RELIABLE PARAMETER ESTIMATION IN PRESENCE OF UNCERTAIN VARIABLES THAT ARE NOT ESTIMATED

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Abstract: In a bounded-error context, reliable set-inversion algorithms such as Sivia provide guaranteed estimates of the set of all the parameters deemed compatible with the selected model and the collected data, assuming that all the uncertain variables of the model are those to be estimated. In this paper we propose a new approach to estimate the parameters of interest assuming that there are other parameters that will not be estimated. This leads to the idea of set projection. A new algorithm for set projection is proposed and applied to the estimation of thermal quantities via a new experimental device to be calibrated.

Keywords: bounded-error estimation, guaranteed estimation, interval analysis, nuisance parameters, parametric models, set-membership estimation

1. INTRODUCTION

Most methods for the estimation of physical parameters minimize a possibly weighted quadratic norm of the difference between the vector of collected data $\mathbf{y}$ and the corresponding model output. The success of such a minimization, usually performed by local iterative search (by, *e.g.*, the Newton, Gauss-Newton, Levenberg-Marquardt or conjugate gradients algorithm) is uncertain for nonlinear models, for two main reasons. First, the estimate obtained is very sensitive to the initial value given to the parameter vector. Second, the search method may be trapped near a local optimum or stop before reaching the actual global optimum.

Moreover, the estimation of physical parameters should be regarded in a same way as any technique for experimental measurement and an uncertainty region should always be provided for the estimate. For the maximum-likelihood estimator, under the very strong assumption that the structure used for the model is correct and that the measurement noise is additive white and Gaussian with zero mean and known variance, the *asymptotic* variance of the estimate is the inverse of the Fisher information matrix, which can be computed. Unfortunately, such hypotheses are seldom verified as the number of samples used might be small, the measurement error may include some deterministic systematic errors or be far from being normally distributed, and the model is in general a much simplified version of reality.

To face these problems, it has been proposed to describe the parameter vector estimate as a set containing all parameter vectors that are consistent with the experimental data and the model given some bounds on the acceptable errors. The size of the set quantifies the uncertainty of the estimate. The development of this approach, called *set-membership estimation* or *bounded-error estimation* started more than thirty years ago with the seminal work of (Schweppe, 1968) and (Witsenhausen, 1968).

This paper deals with the case where, in addition to the parameters of interest, *i.e.*, the parameters to be estimated, there are some non-essential parameters (or *nuisance* parameters) subject to bounded uncertainty. To keep the dimension of parameter space sufficiently low, these nuisance parameters are not going to be estimated. However, their uncertainty will affect both the estimates of the parameters of interest and the uncertainty of these estimates.

Bounded-error estimation is recalled in Section 2. Before presenting the new set projection algorithm PROJECT, Section 3 briefly describes interval analysis and constraint propagation techniques that will allow PROJECT to be reliable. As an application, Section 4 shows that these guaranteed techniques permit the simultaneous identification of thermal resistance and Fourier time of material by the periodic method developed at CERTES by (Tang-Kwor, 1998).

2. SET-MEMBERSHIP ESTIMATION

In the sequel two types of parameters will be distinguished. The parameters of interest, *i.e.*, those to be identified, are in the *parameter vector* $\mathbf{p}$. The other non-essential parameters are gathered in a vector $\mathbf{q}$ called the *nuisance parameter vector*. It is assumed that $\mathbf{p} \in \mathbb{P}$ and $\mathbf{q} \in \mathbb{Q}$, where $\mathbb{P}$ and $\mathbb{Q}$ are known prior domains.

