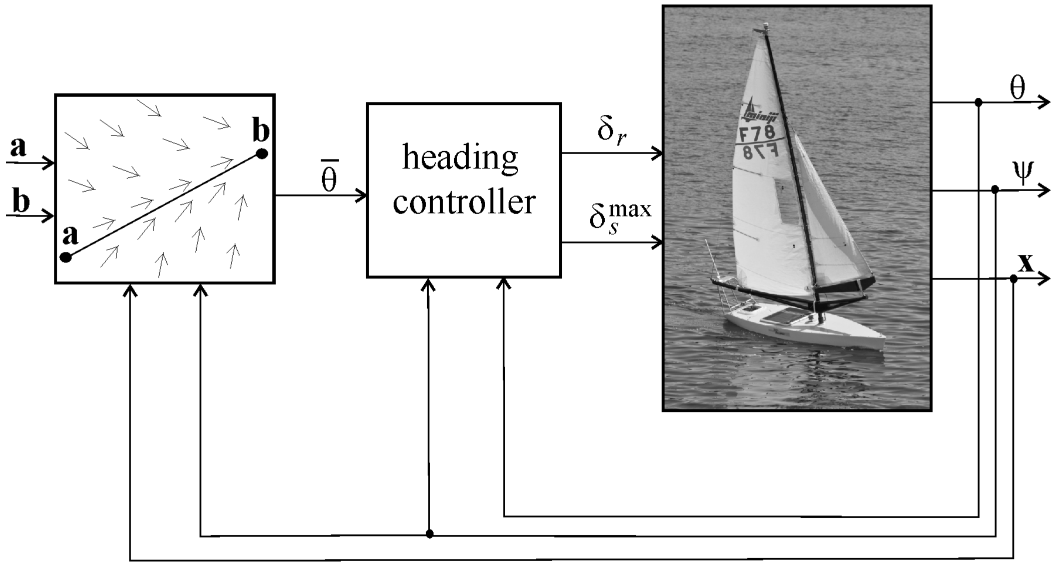


Controller of a sailboat robot

1 Line following



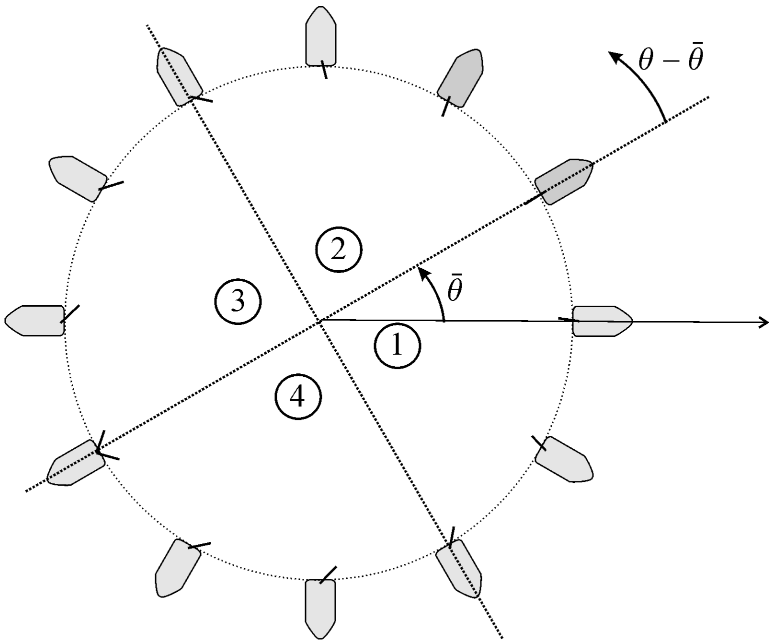
a
b
a
b
θ̄
heading
controller
δr
δs max
θ
ψ
x

Heading controller

$$
\begin{cases}
\delta_r & = \begin{cases} \delta_r^{\max}.\sin\left(\theta - \bar{\theta}\right) & \text{if } \cos\left(\theta - \bar{\theta}\right) \geq 0 \\ \delta_r^{\max}.\operatorname{sign}\left(\sin\left(\theta - \bar{\theta}\right)\right) & \text{otherwise} \end{cases} \\
\delta_s^{\max} & = \frac{\pi}{2}.\left(\frac{\cos(\psi - \bar{\theta}) + 1}{2}\right)^q.
\end{cases}
$$

