Visibility Contractors
Application to Mobile Robot Localization

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Introduction

→ Visibility is studied and used in several fields
  - Computer graphics
  - Telecommunication
  - Robotics...

→ Usually associated to bearing or ranging data

→ We consider the visibility as a boolean information
  - Application to mobile robot localization

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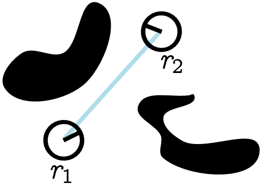

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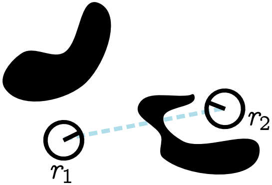

Outlines

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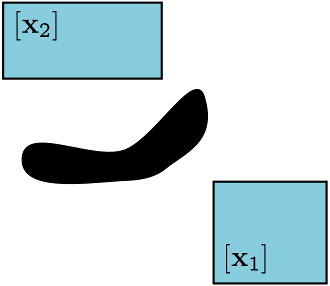
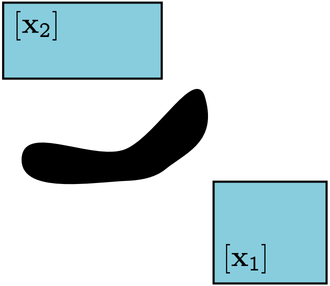
Objectives

- Developing a contractor associated to the constraint r_1 *sees* r_2



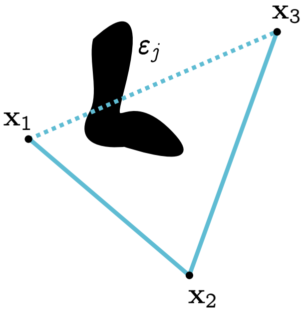
Objectives

- Developing a contractor associated to the constraint r_1 *sees* r_2
- Developing a contractor associated to the constraint r_1 *does not see* r_2



Visibility between two points

- $(\mathbf{x}_1 \mathsf{V} \mathbf{x}_2)_{\varepsilon_j} \Leftrightarrow Seg(\mathbf{x}_1, \mathbf{x}_2) \cap \varepsilon_j = \emptyset$
  - $\rightarrow$ ε_j : connected subset of $\mathbb{R}^n$, with $\mathbf{x}_1 \notin \varepsilon_j$ and $\mathbf{x}_2 \notin \varepsilon_j$



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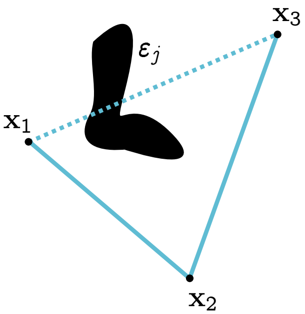
  - $\rightarrow$ Symmetric relation : $(\mathbf{x}_1 \mathsf{V} \mathbf{x}_2)_{\varepsilon_j} \equiv (\mathbf{x}_2 \mathsf{V} \mathbf{x}_1)_{\varepsilon_j}$
  - $\rightarrow$ Non-transitive relation : $(\mathbf{x}_1 \mathsf{V} \mathbf{x}_2)_{\varepsilon_j} \wedge (\mathbf{x}_2 \mathsf{V} \mathbf{x}_3)_{\varepsilon_j} \nRightarrow (\mathbf{x}_1 \mathsf{V} \mathbf{x}_3)_{\varepsilon_j}$



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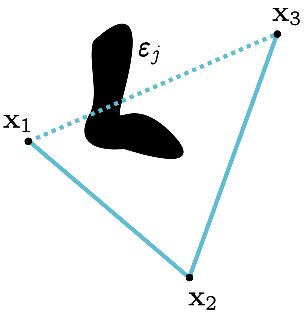
  - $\rightarrow$ $\left((\mathbf{x}_1 \mathsf{V} \mathbf{x}_3)_{\varepsilon_j}\right)^c = (\mathbf{x}_1 \overline{\mathsf{V}} \mathbf{x}_3)_{\varepsilon_j}$



Visibility spaces of a point

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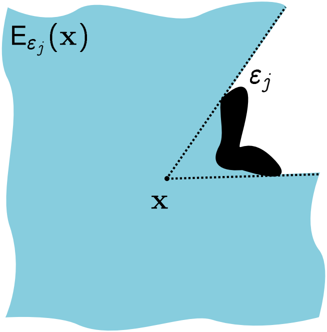

