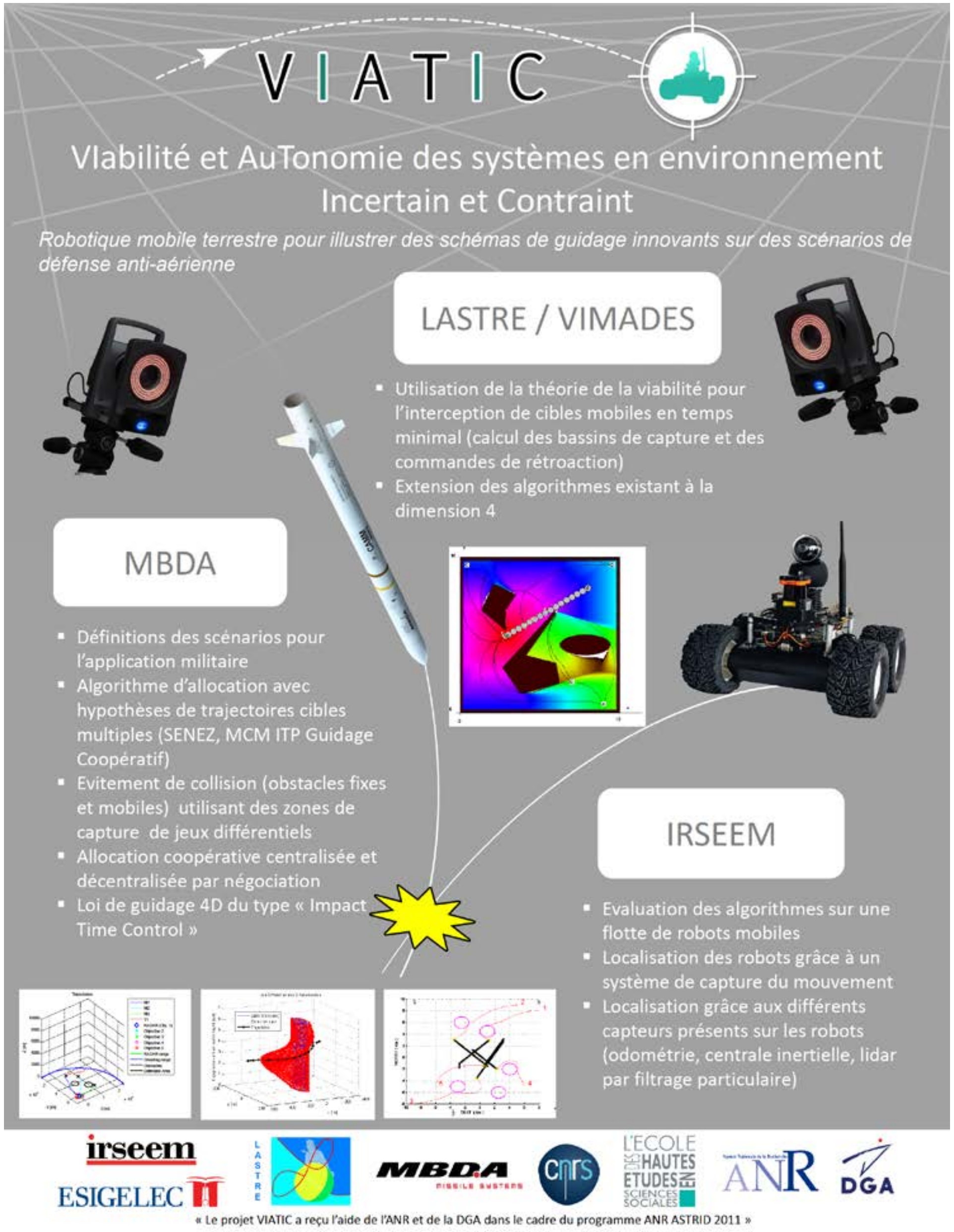
Robotic Demonstration of Collision Avoidance Based on Differential Games

MEA

ENSTA Bretagne

Brest

2013 December 5th



Stéphane Le Ménec, Victor Lecointre

MBDA-F

Guidance Control & Navigation Department

Le Plessis-Robinson (92 358)

+ ANR / ASTRID VIATIC partners

Overview

- Introduction
  - Autonomous Robust Collision Avoidance
  - Unsafe State Sets
- Differential Game of Two Cars
- Interval Analysis *(Using contributions of Luc Jaulin)*
- Computation of Backward Reachable Sets *(Using contributions of Francisco Rego)*
- LIDAR Sensing
- Robotic Demonstration
- More Simulation Results
- Conclusion & Way Forward

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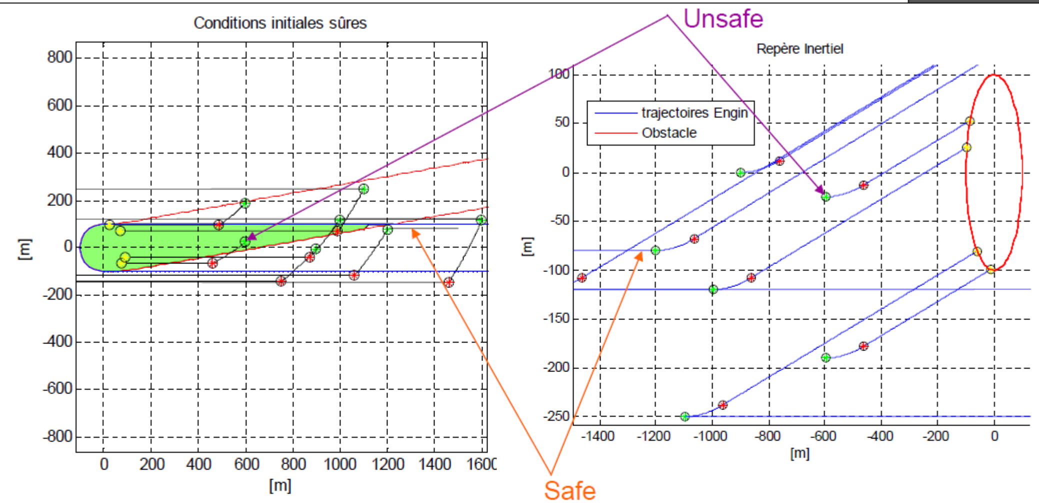
Introduction

- Follow a study about <u>fix obstacle</u> collision avoidance
- Predefined escape manoeuvre
- Computation of safe and unsafe state set

Example of Forward Simulation Trajectories

RTG 12



- On the left side : Reachable Set with trajectories also printed in inertial coordinate frame on the right side
- Green mark: start of mode 2 avoidance protocol
- Red mark: end of turn (after « *tMaxCurved* » seconds)
- Yellow mark: collision

Page 9/10 - N° GED : 2012 SLM 2165930-v1

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Autonomous Robust Collision Avoidance

- Collision avoidance of moving objects is a crucial feature for autonomous flying vehicles such as drones and missiles.