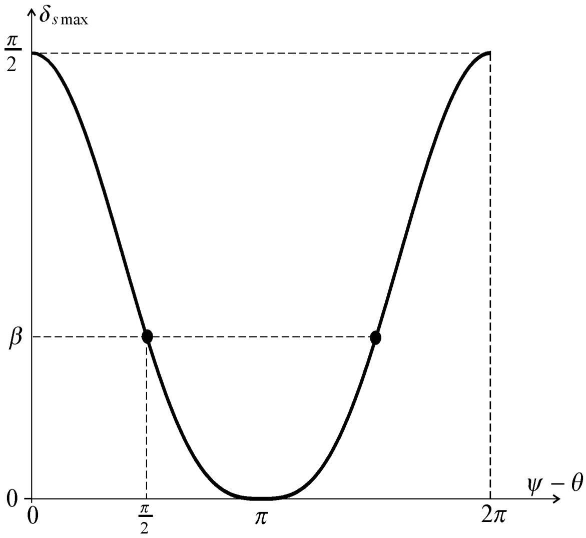
Rudder

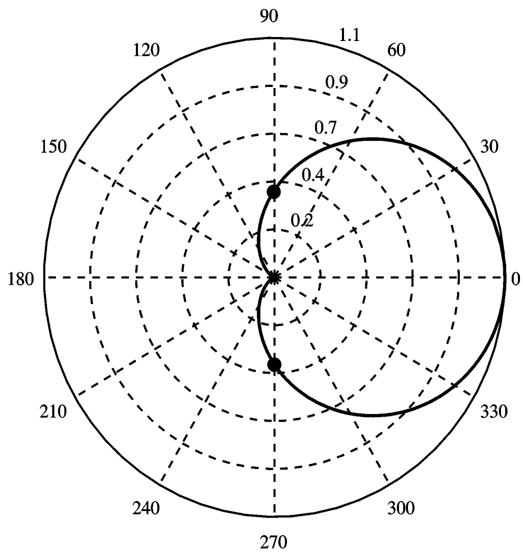
$$\delta_r = \begin{cases} \delta_r^{\max}.\sin\left(\theta-\bar{\theta}\right) & \text{if } \cos\left(\theta-\bar{\theta}\right)\geq 0 \\ \delta_r^{\max}.\text{sign}\left(\sin\left(\theta-\bar{\theta}\right)\right) & \text{otherwise} \end{cases}$$



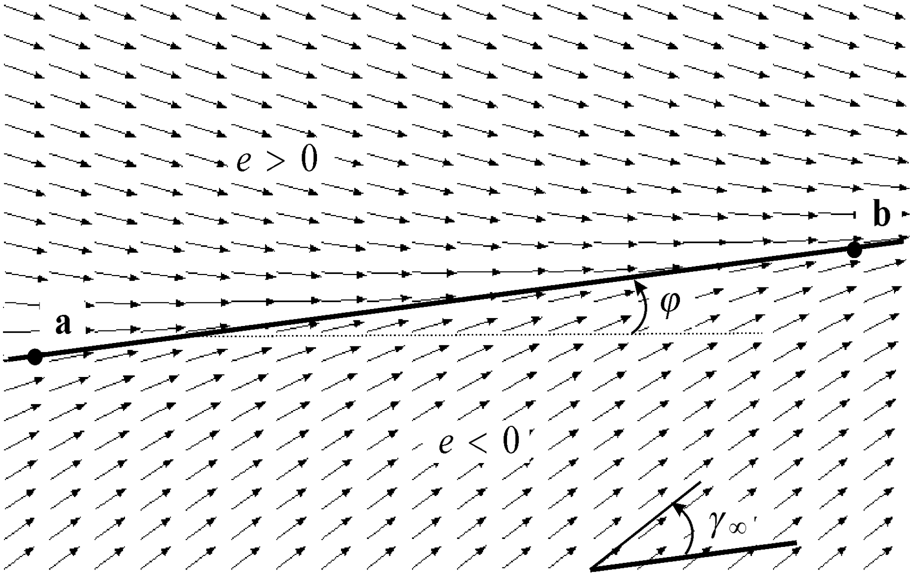
Sail

$$\delta_s^{\max} = \frac{\pi}{2} \cdot \left(\frac{\cos\left(\psi - \bar{\theta}\right) + 1}{2} \right)^q \text{ with } q = \frac{\log\left(\frac{\pi}{2\beta}\right)}{\log(2)}$$