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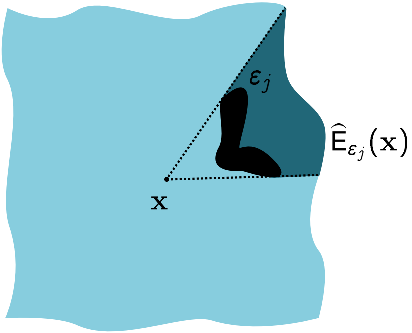

Visibility spaces of a set

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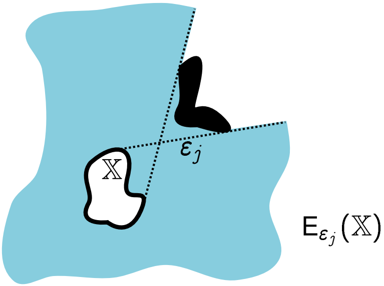

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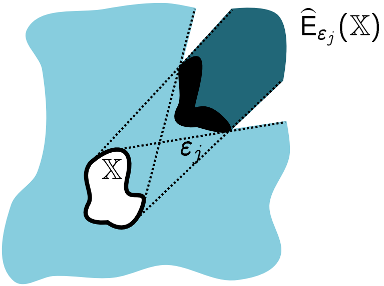

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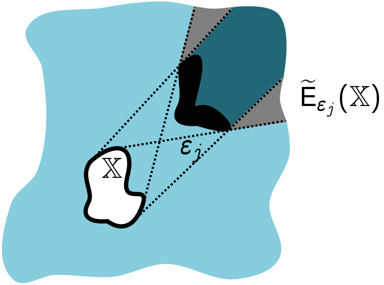
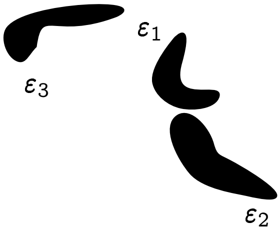
- $\widetilde{\mathsf{E}}_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \exists \mathbf{x}_1 \in \mathbb{X}, \exists \mathbf{x}_2 \in \mathbb{X}, (\mathbf{x}_i \mathsf{V} \mathbf{x}_1)_{\varepsilon_j} \wedge (\mathbf{x}_i \overline{\mathsf{V}} \mathbf{x}_2)_{\varepsilon_j}\}$



Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$



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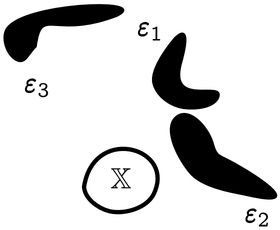

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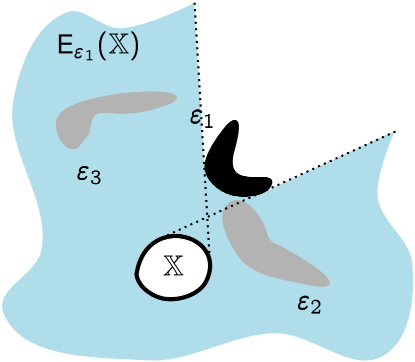

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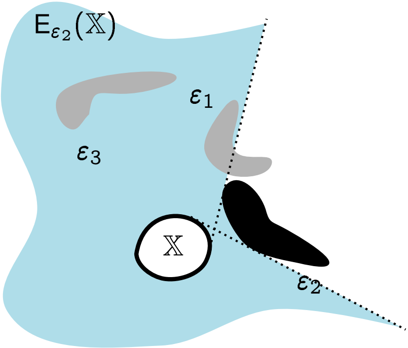

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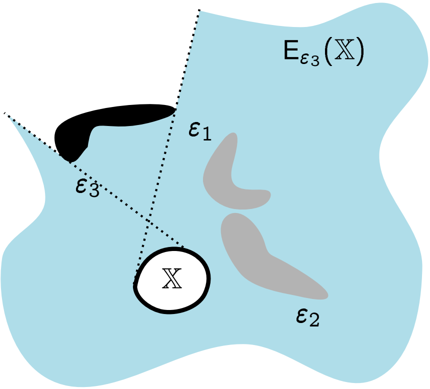

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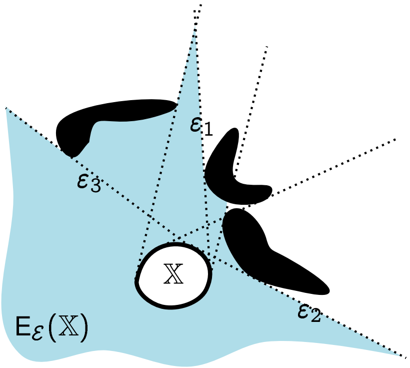