- In particular, validation of collision avoidance capabilities is mandatory when dealing with applications involving several platforms.

- Plenty of missions: satellite formation flying; cooperative search and rescue; air traffic control (civil aircrafts) and raids of cruise missiles require to master the sense and avoid aspects for safety purpose.

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Autonomous Robust Collision Avoidance

- There exist powerful collision avoidance mechanisms which rely on synchronized supervised manoeuvres between cooperative moving vehicles.
- Centralized collision avoidance requires external additional inputs provided through data links.
- However, for robustness reasons (loss of data links; latency), we study autonomous collision avoidance capabilities based on standalone decision making process and on-board sensors only.
- This study is part of VIATIC project (Viability and Autonomy of Systems in Unreliable and Constrained Environment) with funding support of ANR (French National Research Agency).

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- Unsafe state set : set of the initial conditions from where a vehicle (Evader) is not able to guaranty an escape manoeuvre whatever is the behaviour of another vehicle (Pursuer);
- Capture zone : set of all the initial conditions from where a Pursuer is able to catch an Evader whatever is the Evader's behaviour
- In the context of two player zero sum non-cooperative differential games; computing capture zones allows to divide the state space into a safe subset and an unsafe part; moving obstacles are called "Pursuers".
- Computing capture zones (No Escape Zones; interception applications) and unsafe areas (for collision avoidance) are duals / similar problems.
- We also talk about (robust) backward reachable sets.

Other Approaches - Advantages and Drawbacks

- Collision avoidance based on backward reachable set is based on simplified models; however computing backward reachable sets is an off-line process; therefore there is no on-board computational issues
- There exist other approaches also taken into account (target manoeuvre) uncertainties such as:
  - Forward reachable sets: convex hull computation
  - Bundles of probabilistic trajectories
- Exploit on-board realistic simulation models; parameters can be refined on-board
- However, these approaches suffer from on-board computation limitations

Differential Game of Two Cars

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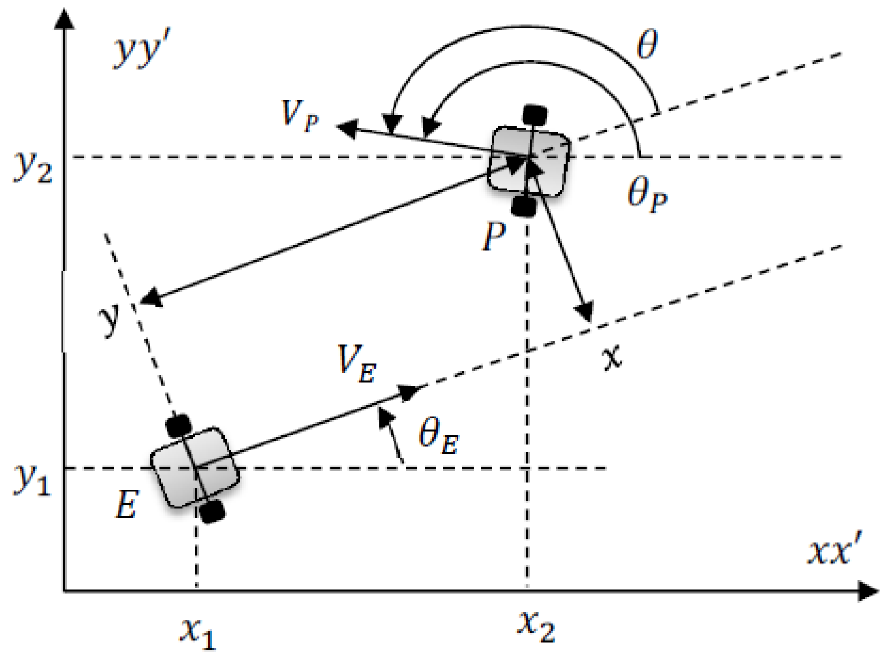
Differential Game of Two Cars

Kinematics with State Vector of Dimension 6

2 non holonomous robots

Problem of dimension 6

(state vector dimension)



$$X_i = (x_i,\, y_i,\, \theta_i);\; i \in \{E, P\}$$

$$\begin{cases} \dot{x}_E = V_E \cos\theta_E \\ \dot{y}_E = V_E \sin\theta_E \\ \dot{\theta}_E = \frac{V_E}{R_E}\, d \end{cases} \quad \text{and} \quad \begin{cases} \dot{x}_P = V_P \cos\theta_P \\ \dot{y}_P = V_P \sin\theta_P \\ \dot{\theta}_P = \frac{V_P}{R_P}\, u \end{cases}$$

$$u \in [-1{,}1]; \qquad d \in [-1{,}1]$$

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Differential Game of Two Cars

Kinematics with State Vector of Dimension 3

$$R_E = R_P = 1$$

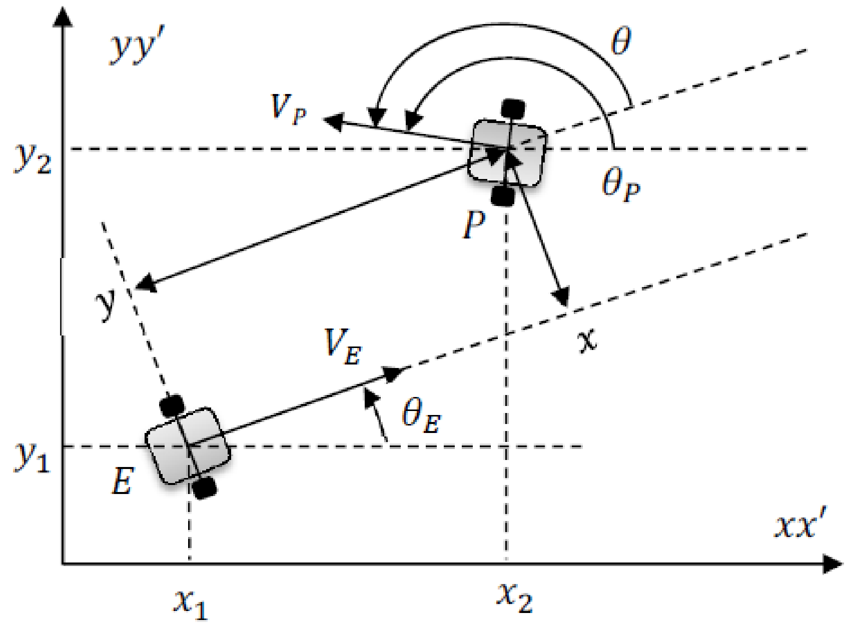
$$X = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta_E & \sin\theta_E \\ -\sin\theta_E & \cos\theta_E \end{pmatrix} \begin{pmatrix} x_P - x_E \\ y_P - y_E \end{pmatrix}$$