Let $\mathbf{e}$ be the model output error $\mathbf{e} = \mathbf{y} - \mathbf{f}(\mathbf{p}, \mathbf{q})$, where $\mathbf{y}$ is the vector of the collected data and $\mathbf{f}(\cdot, \cdot)$ the corresponding model output. In bounded-error estimation (or *set-membership estimation*), one looks for the set of all parameter vectors such that the error stays within some known feasible domain $\mathbb{E}$, *i.e.*, $\mathbf{e} \in \mathbb{E}$ (see *e.g.* (Milanese *et al.*, 1996), (Norton, 1994), (Norton, 1995) and the references therein). The set estimate then contains all values of the parameter vector that are *acceptable*, *i.e.*, consistent with the model and the collected data $\mathbf{y}$, given what is deemed an acceptable error. The size of this set quantifies the uncertainty associated with the estimated parameters.

Assume first that the value $\mathbf{q}^*$ taken by the nuisance parameter vector $\mathbf{q}$ is known. The set $\mathbb{C}$ to be estimated is the set of all the acceptable parameter vectors $\mathbf{p}$

$$\mathbb{C} = \{\mathbf{p} \in \mathbb{P}, \quad \mathbf{f}(\mathbf{p}, \mathbf{q}^*) \in \mathbb{Y}\}, \tag{1}$$

where $\mathbb{Y} = \mathbf{y} + \mathbb{E}$. Characterizing $\mathbb{C}$ is a set-inversion problem, as (1) can be rewritten as

$$\mathbb{C} = \mathbf{g}^{-1}(\mathbb{Y}) \cap \mathbb{P}, \tag{2}$$

where $\mathbf{g}(\cdot) = \mathbf{f}(\cdot, \mathbf{q}^*)$. It can be solved in a guaranteed way using the algorithm SIVIA (Jaulin *et al.*, 2001), see Section 3.

Suppose now that $\mathbf{q}^*$ is unknown. One may of course choose to estimate the set

$$\mathbb{S} = \{(\mathbf{p}, \mathbf{q}) \in \mathbb{P} \times \mathbb{Q} \mid \mathbf{f}(\mathbf{p}, \mathbf{q}) \in \mathbb{Y}\}, \tag{3}$$

which can again be seen as a set-inversion problem. However, characterizing $\mathbb{S}$ will be much more difficult than estimating $\mathbb{C}$, since the dimension of $\mathbb{S}$ is larger than that of $\mathbb{C}$ and the volume of $\mathbb{S}$ may be very large, if the parameters in $(\mathbf{p}, \mathbf{q})$ are not identifiable.

Since the value of $\mathbf{q}$ is not considered essential, an alternative simpler approach is to characterize the set Π of all the acceptable parameter vectors $\mathbf{p}$ under the assumption that $\mathbf{q}$ belongs to its prior domain, *i.e.*,

$$\Pi = \{\mathbf{p} \in \mathbb{P} \mid \exists \mathbf{q} \in \mathbb{Q}, \mathbf{f}(\mathbf{p}, \mathbf{q}) \in \mathbb{Y}\}. \tag{4}$$

The estimation of the acceptable values of $\mathbf{q}$ is then given up to simplify computation.

While $\mathbb{C}$ is a cut of $\mathbb{S}$, Π is the projection of $\mathbb{S}$ onto the $\mathbf{p}$-space (see Figure 1)

$$\Pi = proj_{\mathbb{P}}\mathbb{S}. \tag{5}$$

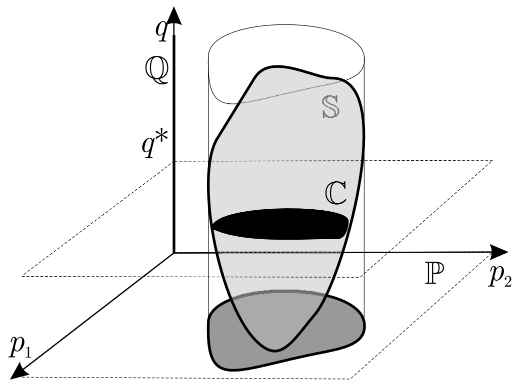

Fig. 1. The various set estimates with $dim\mathbf{p} = 2$ and $dim\mathbf{q} = 1$

Remark 1. The inclusion $\mathbb{C} \subset \Pi$ illustrates the fact that when $\mathbf{q}$ is uncertain, the uncertainty on $\mathbf{p}$ increases.