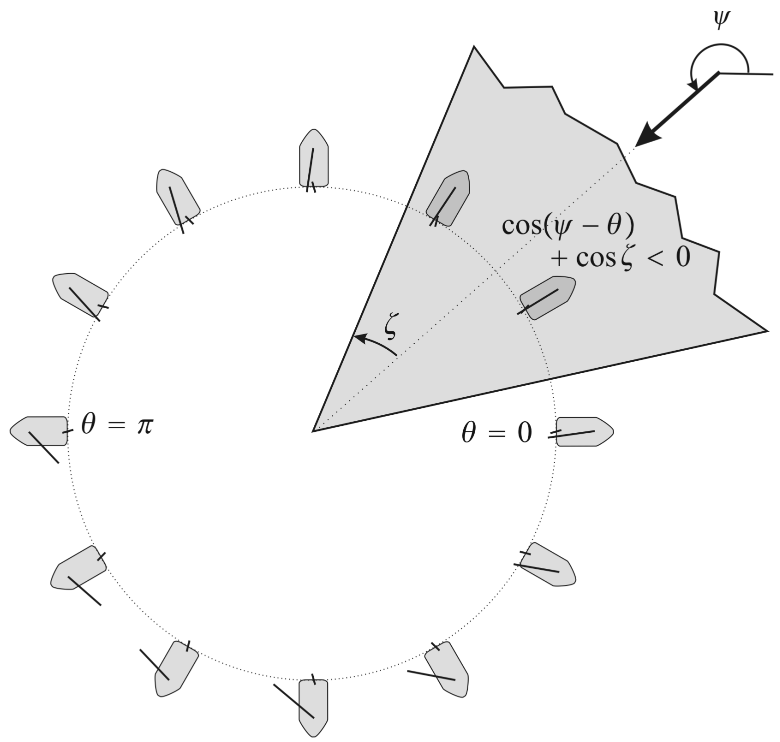


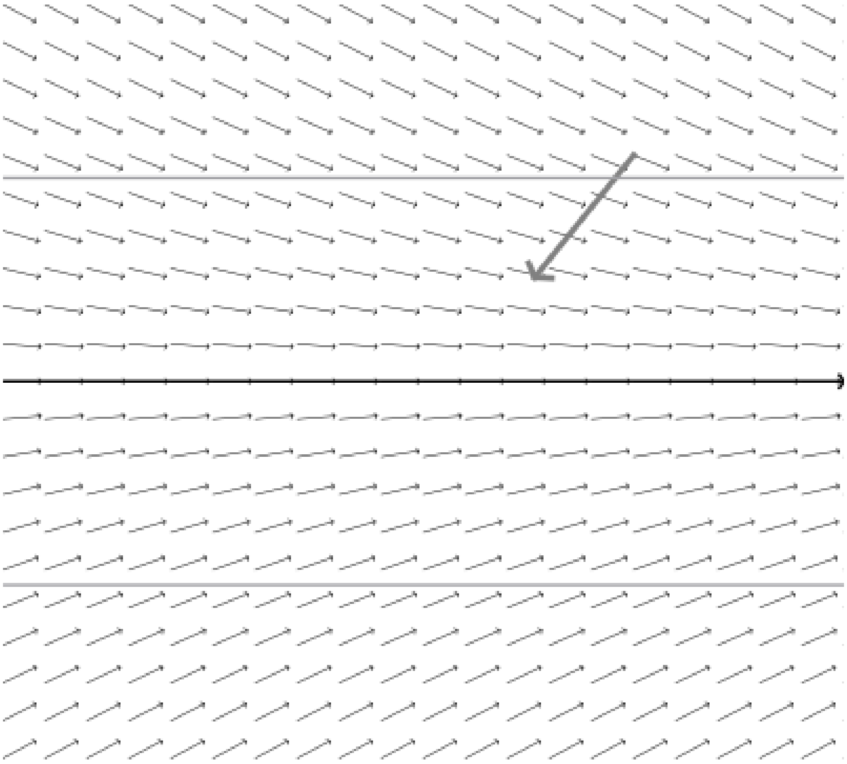
2 Vector field



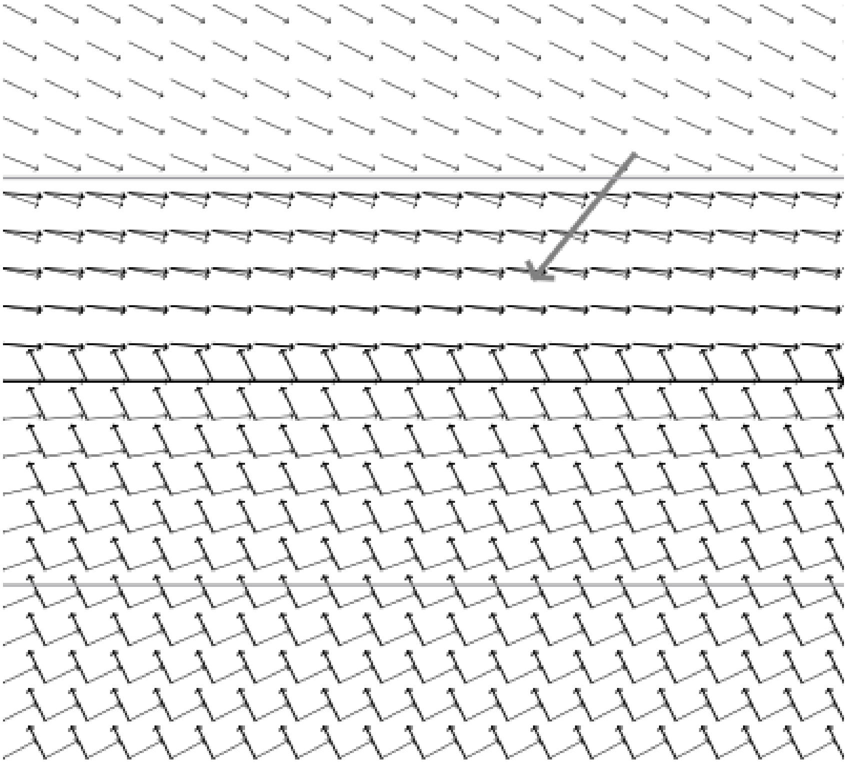
Nominal vector field: $\theta^* = \varphi - \frac{2.\gamma_\infty}{\pi}.\text{atan}\left(\frac{e}{r}\right).$

A course θ^* may be unfeasible





$$\theta^* = -\frac{2.\gamma_\infty}{\pi}.\mathsf{atan}\left(\frac{e}{r}\right)$$



Keep close hauled strategy.

http://youtu.be/pHteidmZpnY

3 Controller

Controller $\bar{\theta}\left(\mathbf{m},\mathbf{a},\mathbf{b},\psi,\gamma_{\infty},r,\zeta\right)$