$$\theta = \theta_P - \theta_E$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \dot{\theta}_E \begin{pmatrix} -\sin\theta_E & \cos\theta_E \\ -\cos\theta_E & -\sin\theta_E \end{pmatrix} \left(\begin{pmatrix} x_P \\ y_P \end{pmatrix} - \begin{pmatrix} x_E \\ y_E \end{pmatrix} \right) + \begin{pmatrix} \cos\theta_E & \sin\theta_E \\ -\sin\theta_E & \cos\theta_E \end{pmatrix} \begin{pmatrix} \dot{x}_P - \dot{x}_E \\ \dot{y}_P - \dot{y}_E \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} -V_E + V_P \cos\theta + V_E\, d\, y \\ V_P \sin\theta - V_E\, d\, x \\ V_P\, u - V_E\, d \end{pmatrix}$$



→ Minimal representation of the game of two cars : dimension 3

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Interval Analysis

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- Computing intervals instead of numbers
- If $\circ \in \{+, -, ., /, max, min\}$

$$\mathbb{A} = \{x \circ y | x \in [x], y \in [y]\}$$

$$[\mathbb{A}] = [x] \circ [y] = [\{x \circ y | x \in [x], y \in [y]\}]$$

where $[\mathbb{A}]$ is the smallest interval which encloses $\mathbb{A} \subset \mathbb{R}$

- Rules

$$[x^-, x^+] + [y^-, y^+] = [x^- + y^-, x^+ + y^+]$$

$$[x^-, x^+].[y^-, y^+] =$$

$$[x^- y^- \wedge x^+ y^- \wedge x^- y^+ \wedge x^+ y^+, x^- y^- \vee x^+ y^- \vee x^- y^+ \vee x^+ y^+]$$

Exemples:

$$[-1, 3] + [2, 5] = [1, 8]$$

$$[-1, 3].[2, 5] = [-5, 15]$$

$$[-2, 6]/[2, 5] = [-1, 3]$$

$$[-2, 2]/[-1, 1] = [-\infty, \infty]$$

If $f \in \{cos, sin, sqr, sqrt, log, exp, \ldots\}$

$$f([x]) = [\{f(x) | x \in [x]\}]$$

Exemples:

$$sin([0, \pi]) = [0, 1]$$
$$sqr([-1, 3]) = [-1, 3]^2 = [0, 9]$$
$$abs([-7, 1]) = [0, 7]$$
$$log([-2, -1]) = \emptyset$$

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Boxes and Inclusion Functions

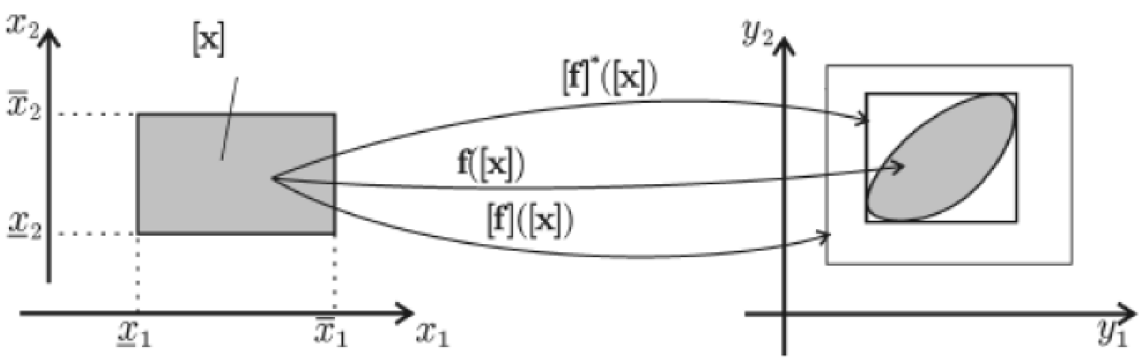
A box, or interval vector [x] of $\mathbb{R}^n$ is

$$[x] = [x_1^-, x_1^+] * \cdots * [x_n^-, x_n^+] = [x_1] * \cdots * [x_n]$$

The set of all boxes of $\mathbb{R}^n$ will be denoted by $\mathbb{IR}^n$

The interval function $[f]$ from $\mathbb{IR}^n$ to $\mathbb{IR}^n$, is an inclusion function of f if

$$\forall [x] \in \mathbb{IR}^n, f([x]) \subset [f]([x])$$



Interval Analysis

Set Inversion Via Interval Analysis

SIVIA is an algorithm which find $\mathbb{X}$ such as

$$\mathbb{X} = \{x \in \mathbb{R}^n | f(x) \in \mathbb{Y}\} = f^{-1}(\mathbb{Y})$$

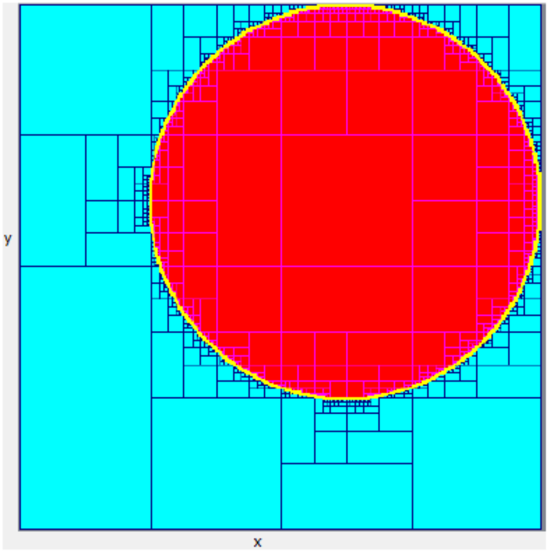
Algorithm Sivia(in: $[\mathbf{x}](0), \mathbf{f}, \mathbb{Y}$)