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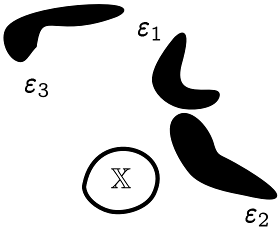

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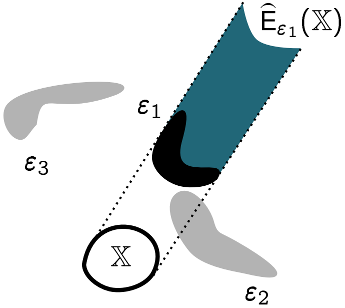

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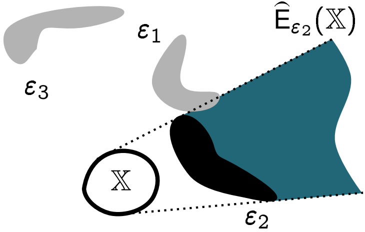

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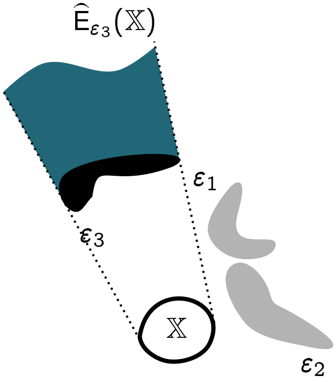

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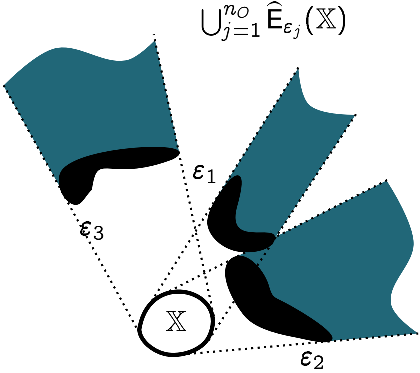

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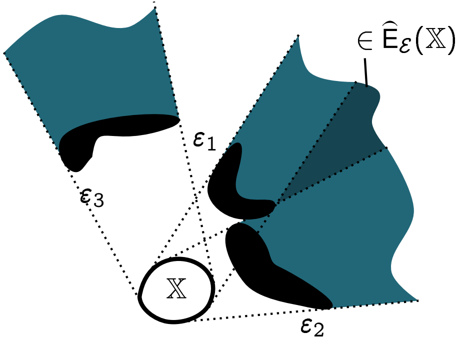

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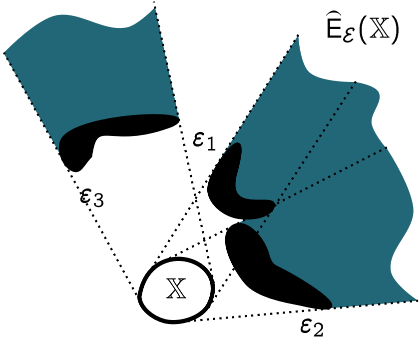

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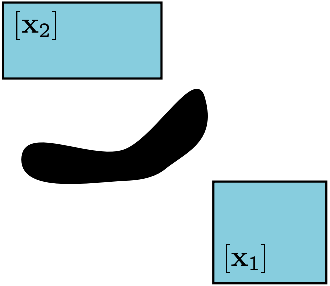

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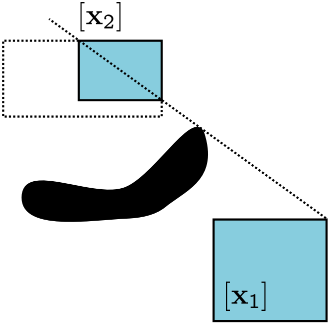

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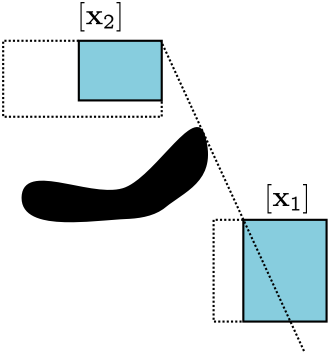