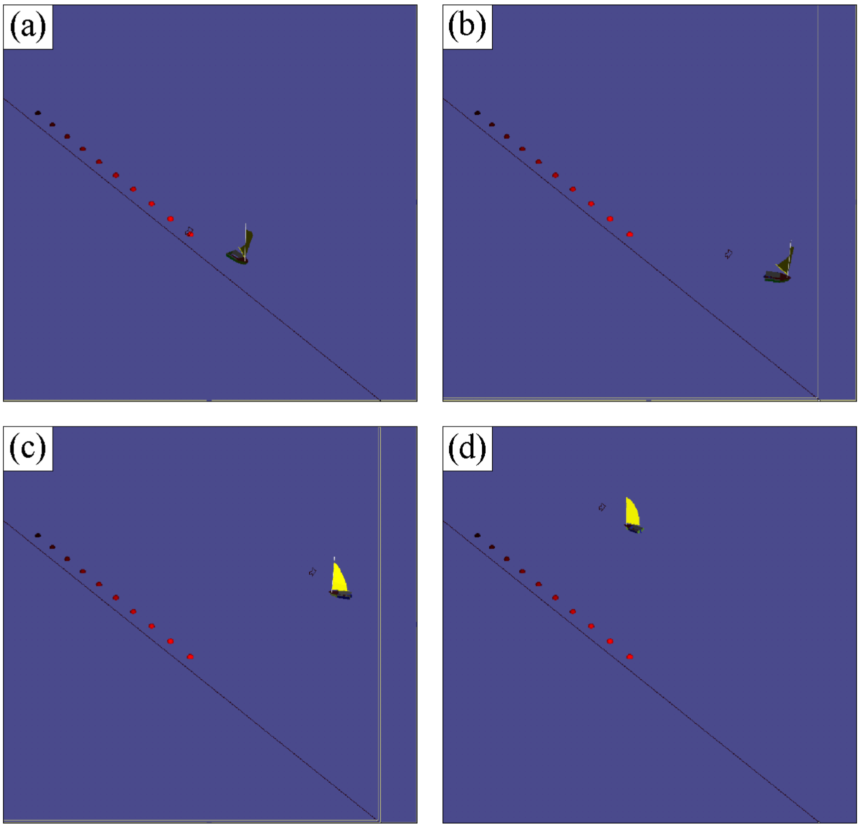
$e = \det\left(\frac{\mathbf{b}-\mathbf{a}}{\|\mathbf{b}-\mathbf{a}\|},\mathbf{m}-\mathbf{a}\right)$
$\varphi =$atan2$(\mathbf{b}-\mathbf{a})$
$\theta^{*} = \varphi - \frac{2.\gamma_{\infty}}{\pi}$.atan$\left(\frac{e}{r}\right)$
if $\cos\left(\psi-\theta^{*}\right)+\cos\zeta<0$
    or $(|e|<r$ and $(\cos(\psi-\varphi)+\cos(\zeta)<0))$
      then $\bar{\theta} = \pi + \psi - \zeta$.sign$(e)$;
      else $\bar{\theta} = \theta^{*}$;
end

Without hysteresis

Controller in: $\mathbf{m}, \theta, \psi, \mathbf{a}, \mathbf{b}$; out: $\delta_r, \delta_s^{\max}$; inout: q
$e = \det\left(\frac{\mathbf{b}-\mathbf{a}}{\|\mathbf{b}-\mathbf{a}\|}, \mathbf{m}-\mathbf{a}\right)$
if $\lvert e \rvert > \frac{r}{2}$ then $q = \text{sign}(e)$
$\varphi = \text{atan2}(\mathbf{b}-\mathbf{a})$
$\theta^* = \varphi - \frac{2.\gamma_\infty}{\pi}.\text{atan}\left(\frac{e}{r}\right)$
if $\cos(\psi - \theta^*) + \cos\zeta < 0$
or $(\lvert e \rvert < r$ and $(\cos(\psi - \varphi) + \cos\zeta < 0))$
then $\bar{\theta} = \pi + \psi - q.\zeta.$
else $\bar{\theta} = \theta^*$
end
if $\cos\left(\theta - \bar{\theta}\right) \geq 0$ then $\delta_r = \delta_r^{\max}.\sin\left(\theta - \bar{\theta}\right)$
else $\delta_r = \delta_r^{\max}.\text{sign}\left(\sin\left(\theta - \bar{\theta}\right)\right)$
$\delta_s^{\max} = \frac{\pi}{2}.\left(\frac{\cos(\psi - \bar{\theta}) + 1}{2}\right)^q.$

With hysteresis

4 Validation by simulation


(a)
(b)
(c)
(d)