1 $\mathcal{L} := \{[\mathbf{x}](0)\}$;
2 pull $[\mathbf{x}]$ from $\mathcal{L}$;
3 if $[\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y}$, draw($[\mathbf{x}]$, 'red');
4 elseif $[\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset$, draw($[\mathbf{x}]$, 'blue');
5 elseif $w([\mathbf{x}]) < \varepsilon$, {draw ($[\mathbf{x}]$, 'yellow')};
6 else bisect $[\mathbf{x}]$ and push into $\mathcal{L}$;
7 if $\mathcal{L} \neq \emptyset$, go to 2

Example with identity function;

Find $X = (x, y)$; $X \in \mathbb{R}^2$ such as $f(X) = X \in \mathbb{Y}$

$$\mathbb{Y} = \{(x, y) \in \mathbb{R}^2 | (x - 0.5)^2 + (y - 0.5)^2 < (1.5)^2\}$$



Color Code:

Blue: The box is completely outside, $f[x] \cap \mathbb{Y} = \emptyset$

Red: The box is completely inside, $f[x] \in \mathbb{Y}$

Yellow: No conclusion can be made on the box

Accurancy=0.01. 7779 boxes computed.

Computation of Backward Reachable Sets

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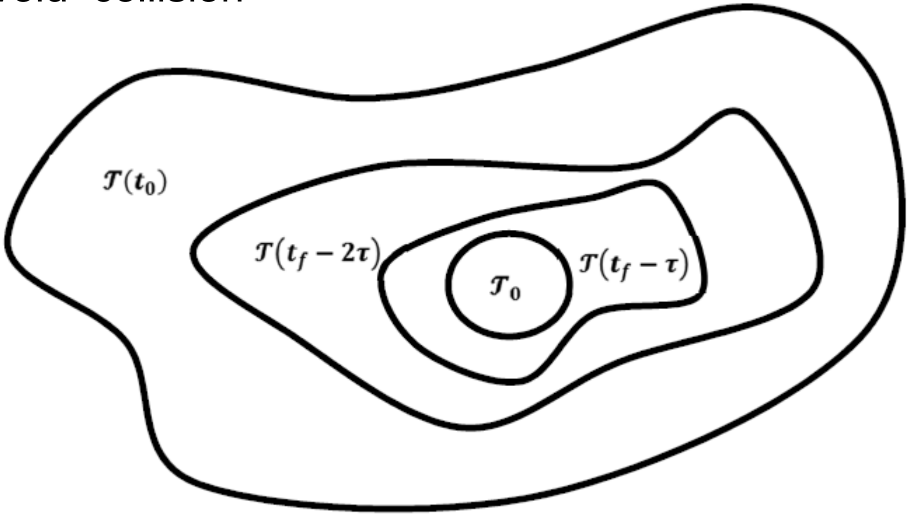
Computation of Backward Reachable Sets

- Time dependent reachable sets are computed in a backward / recursive manner.
- The initial reachable set is the target set $\Im_0 = \Im(t_f)$ corresponding to collision (immediate collision; no move); typically a circle of radius R_0.
- Then, a new reachable set $\Im(t_f - \tau)$ of initial conditions leading to collision is computed considering an interval of time τ and so on replacing the target set by the reachable set computed the iteration before.
- $t_f - t_0$ is the time horizon the Evader may avoid collision
- If $t_f - t_0$ is enough large we may have no more growing reachable sets
- Maximum range corresponding to this capture zone can be seen as a maximum (ideal) specification for on-board sensor design.



Computation of Backward Reachable Sets

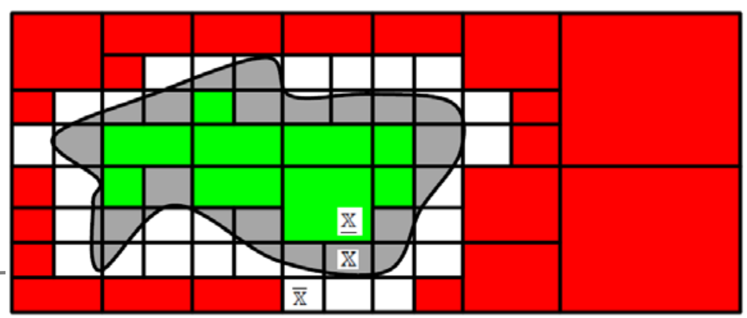
Inner and Outer Approximations

- System evolution described by an ODE φ
- u and d are controls respectively of the Pursuer and the Evader
- Perfect information game (Each player knows X)
- $u(t)$ and d(t) are non-anticipative controls

$$\dot{X} = \frac{dX}{dt} = \varphi\big(X(t), u^*(t), d(t)\big)$$

$$X(t_0) \in \mathfrak{I}(t_0), \qquad \exists t \mid X(t) \in \mathfrak{I}_0, \qquad t \in \left[t_0, t_f\right]$$

- A reachability problem can be transcribed as a set inversion problem for computing inner and outer bounds of backward reachable sets
- $\mathfrak{I}^-(t)$ (inner approximation); $\mathfrak{I}^+(t)$ (outer approximation) of reachable set $\mathfrak{I}(t)$ are such that $\mathfrak{I}^-(t) \subseteq \mathfrak{I}(t) \subseteq \mathfrak{I}^+(t)$



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- Computing backward reachable can be summarized by the following statement:

$$X \in \Im\,(t-\tau)\ \ iff\ \exists u \in \mathbb{U},\ \forall d \in \mathbb{D}\ |\ \varphi\,(\tau, X, u, d) \in \Im^{-}\,(t)$$