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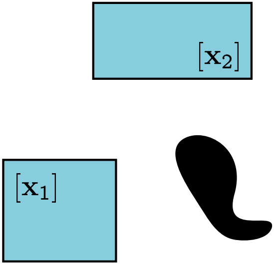

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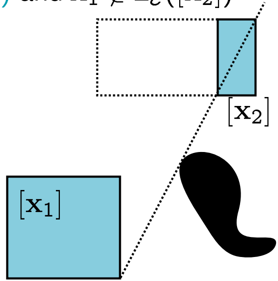

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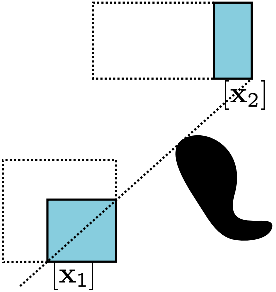
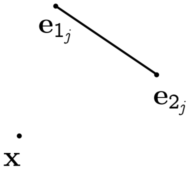


Outlines

Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$



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  $\rightarrow$ Visible space

$$
\begin{aligned}
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& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) > 0 \vee \\
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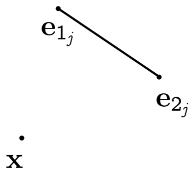

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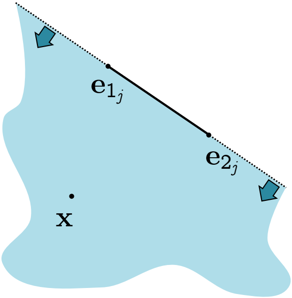
& [\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1_j} \cup \mathbf{e}_{2_j}] = \emptyset \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) > 0 \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{x} - \mathbf{e}_{1_j}) > 0 \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{2_j} | \mathbf{x} - \mathbf{e}_{2_j}) < 0 \}
\end{aligned}
$$

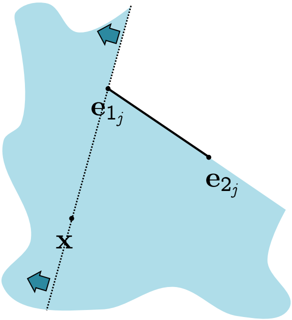


Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$

  $\rightarrow$ Visible space

$$
\begin{aligned}
\mathsf{E}_{\varepsilon_j^s}(\mathbf{x}) = \{ \mathbf{x}_i \in \mathbb{R}^2 \mid \; & \\
& [\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1_j} \cup \mathbf{e}_{2_j}] = \emptyset \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) > 0 \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{x} - \mathbf{e}_{1_j}) > 0 \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{2_j} | \mathbf{x} - \mathbf{e}_{2_j}) < 0 \}
\end{aligned}
$$

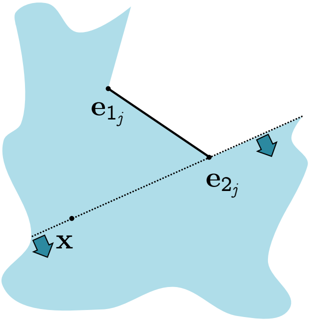


Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$

  $\rightarrow$ Visible space

$$\mathsf{E}_{\varepsilon_j^s}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$
$$[\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1_j} \cup \mathbf{e}_{2_j}] = \emptyset \vee$$
$$\zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) > 0 \vee$$
$$\zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{x} - \mathbf{e}_{1_j}) > 0 \vee$$
$$\zeta_x \det(\mathbf{x}_i - \mathbf{e}_{2_j} | \mathbf{x} - \mathbf{e}_{2_j}) < 0\}$$

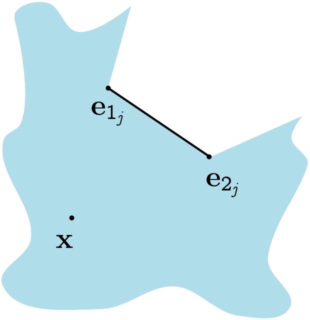


Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$

  $\rightarrow$ Visible space

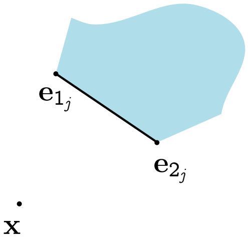
$$
\begin{aligned}
E_{\varepsilon_j^s}(\mathbf{x}) = \{ \mathbf{x}_i \in \mathbb{R}^2 \mid \\
& [\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1_j} \cup \mathbf{e}_{2_j}] = \emptyset \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) > 0 \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{x} - \mathbf{e}_{1_j}) > 0 \vee \\
& \zeta_x \det(\mathbf{x}_i - \mathbf{e}_{2_j} | \mathbf{x} - \mathbf{e}_{2_j}) < 0 \}
\end{aligned}
$$