5 Theoretical validation

When the wind is known, the sailboat with the heading controller is described by

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}\right).$$

The system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

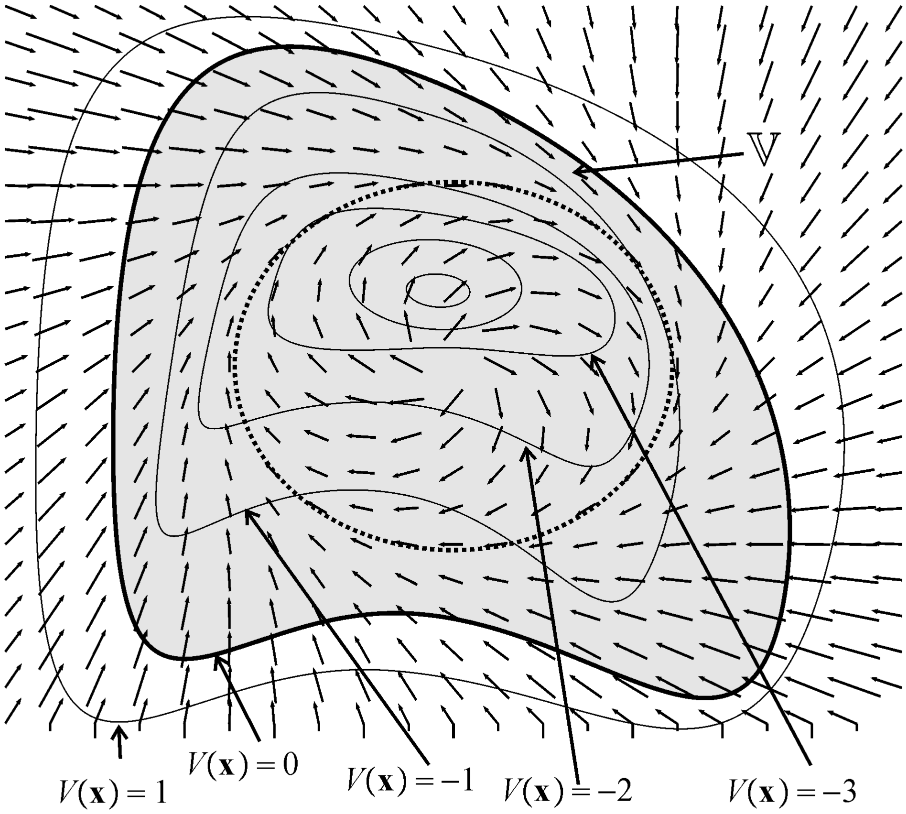
is Lyapunov-stable (1892) is there exists $V(\mathbf{x}) \geq 0$ such that

$$\begin{aligned} \dot{V}(\mathbf{x}) &< 0 \text{ if } \mathbf{x} \neq \mathbf{0}. \\ V(\mathbf{x}) &= 0 \text{ iff } \mathbf{x} = \mathbf{0} \end{aligned}$$

Definition. Consider a differentiable function $V(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$. The system is V-stable* if

$$\left(V(\mathbf{x}) \geq 0 \Rightarrow \dot{V}(\mathbf{x}) < 0\right).$$

*Jaulin, Le Bars (2012). An interval approach for stability analysis; Application to sailboat robotics. IEEE TRO.



$\mathbb{V}$
$V(\mathbf{x}) = 1$
$V(\mathbf{x}) = 0$
$V(\mathbf{x}) = -1$
$V(\mathbf{x}) = -2$
$V(\mathbf{x}) = -3$

Theorem. If the system is V-stable then

(i) $\forall \mathbf{x}(0), \exists t \geq 0$ such that $V(\mathbf{x}(t)) < 0$
(ii) if $V(\mathbf{x}(t)) < 0$ then $\forall \tau > 0,\ V(\mathbf{x}(t+\tau)) < 0$.