The basic tools for the characterization of Π will now be presented.

3. INTERVAL ANALYSIS

Several techniques have been developed to describe estimated sets, using various objects, such as zonotopes, ellipsoids, or vector intervals (Milanese *et al.*, 1996). When $\mathbf{g}$ is non-linear, the interval analysis techniques, using intervals and unions of intervals are particularly well suited. For any given function $\mathbf{g}(\mathbf{p})$, interval analysis permits the computation of an outer approximation $[\mathbf{g}]([\mathbf{p}])$ of $\mathbf{g}([\mathbf{p}])$ where $[\mathbf{p}]$ and $[\mathbf{g}]([\mathbf{p}])$ are boxes (or vector intervals), *i.e.*, Cartesian products of intervals, and where the image of a box $[\mathbf{p}]$ by $\mathbf{g}$ is

$$\mathbf{g}([\mathbf{p}]) \triangleq \{\mathbf{y} \mid \exists \mathbf{p} \in [\mathbf{p}],\ \mathbf{y} = \mathbf{g}(\mathbf{p})\}.$$

The function $[\mathbf{g}](\cdot)$ satisfying

$$\forall [\mathbf{p}],\ \mathbf{g}([\mathbf{p}]) \subset [\mathbf{g}]([\mathbf{p}]), \tag{6}$$

is called an *inclusion function* associated with $\mathbf{g}$. Inclusion functions provide reliable outer approximations of images of boxes, while other numerical approaches just evaluate the images of discrete sets of values.

3.1 *Set inverter*

Assume that the set to be estimated is $\mathbb{C}$ as defined by (2). Interval analysis allows us to obtain a reliable enclosure of $\mathbb{C}$ as defined by

$$\underline{\mathbb{C}} \subset \mathbb{C} \subset \overline{\mathbb{C}}. \tag{7}$$

The inner approximation $\underline{\mathbb{C}}$ of $\mathbb{C}$ consists of boxes $[\mathbf{p}]$ that have been proved acceptable, *i.e.* such that $\forall \mathbf{p} \in [\mathbf{p}]$, $\mathbf{p}$ is acceptable. To prove that $[\mathbf{p}]$ is acceptable, it suffices to check that $[\mathbf{g}]([\mathbf{p}]) \subset \mathbb{Y}$ and use (6). Else, if $[\mathbf{g}]([\mathbf{p}]) \cap \mathbb{Y} = \varnothing$, then the whole box $[\mathbf{p}]$ can be rejected. Otherwise, no conclusion is reached and the box $[\mathbf{p}]$ is said undetermined. The recursive algorithm SIVIA (*Set Inverter Via Interval Analysis*) (Jaulin *et al.*, 2001) partitions the prior space $\mathbb{P}$ into boxes $[\mathbf{p}]$ to be submitted to these tests. Any undetermined box is bisected and tested again, unless its size is less than a precision parameter ε to be tuned by the user, which ensures that the algorithm terminates after a finite number of iterations. The outer approximation is then computed as $\overline{\mathbb{C}} = \underline{\mathbb{C}} \cup \Delta\mathbb{C}$ where $\Delta\mathbb{C}$ is the union of all remaining undetermined boxes.