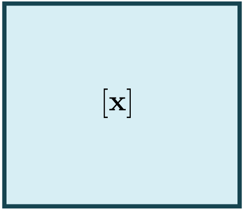
Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$
  - $\rightarrow$ Visible space
  - $\rightarrow$ Non-visible space

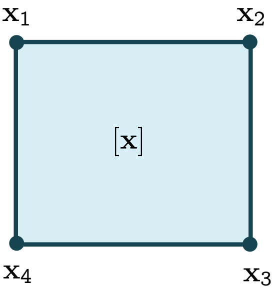
$$\widehat{\mathsf{E}}_{\varepsilon_j^s}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$
$$[\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1_j} \cup \mathbf{e}_{2_j}] \neq \emptyset \wedge$$
$$\zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) \leq 0 \wedge$$
$$\zeta_x \det(\mathbf{x}_i - \mathbf{e}_{1_j} | \mathbf{x} - \mathbf{e}_{1_j}) \leq 0 \wedge$$
$$\zeta_x \det(\mathbf{x}_i - \mathbf{e}_{2_j} | \mathbf{x} - \mathbf{e}_{2_j}) \geq 0\}$$



Visibility of a segment

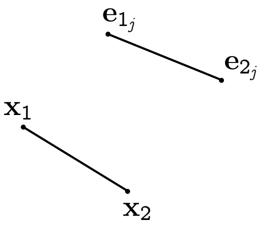


Visibility of a segment



Visibility of a segment

- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$



Visibility of a segment

- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$

  $\rightarrow$ Visible space - $\mathrm{E}_{\varepsilon_j^s}(Seg(\mathbf{x}_1, \mathbf{x}_2))$

$$\mathrm{E}_{\varepsilon_j^s}(Seg(\mathbf{x}_1,\mathbf{x}_2)) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$

$$(\zeta_{x_1} = \zeta_{x_2}) \wedge \Big(\zeta_{x_1}\det(\mathbf{x}_i - \mathbf{e}_{1_j}|\mathbf{e}_{2_j} - \mathbf{e}_{1_j}) > 0 \vee$$

$$\zeta_{x_1}\det(\mathbf{x}_i - \mathbf{e}_{1_j}|\mathbf{x}_1 - \mathbf{e}_{1_j}) > 0 \wedge \zeta_{x_2}\det(\mathbf{x}_i - \mathbf{e}_{1_j}|\mathbf{x}_2 - \mathbf{e}_{1_j}) > 0 \vee$$

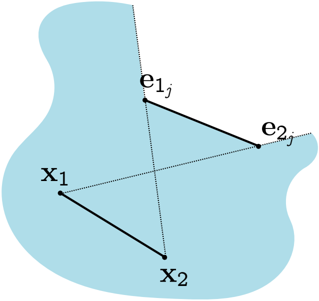
$$\zeta_{x_1}\det(\mathbf{x}_i - \mathbf{e}_{2_j}|\mathbf{x}_1 - \mathbf{e}_{2_j}) < 0 \wedge \zeta_{x_2}\det(\mathbf{x}_i - \mathbf{e}_{2_j}|\mathbf{x}_2 - \mathbf{e}_{2_j}) < 0\Big) \vee$$

$$(\zeta_{x_1} = -\zeta_{x_2}) \wedge \Big($$

$$\Big(\zeta_{e_1}\det(\mathbf{x}_i - \mathbf{e}_{1_j}|\mathbf{x}_1 - \mathbf{e}_{1_j}) > 0 \vee \zeta_{e_1}\det(\mathbf{x}_i - \mathbf{e}_{1_j}|\mathbf{x}_2 - \mathbf{e}_{1_j}) < 0\Big) \wedge$$

$$\Big(\zeta_{e_2}\det(\mathbf{x}_i - \mathbf{e}_{2_j}|\mathbf{x}_1 - \mathbf{e}_{2_j}) > 0 \vee \zeta_{e_2}\det(\mathbf{x}_i - \mathbf{e}_{2_j}|\mathbf{x}_2 - \mathbf{e}_{2_j}) < 0\Big)\Big) \vee$$