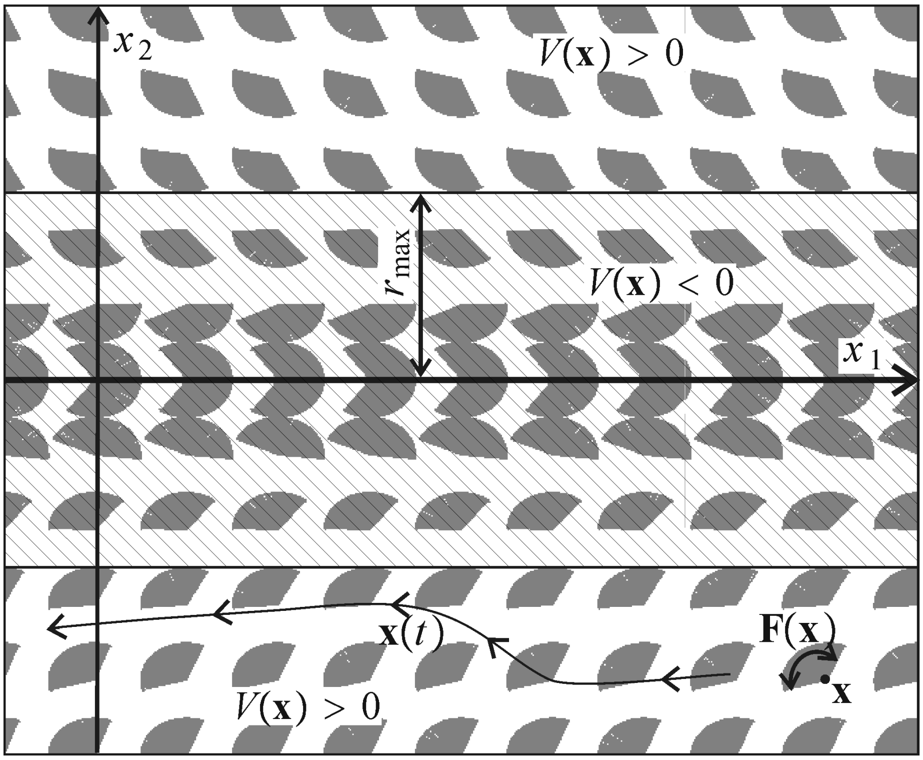
When some parameters are unknown but bounded, the state equation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

becomes a differential inclusion

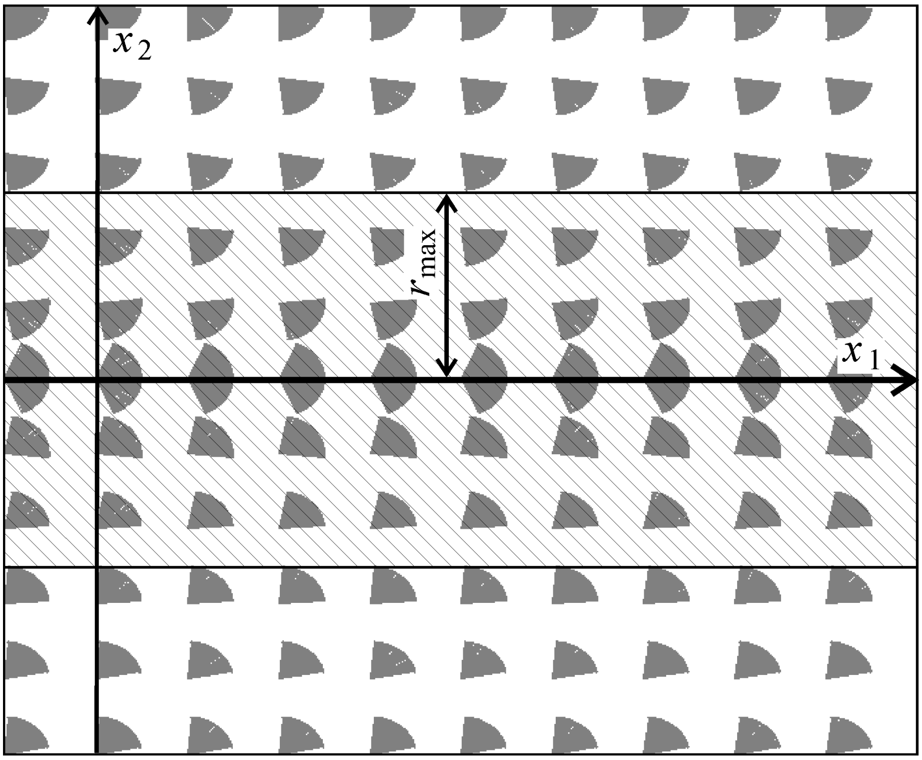
$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x}).$$

where $\mathbf{F}$ is a set-membership function.