As SIVIA is a branch-and-bound algorithm, the computational time and memory space required increase exponentially with the dimension of $\mathbf{p}$. *Contractors* may also be used to reduce the size of the box to be tested without bisection (Jaulin *et al.*, 2001). For any given set $\mathbb{C}$, a contractor $\mathcal{C}_{\mathbb{C}}$ is an algorithm computing a box $[\mathbf{p}_0'] = \mathcal{C}_{\mathbb{C}}([\mathbf{p}_0])$ from any given box $[\mathbf{p}_0]$ in $\mathbb{P}$, such that the following properties hold

$$[\mathbf{p}_0'] \subset [\mathbf{p}_0], \tag{8}$$

$$[\mathbf{p}_0'] \cap \mathbb{C} = [\mathbf{p}_0] \cap \mathbb{C}. \tag{9}$$

(see Figure 2). When used in SIVIA before the tests, contractors may eliminate boxes such that $[\mathbf{p}] \cap \mathbb{C} = \varnothing$ without any bisection and may reduce the size of undetermined boxes for which $[\mathbf{p}] \cap \mathbb{C} \neq \varnothing$. Contractors are especially useful when the set to be estimated is small, or when the large dimension of $\mathbf{p}$ makes the use of the basic version of SIVIA intractable. The resulting algorithm to be used here is called SIVIAP and is a direct variation of SIVIA using contractors (see (Jaulin *et al.*, 2001) and the references therein).

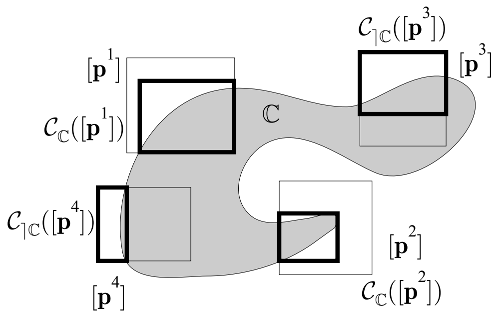

Fig. 2. Contractors $\mathcal{C}_{\mathbb{C}}$ for a given set $\mathbb{C}$ and $\mathcal{C}_{\rceil\mathbb{C}}$ for the complementary set $\rceil\mathbb{C}$; $\mathcal{C}_{\mathbb{C}}$ and $\mathcal{C}_{\rceil\mathbb{C}}$ are respectively pessimistic on $[\mathbf{p}^1]$ and $[\mathbf{p}^3]$, but optimal on $[\mathbf{p}^2]$ and $[\mathbf{p}^4]$

3.2 *Project*

When only Π is to be characterized, one can use another algorithm called PROJECT (Braems, 2002), (Jaulin *et al.*, 2002). This algorithm computes inner and outer approximations $\underline{\Pi}$ and $\overline{\Pi}$ of the set Π defined by (4). As only the $\mathbf{p}$-space is partitioned, the memory and computational time required are much smaller than for a full characterization of $\mathbb{S}$. Obviously, the main difference between PROJECT and SIVIA lies in the tests to be implemented. In SIVIA, the outer approximation $[\mathbf{g}]([\mathbf{p}])$ is directly used to test the acceptability of *all* elements of $[\mathbf{p}]$. Here, to characterize Π, $[\mathbf{p}]$ will be said acceptable if *there exists* $\mathbf{q} \in \mathbb{Q}$ such that $\mathbf{f}([\mathbf{p}], \mathbf{q}) \subset \mathbb{Y}$. Feasible point finders then require specific approaches. An algorithm based on partitioning can be found in (Jaulin and Walter, 1996). In order to allow consideration of higher dimensions, the procedure implemented in PROJECT uses contractors. Because of lack of space, PROJECT will not be presented in detail, and we shall only describe one of its constituants, INSIDE (Table 1), which for a given box $[\mathbf{p}_0]$ evaluates if there is a feasible $\mathbf{q}$ in $[\mathbf{q}_0]$.