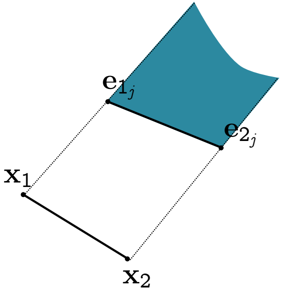
$$\Big([\mathbf{x}_i \cup \mathbf{x}_1 \cup \mathbf{x}_2] \cap [\mathbf{e}_{1_j} \cup \mathbf{e}_{2_j}] = \emptyset\Big)\}.$$



Visibility of a segment

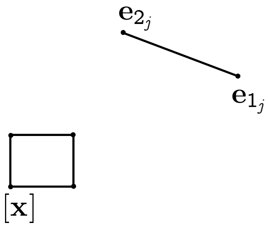
- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$
  - → Visible space - $\mathsf{E}_{\varepsilon_j^s}(Seg(\mathbf{x}_1, \mathbf{x}_2))$
  - → Non-visible space - $\widehat{\mathsf{E}}_{\varepsilon_j^s}(Seg(\mathbf{x}_1, \mathbf{x}_2))$

$$\widehat{\mathsf{E}}_{\varepsilon_j^s}(Seg(\mathbf{x}_1, \mathbf{x}_2)) = \widehat{\mathsf{E}}_{\varepsilon_j^s}(\mathbf{x}_1) \cap \widehat{\mathsf{E}}_{\varepsilon_j^s}(\mathbf{x}_2)$$



Visibility of a box

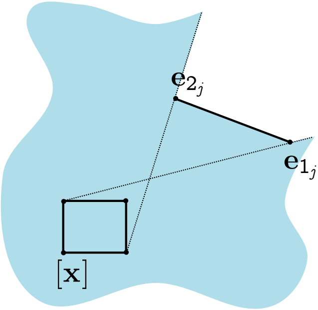
- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$



Visibility of a box

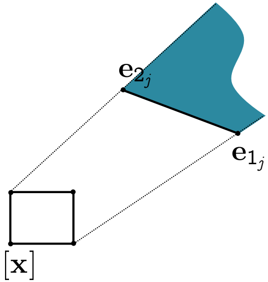
- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$

  → Visible space - $\mathrm{E}_{\varepsilon_j^s}([\mathbf{x}])$



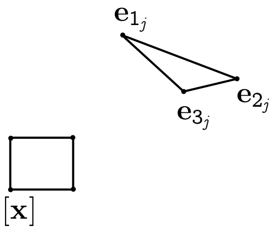
Visibility of a box

- With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$
  - $\rightarrow$ Visible space - $\mathsf{E}_{\varepsilon_j^s}([\mathbf{x}])$
  - $\rightarrow$ Non-visible space - $\widehat{\mathsf{E}}_{\varepsilon_j^s}([\mathbf{x}])$



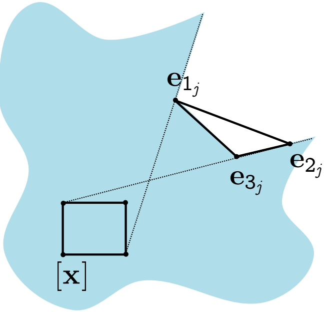
With a convex polygon as obstacle

- Convex polygon : set of segments
  $\rightarrow \; \varepsilon_j^p = \bigcup_{k=1}^{n_{P_j}} \varepsilon_k^s$



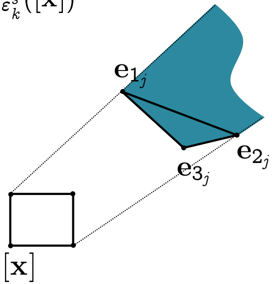
With a convex polygon as obstacle

- Convex polygon : set of segments
  - $\rightarrow \; \varepsilon_j^p = \bigcup_{k=1}^{n_{P_j}} \varepsilon_k^s$
  - $\rightarrow \; \mathsf{E}_{\varepsilon_j^p}([\mathbf{x}]) = \bigcap_{k=1}^{n_{P_j}} \mathsf{E}_{\varepsilon_k^s}([\mathbf{x}])$