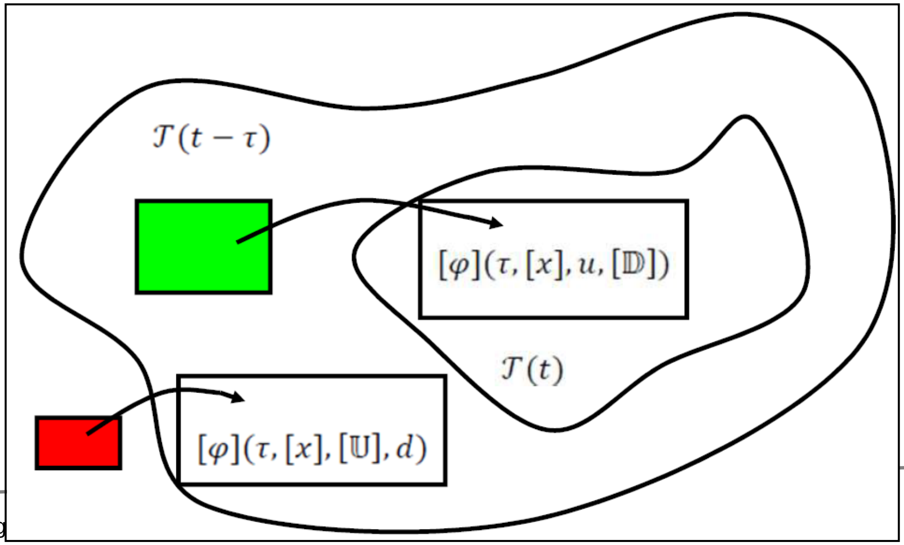
$$X \notin \Im\,(t-\tau)\ \ iff\ \exists d \in \mathbb{D},\ \forall u \in \mathbb{U}\ |\ \varphi\,(\tau, X, u, d) \notin \Im^{+}\,(t)$$

- Which is equivalent to the following equation when using interval analysis:

$$[\varphi]\,(\tau, [x], u, [\mathbb{D}]) \subseteq \Im^{-}\,(t) \ \Rightarrow\ [x] \subseteq \Im\,(t-\tau)$$

$$[\varphi]\,(\tau, [x], [u], d) \cap \Im^{+}\,(t) = \emptyset \ \Rightarrow\ [x] \cap \Im\,(t-\tau) = \emptyset$$

With [ ] describing boxes; $[\varphi]\,(\tau, [x], u, [d])$ being an inclusion function



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LIDAR Sensing

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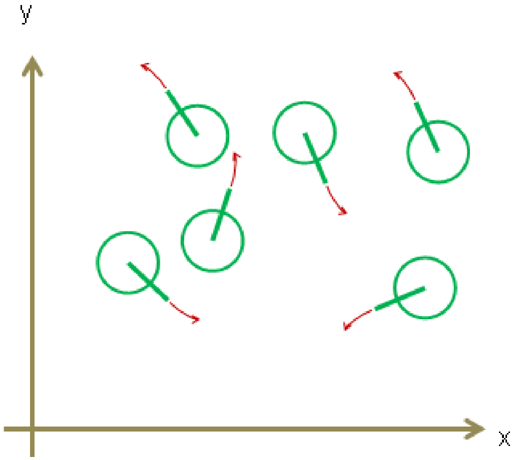
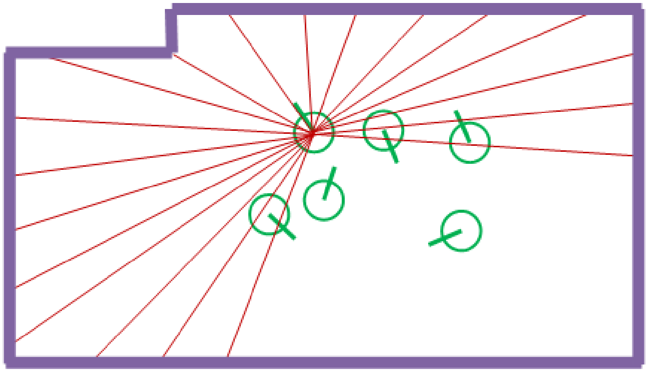
LIDAR Sensing

Use of WiFiBot Light Detection And Ranging Sensors and particle filters for

- Geo Localization purpose
- And for surrounding robot tracking
- Use LIDAR sensors only; no IMU

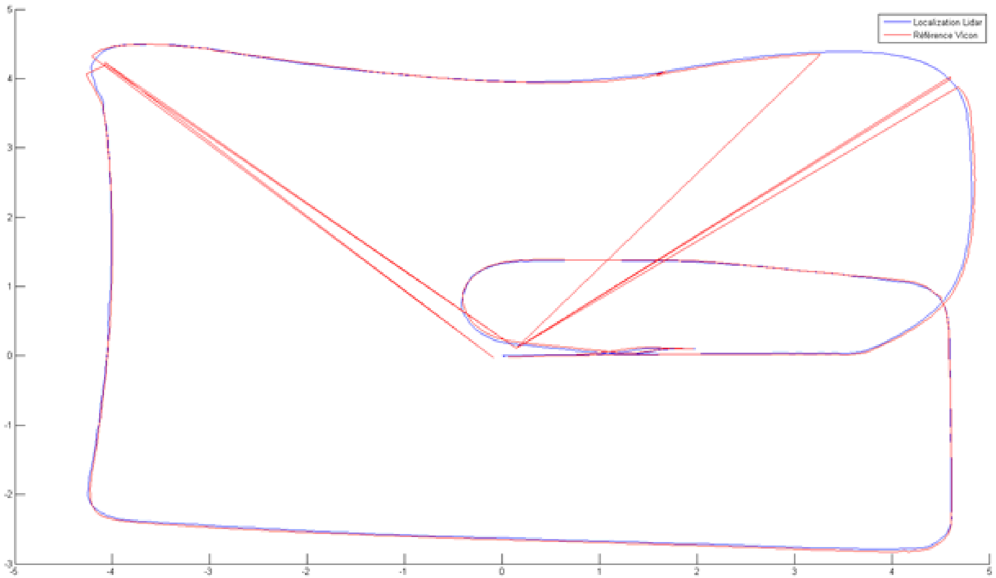




LIDAR Sensing

VICON Cameras + Markers

- Alternative method for localization;
  Available in lab only; for testing purpose
- Comparison between VICON cameras and LiDAR
- In Red: VICON; In Blue: LiDAR