In Table 1, $\mathcal{C}_{\mathbb{S}}$ is a contractor associated with $\mathbb{S}$ and $\mathcal{C}_{\rceil\mathbb{S}}$ a contractor associated with the complementary set $\rceil\mathbb{S}$ (see Figure 2). At Step 6, if $[\overline{\mathbf{p}}] \neq [\mathbf{p}_0]$, then the part $\mathcal{P}$ of $[\mathbf{p}_0]$ that has been eliminated

Table 1. INSIDE

Algorithm INSIDE (in:$[\mathbf{p}_0]$,$[\mathbf{q}_0]$; out: $[\overline{\mathbf{p}}]$)

1	$\mathcal{L} := \{[\mathbf{q}_0]\}$; $[\overline{\mathbf{p}}] := [\mathbf{p}_0]$;
2	do
3	take the first box $[\mathbf{q}]$ out of $\mathcal{L}$;
4	do
5	$\mathbf{q}_0 :=$ center $([\mathbf{q}])$;
6	$[\overline{\mathbf{p}}] := proj_{\mathbb{P}}\mathcal{C}_{\rceil\mathbb{S}}([\overline{\mathbf{p}}], \mathbf{q}_0)$;
7	$[\mathbf{p}_1] := [\overline{\mathbf{p}}]$;
8	$\left([\mathbf{p}_1'], [\mathbf{q}]\right) := \mathcal{C}_{\mathbb{S}}([\mathbf{p}_1], [\mathbf{q}])$;
9	while contraction of $[\overline{\mathbf{p}}]$ is significant;
10	if $([\overline{\mathbf{p}}] = \emptyset)$, return;
11	if $w([\mathbf{q}]) > w([\mathbf{p}_0])$,
12	bisect $[\mathbf{q}]$ into $[\mathbf{q}_1]$ and $[\mathbf{q}_2]$;
13	store $[\mathbf{q}_1]$ and $[\mathbf{q}_2]$ at the end of $\mathcal{L}$;
14	while $\mathcal{L} \neq \varnothing$.

by the contractor for the complementary set satisfies $(\mathcal{P}, \mathbf{q}_0) \cap \rceil\mathbb{S} = \emptyset$, so $(\mathcal{P}, \mathbf{q}_0) \subset \mathbb{S}$, $\mathbf{q}_0$ is feasible and $\mathcal{P} \subset \Pi$. Otherwise, $\mathbf{q}_0$ has not been proved feasible, and other feasible points have to be found in $[\mathbf{q}]$. Step 8 then allows a contraction of the domain $[\mathbf{q}]$, and both contractors are applied as long as contraction of $[\overline{\mathbf{p}}]$ is significant. If the resulting box $[\overline{\mathbf{p}}]$ is empty, then the whole box $[\mathbf{p}_0]$ has been proved feasible. Otherwise, it is still undetermined, as some points in $[\overline{\mathbf{p}}]$ may either belong to Π or to $\rceil\Pi$. The box $[\mathbf{q}]$ is then bisected unless its size is less than some precision parameter, here chosen as the width of the initial box $[\mathbf{p}_0]$ to be tested. Finally, the resulting algorithm INSIDE returns a box $[\overline{\mathbf{p}}]$ that satisfies $[\mathbf{p}_0] \cap \rceil\Pi = [\overline{\mathbf{p}}] \cap \rceil\Pi$, so INSIDE is a contractor for $\rceil\Pi$. As it does not modify $[\mathbf{p}_0]$, another routine also based on contractors can be called before INSIDE to reduce the size of the boxes to be tested (see (Jaulin *et al.*, 2002)).

4. APPLICATION

Techniques such as the flash method (see (Navarette *et al.*, 2000), (Thermitus and Laurent, 1997) and the references therein) and the periodic methods (Mattei and Tang Kwor, 1999) have recently been developed for the simultaneous identification of several thermo-physical parameters. To test the set-up developed in (Tang-Kwor, 1998) and based on a periodic method, reliable set estimation will be used to identify the thermo-physical characteristics of a sample under study. For that purpose, the uncertainty intrinsic to this experimental set-up will have to be taken into account.