With a convex polygon as obstacle

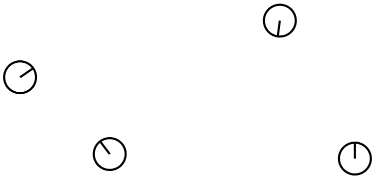
- Convex polygon : set of segments
  - $\rightarrow \varepsilon_j^p = \bigcup_{k=1}^{n_{P_j}} \varepsilon_k^s$
  - $\rightarrow \mathsf{E}_{\varepsilon_j^p}([\mathbf{x}]) = \bigcap_{k=1}^{n_{P_j}} \mathsf{E}_{\varepsilon_k^s}([\mathbf{x}])$
  - $\rightarrow \widehat{\mathsf{E}}_{\varepsilon_j^p}([\mathbf{x}]) = \bigcup_{k=1}^{n_{P_j}} \widehat{\mathsf{E}}_{\varepsilon_k^s}([\mathbf{x}])$



Outlines

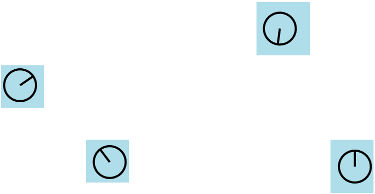
Problem presentation

- Team of robots



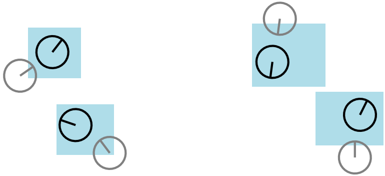
Problem presentation

- Team of robots
- Initial poses known



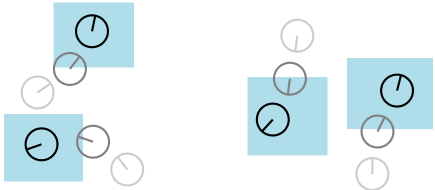
Problem presentation

- Team of robots
- Initial poses known



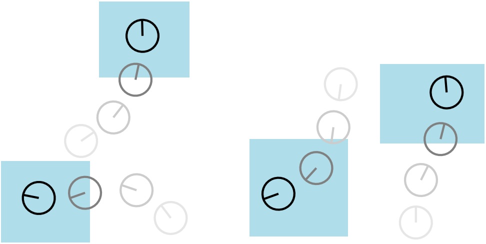
Problem presentation

- Team of robots
- Initial poses known



Problem presentation

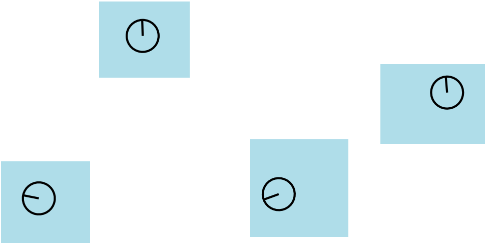
- Team of robots
- Initial poses known



Problem presentation

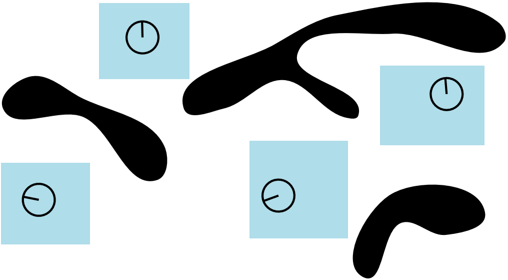
- Team of robots
- Initial poses known

→ The robots are drifting



Problem presentation

- Team of robots
- Initial poses known
- Known environment



Problem presentation

- Team of robots
- Initial poses known
- Known environment
- Is it possible to avoid the drifting of the robot by using a boolean information : the visibility between the robots ?

The robots

- Bounded error context

  $\rightarrow \mathbf{q}_{i,0} \in [\mathbf{q}_{i,0}]$

The robots

- Bounded error context
- Evaluation of the orientation by using a compass

  $\rightarrow \; \theta_{i,k} \in [\theta_{i,k}]$

Compass CMPS10

Compass SEN12753P

The robots

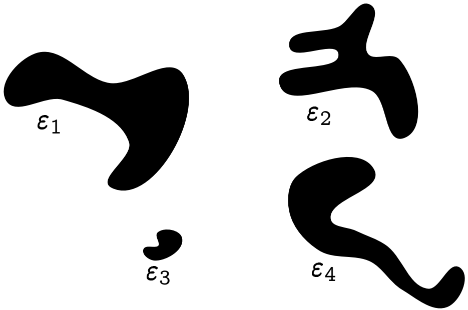
- Bounded error context
- Evaluation of the orientation by using a compass
- Inter-robot communication
  - $\rightarrow$ At each time step k each robot knows the position estimation of all the robots