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Robotic Demonstration

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  - Autonomous Robust Collision Avoidance
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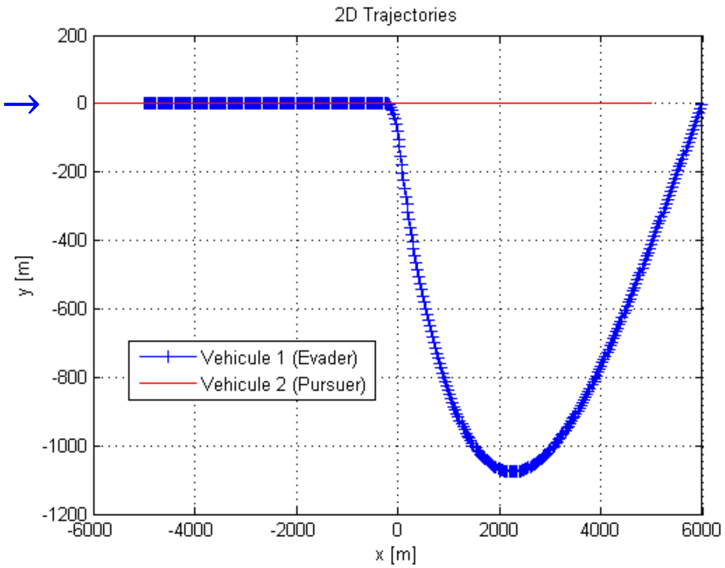
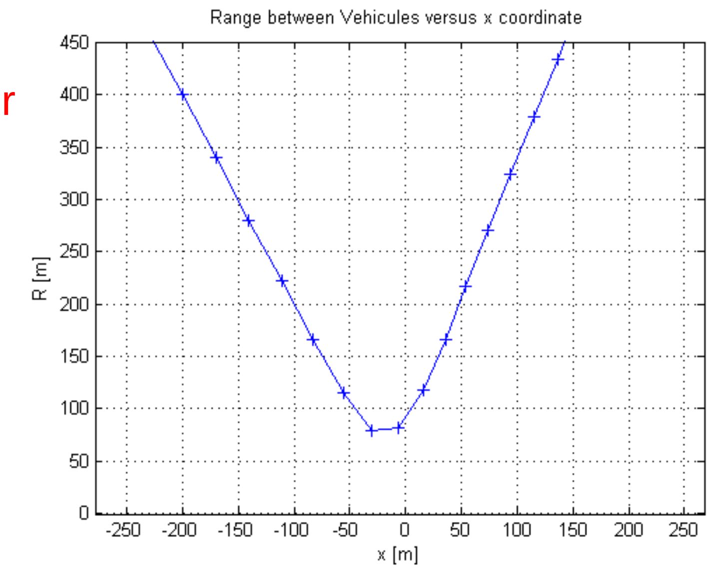
Scenario - Simulation Results

- Trajectory of Pursuer and Evader in head on situation

Evader →



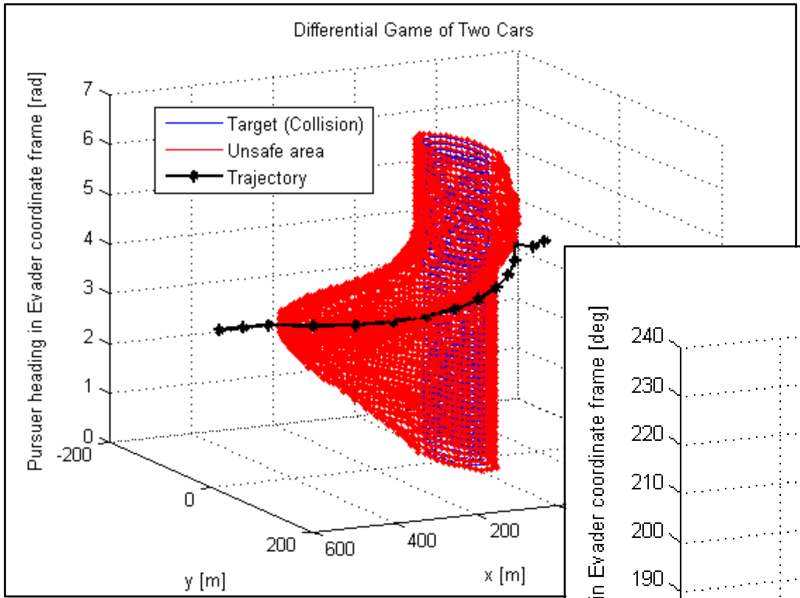
← Pursuer

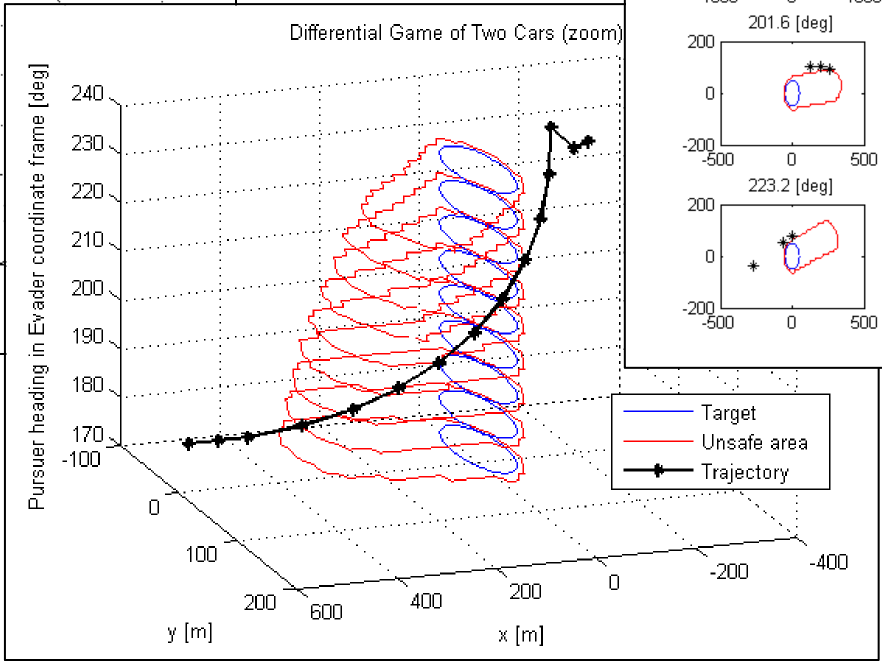


Variable	Value
V_E (Evader velocity)	30 m/sec
V_P (Pursuer velocity)	30 m/sec
R_E (Evader minimum curvature)	300 m
R_P (Pursuer minimum curvature)	300 m
R_0 (target radius)	50 m
$t_f - t_0$ (maximum time horizon for computing backward reachable set)	5 sec