The experimental set-up is as shown on Figure 3. The sample under study is fixed within a metallic rack by a glue with very large conductivity. While the front side of the rack is fixed to a heating device, the rear side is in contact with air at ambient temperature. Radiative shields are used to reduce lateral heat losses. The heating sequence consists of five sinusoids of angular frequency ω_i, $i = 1, \ldots, 5$ and the temperatures of the rear and front sides are measured with thermocouples. The experimental temperature spectra are used

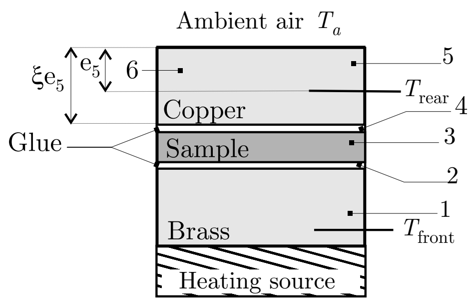

Fig. 3. The experimental set-up: 1: brass, 2: glue, 3: PVC, 4: glue, 5: copper

to estimate the experimental frequency response, taken as the ratio of the Fourier transforms of the output and the input

$$H^{\text{mes}}(j\omega_i) = H_i^{\text{mes}} = \frac{T_{\text{rear}}^{\text{mes}}(j\omega_i)}{T_{\text{front}}^{\text{mes}}(j\omega_i)}. \qquad (10)$$

The data are corrupted by several measurement errors, including the device error, the error in the computation of the spectrum, and the reading error. As only 30 measurements have been performed, no statistical information can be reliably inferred, and a bounded-error context will be considered. For each input angular frequencies ω_i, $i = 1, \ldots, 5$ the output interval $[H_i^{\text{mes}}]$, is defined as

$$[H_i^{\text{mes}}] = \left[\inf_{j=1..30} H_i^{\text{mes}j}, \sup_{j=1..30} H_i^{\text{mes}j}\right].$$

to be gathered in the interval data vector $[\mathbf{H}^{\text{mes}}]$.

This system is modeled with a series of one-dimensional quadrupoles. For the k-th layer of homogeneous material, the relationship between the front pair temperature/flux $(T_{k-1}(s), \varphi_{k-1}(s))$ and the rear one $(T_k(s), \varphi_k(s))$ is given by

$$\begin{bmatrix} T_{k-1}(s) \\ \varphi_{k-1}(s) \end{bmatrix} = Z_k(s) \begin{bmatrix} T_k(s) \\ \varphi_k(s) \end{bmatrix},$$

where

$$Z_k(s) = \begin{bmatrix} \cosh\sqrt{\tau_k s} & \frac{R_k}{\sqrt{\tau_k s}} \sinh\sqrt{\tau_k s} \\ \frac{R_k}{\sqrt{\tau_k s}} \sinh\sqrt{\tau_k s} & \cosh\sqrt{\tau_k s} \end{bmatrix},$$

and R_k and τ_k are respectively the thermal resistance and the Fourier time of the k-th layer. For $k = 2, 4$, the glue is supposed with no inertia, so $Z_k(s)$ just depends on the resistance $R_2 = R_4 = R_g$. The i-th component of the output vector associated with $[\mathbf{H}^{\text{mes}}]$ is then given by

$$H_i(\mathbf{p}, \mathbf{q}) = \left. \frac{T_{\text{rear}}(s)}{T_{\text{front}}(s)} \right|_{s=j\omega_i}, \quad i = 1, \ldots, 5, \quad (11)$$

where the front temperature/flux pair is related to the ambient temperature/convective flux pair (T_a, φ_a) by

$$\begin{bmatrix} T_{\text{front}}(s) \\ \varphi_{\text{front}}(s) \end{bmatrix} = \bigotimes_{k=1,\ldots,5} Z_k(s) \begin{bmatrix} T_a(s) \\ \varphi_a(s) \end{bmatrix},$$

and the rear pair temperature/flux is given by

$$\begin{bmatrix} T_{\text{rear}}(s) \\ \varphi_{\text{rear}}(s) \end{bmatrix} = Z_6(s) \begin{bmatrix} T_a(s) \\ \varphi_a(s) \end{bmatrix}.$$

Finally, heat transfers at ambient temperature are modeled by $\varphi_a = hT_a$, where the heat surface exchange coefficient h is generally assumed to be constant.