x_2
$V(\mathbf{x}) > 0$
r_{max}
$V(\mathbf{x}) < 0$
x_1
$\mathbf{x}(t)$
$\mathbf{F}(\mathbf{x})$
$\mathbf{x}$
$V(\mathbf{x}) > 0$

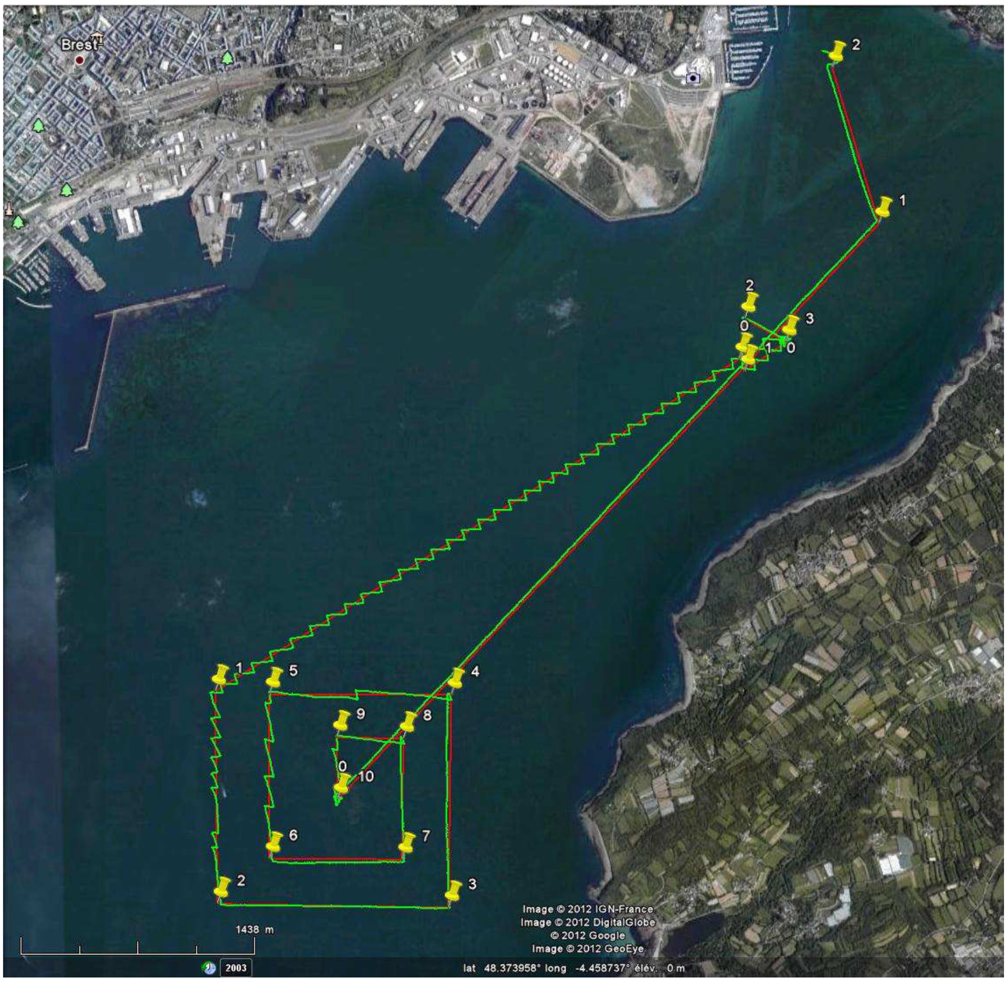
.



x_2
r_{max}
x_1

6 Experimental validation


F7
8



Brest
1438 m
Image © 2012 IGN-France
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Image © 2012 GeoEye
2003
lat 48.373958° long -4.458737° élév. 0 m

7 Conclusion

It is possible for a sailboat robot to move inside a corridor.

Essential, to create circulation rules when robot swarms are considered.

Essential to determine the responsible robot in case of accident.