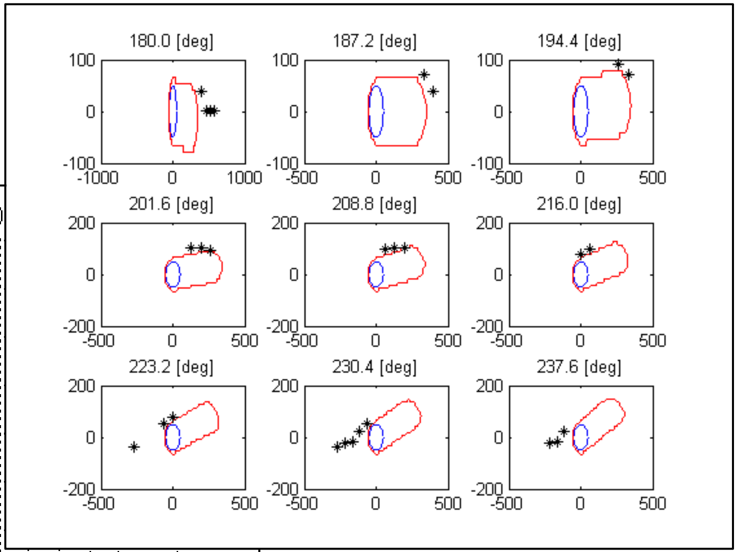
Simulation – Trajectory Respect to the Unsafe Set Boundaries

- Backward reachable set of the two car differential game with trajectory corresponding to the head on example; use of Delaunay / Voronoï techniques



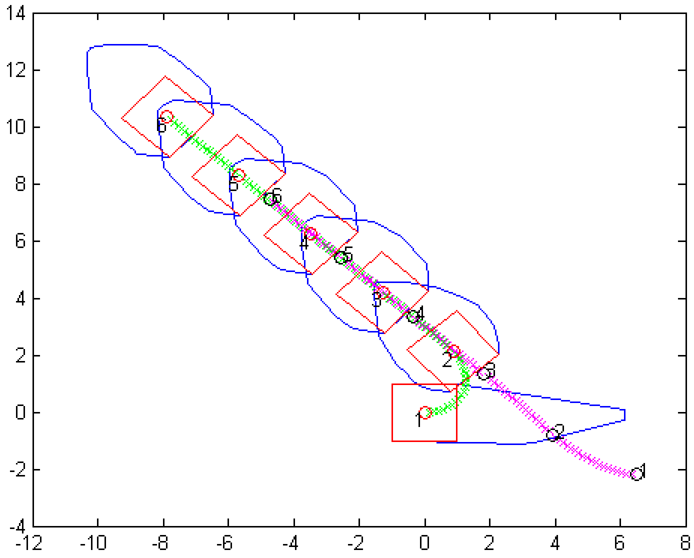


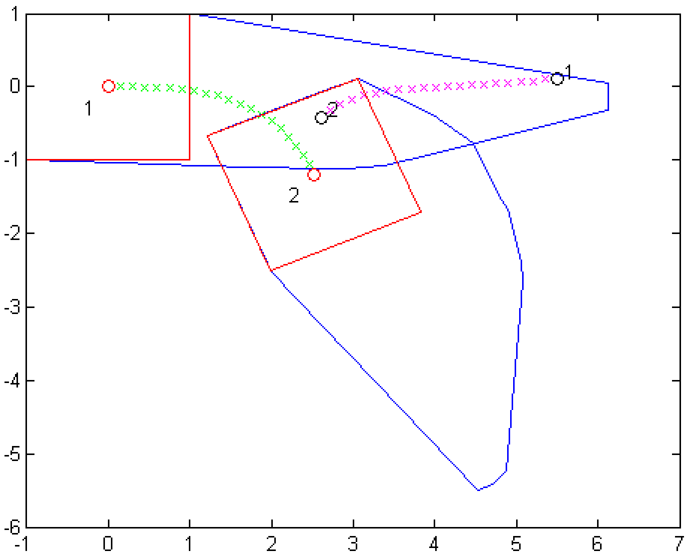
Zoom →



More Simulations

- 2D trajectories with projections of unsafe sets around the Evader (similar vehicles)
- Green line is Evader; magenta line is Pursuer
- Blue sets are unsafe sets: (x, y) frontiers corresponding to current θ (relative heading)
- Red squares are the target (collision)
- Evasion initiated outside allows the Evader staying in the safe part (left drawing)
- Evasion initiated inside leads to collision (right drawing)