The thermal resistances R_k and Fourier time τ_k of each layer intervening in $H_i(\mathbf{p}, \mathbf{q})$ as defined by (11) depend on several uncertain physical quantities:
(i) the thermo-physical characteristics of the non-inertial materials (copper, brass and sample) cannot be identified without destroying the device because of the glue; their prior uncertainty then corresponds to the range of values found in literature;
(ii) the thermo-physical characteristic of the glue is provided with its uncertainty by the manufacturer;
(iii) the thickness e_k of each layer is measured with uncertainty;
Moreover, the exchange heat coefficient h is assumed to be constant but inside an interval range to take into account model error.

The parameter vector $\mathbf{p}$ to be identified is then

$$\mathbf{p} = \left(R_3 \ \sqrt{\tau_3} \right), \qquad (12)$$

while the nuisance parameter vector $\mathbf{q}$ contains all other uncertain quantities

$$\mathbf{q} = \left(R_1 \ \sqrt{\tau_1} \ R_g \ R_5 \ \sqrt{\tau_5} \ \xi \ h \right), \qquad (13)$$

where ξ is the position of the thermocouple in the copper layer (see Figure 3). Prior domains $\mathbb{Q}_i$ for the uncertain quantities q_i's are presented in Table 2, specifying their center q_{i0} and relative radius $\Delta_r q_i$.

Table 2. Uncertainty interval associated with each nuisance parameter

i	q_i	q_{i0}	$\Delta_r q_i$
1	$R_1(10^{-5}\text{SI})$	2.90	**38%**
2	$\sqrt{\tau_1}$	5.19×10^{-4}	**39%**
3	$R_g(10^{-5}\text{SI})$	3.05	**41%**
4	$R_5(10^{-5}\text{SI})$	2.5	4%
5	$\sqrt{\tau_5}$	0.92	2%
6	ξ	0.5005×10^{-3}	**40%**
7	h	7.5	**33%**

The sample under study is made of PVC for which only rough prior values for the thermo-physical values are available. The prior space for $\mathbf{p}$ is thus taken as $\mathbb{P} = [0.014, 0.047] \times [7.2, 23]$.

4.1 *Known nuisance parameters*

Assuming that the nuisance parameter q_i is known amounts to fixing its numerical value $\widehat{q}_i$. For the nuisance parameters q_i, $i = 1, 2, 4, 5, 6$ there exists an actual constant value q_i. As in (Tang-Kwor, 1998), it will be assumed for the time being that this value is equal to q_{i0}. The other nuisance parameters may actually vary during the experiment or characterize a structural uncertainty. Fixing their value is thus a strong hypothesis: in (Tang-Kwor, 1998), q_7 and q_3 have been arbitrarily chosen as $\widehat{q}_7 = \underline{\mathbb{Q}}_7$ and $\widehat{q}_3 = 0$, which corresponds to neglecting the glue layer. Contrary to the assumption $\widehat{q}_3 = 0$, the hypothesis $\widehat{q}_7 = \underline{\mathbb{Q}}_7$ has been invalidated as it could be proved that no acceptable solution exists in a bounded-error context (Braems, 2002). In the following, it will be assumed that

$$\widehat{q}_3 = 0 \text{ and } \widehat{q}_7 = \overline{\mathbb{Q}}_7.$$

Since $\widehat{\mathbf{q}}$ is fixed at a known numerical value,

$$\mathbb{C} = \{\mathbf{p} \in \mathbb{P} \mid \mathbf{H}(\mathbf{p}, \widehat{\mathbf{q}}) \in [\mathbf{H}^{\text{mes}}]\}.$$

can now be characterized by set inversion.