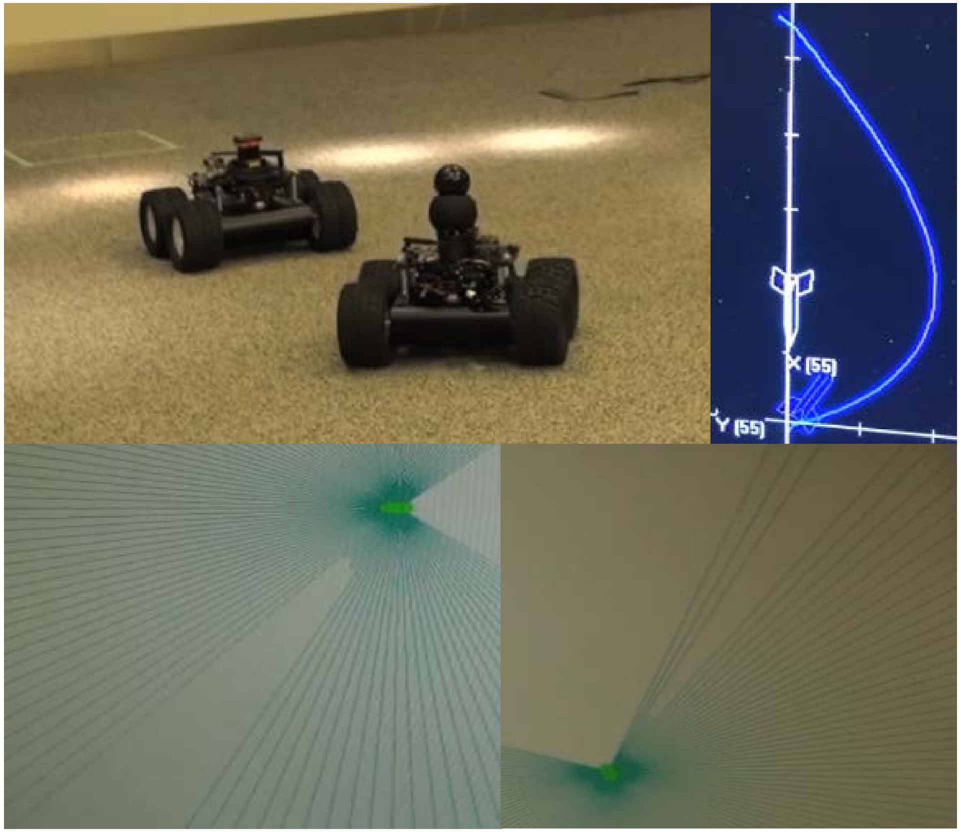




Robotic Demonstration

Robotic Results - Video 1

- Robotic experimentation with on-board Hokuyo LIDAR sensors on top of each platform; bottom left is the LIDAR pattern of the rear robot (Pursuer); bottom right is the LIDAR pattern of the robot performing the escape manoeuvre (Evader)



Collision avoidance using LiDAR sensors only; demo in MBDA-F Le Plessis-Robinson meeting room

↓

RoboticDemo_2203D.mov

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Robotic Demonstration

Video 2

- Time arrival and terminal slope synchronisation (trajectory shaping; feedback control laws; constant speed); localization using VICON cameras; demo in Lab facilities

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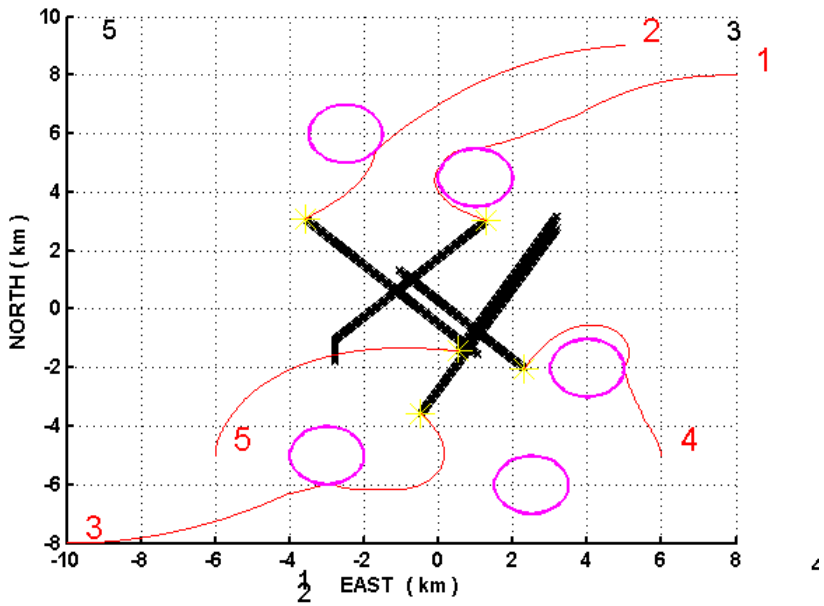
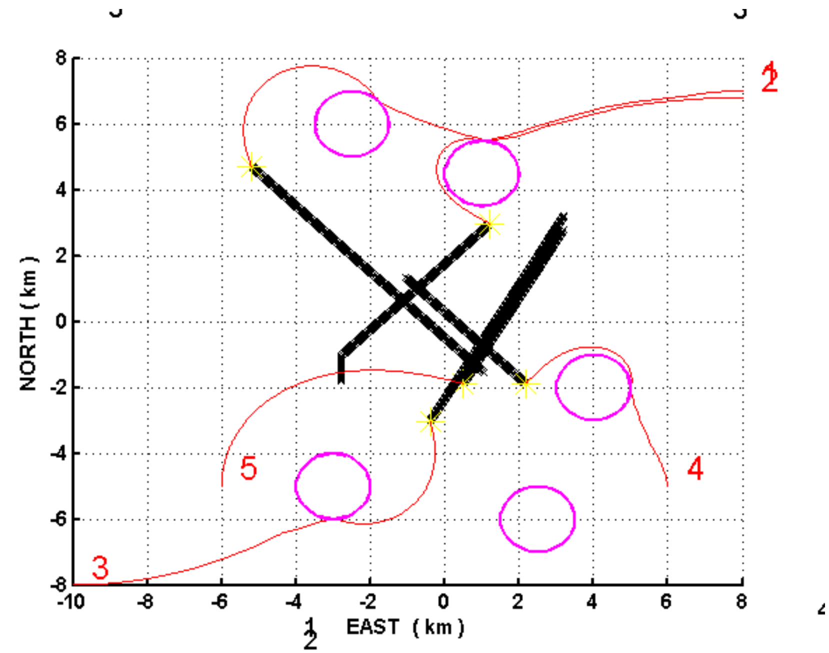
More Simulations

In Multi-Agent coordination context



Fix obstacle collision avoidance thanks to safety bags running in parallel of a multi-agent target (re)allocation algorithm; vehicles depicted using thin red lines reach black vehicles; the obstacles are the magenta circles; red vehicles run safety bags for collision avoidance purpose



Same scenario; predefined allocation plan; no in flight reallocation allowed; demonstration of collision avoidance between two red moving vehicles (red lines top right of the figure)

Conclusion & Way Forward

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Conclusion & Way Forward

On going Work

- Finalize algorithm integration into the robotic platforms
- New Algorithms for computing backward reachable sets
  - Validate evasion strategies rather than optimal strategy computation
  - Reduce the state space to explore
  - Use new tools of interval analysis
  - Description of backward reachable sets as level sets; i.e. computation of scalar products between the system dynamics and gradients of level sets in place of integrating trajectories
  - Only over approximations (under approximations) are needed for collision avoidance (for interception)
  - Increase robustness: uncertain Pursuer trajectory and uncertain model parameters
  - → Faster computations and models up to dimension 10

Conclusion & Way Forward

Summary

- Summary
  - Autonomous robust collision avoidance of moving vehicles
  - Based on non cooperative differential games
  - Off line computation of safe / unsafe state sets
  - Demonstration using robotic platforms using LiDAR sensors for geo localization and for sensing vehicles moving around
- Next step
  - VIATIC project is now looking for:
    - Industrial applications (Unmanned Autonomous Systems; aeronautical industry; car industry; public transportation …)
    - Framework for working at industrial level
    - Resources for more experimentations and validation

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