In 17.8 s on a Pentium IV at 1.7 GHz, for $\varepsilon = 0.01$, SIVIAP provides two non-empty sets $\underline{\mathbb{C}}$ and $\overline{\mathbb{C}}$ (see Figure 4). The projection of $\overline{\mathbb{C}}$ onto the p_1 and p_2 axes provides an outer approximation of the uncertainty interval associated with each parameter $p_1 \in [0.0290, 0.0308]$ and $p_2 \in [14.0468, 15.0469]$. So

$$p_{10} = 0.030, \ \Delta_r p_1 = 3.2\%,$$
$$p_{20} = 14.547, \ \Delta_r p_2 = 3.5\%.$$

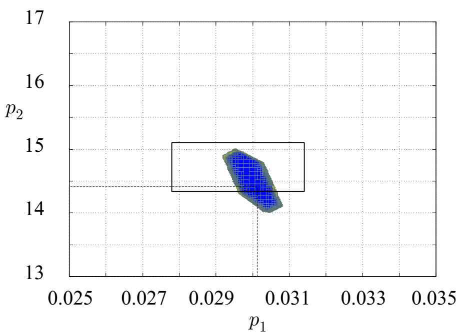

Fig. 4. Inner and outer approximations $\underline{\mathbb{C}}$ (black) et $\overline{\mathbb{C}}$ (grey) obtained by SIVIAP when the nuisance parameters are assumed to be known; the frame in black corresponds to the range found in the literature for the two parameters

4.2 *Unknown nuisance parameters*

Assume now that $\mathbf{q}$ is only known to belong to $\mathbb{Q}$ and take

$$\Pi = \{\mathbf{p} \in \mathbb{P} \mid \exists \mathbf{q} \in \mathbb{Q}, \mathbf{H}(\mathbf{p}, \mathbf{q}) \in [\mathbf{H}^{\text{mes}}]\}. \qquad (14)$$

PROJECT provides inner and outer approximations of Π (see Figure 5) in 1h54mn on a Pentium IV at 1.7 GHz, for $\varepsilon = 0.05$. The smallest box containing the outer approximation of Π is

$$[\overline{\Pi}] = [0.0265, 0.0336] \times [13.125, 15.5625]$$

and

$$p_{10} = 0.031,\ \Delta_r p_1 = 11.9\%,$$
$$p_{20} = 14.354,\ \Delta_r p_2 = 8.5\%.$$

Taking into account the uncertainty associated with $\mathbf{q}$ has thus significantly increased the uncertainty associated with the estimate of $\mathbf{p}$, from 3.2% to 11.9% for R_3 and from 3.5% to 8.5% for $\sqrt{\tau_3}$.

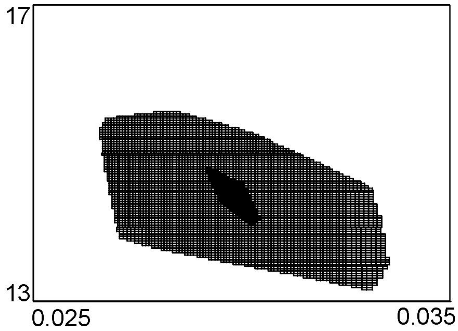

Fig. 5. Inner and outer approximations of the set Π obtained by PROJECT)

5. CONCLUSIONS

A new method for computing inner and outer approximations of the projection of a set over a subspace has been briefly presented. This method is based on interval analysis, which allows guaranteed results to be derived.

The bounded-error estimation of a vector $\mathbf{p}$ of parameters of interest when another vector $\mathbf{q}$ of nuisance parameters is only known to belong to a set is a direct application of this new method. The fact that the resulting set-estimation technique is guaranteed makes it possible to bypass any prior identifiability study and to characterize the set of all possible values for the parameters, whether these parameters are identifiable or not.

The feasibility of the approach has been demonstrated on a real-life problem of estimation of physical parameters, involving nine uncertain quantities. This would have been impossible without the use of contractors and of the new algorithm PROJECT that makes it possible to concentrate on the parameter of interest.

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