The robots

- Bounded error context
- Evaluation of the orientation by using a compass
- Inter-robot communication
- Boolean measurements
  - → r_1 sees $r_2 \Leftrightarrow (\mathbf{x}_1 \mathrm{V} \mathbf{x}_2)_{\mathcal{E}}$
  - → r_1 does not see $r_2 \Leftrightarrow (\mathbf{x}_1 \overline{\mathrm{V}} \mathbf{x}_2)_{\mathcal{E}}$
  - → $\mathbf{z}_{i,k} = \{0, 1, \cdots, 1\}$
    - 0 : the robot r_i does not see the first robot
    - 1 : the robot r_i sees the second robot
    - $\cdots$
    - 1 : the robot r_i sees the last robot

Environment characterisations

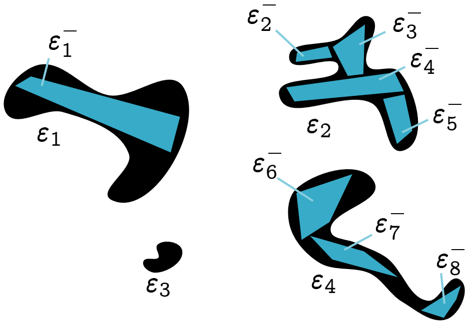
- Environment $\mathcal{E}$
  - $\rightarrow$ $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
  - $\rightarrow$ Sets of convex polygons



Environment characterisations

- Environment $\mathcal{E}$
- Inner characterisation $\mathcal{E}^-$

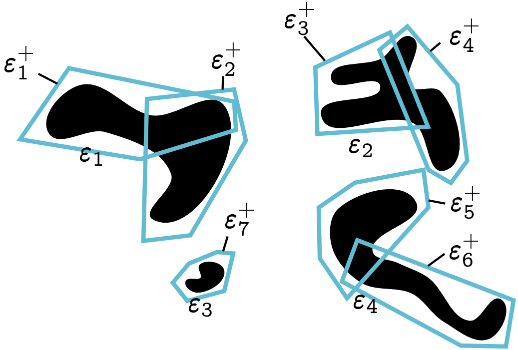
$\rightarrow \mathcal{E}^- \subseteq \mathcal{E}$



Environment characterisations

- Environment $\mathcal{E}$
- Inner characterisation $\mathcal{E}^-$
- Outer characterisation $\mathcal{E}^+$

$\rightarrow \ \mathcal{E} \subseteq \mathcal{E}^+$



Environment characterisations

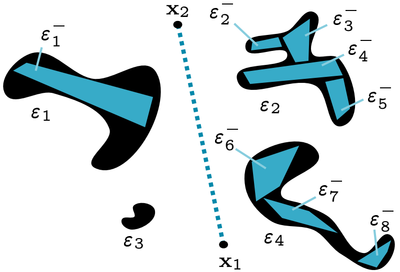
- Environment $\mathcal{E}$
- Inner characterisation $\mathcal{E}^-$
- Outer characterisation $\mathcal{E}^+$
- Environment/Characterisations

Environment characterisations

- Environment $\mathcal{E}$
- Inner characterisation $\mathcal{E}^-$
- Outer characterisation $\mathcal{E}^+$
- Environment/Characterisations

$\rightarrow$ r_1 sees $r_2 \Rightarrow (\mathbf{x}_1 \mathsf{V} \mathbf{x}_2)_{\mathcal{E}^-}$

- $\mathbf{x}_1 \in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_2)$, $\mathbf{x}_1 \notin \widehat{\mathsf{E}}_{\mathcal{E}^-}(\mathbf{x}_2)$
- $\mathbf{x}_2 \in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_1)$, $\mathbf{x}_2 \notin \widehat{\mathsf{E}}_{\mathcal{E}^-}(\mathbf{x}_1)$



Environment characterisations

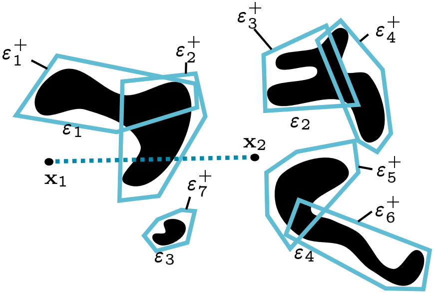
- Environment $\mathcal{E}$
- Inner characterisation $\mathcal{E}^-$
- Outer characterisation $\mathcal{E}^+$
- Environment/Characterisations

→ r_1 sees $r_2 \Rightarrow (\mathbf{x}_1 \mathsf{V} \mathbf{x}_2)_{\mathcal{E}^-}$

- $\mathbf{x}_1 \in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_2)$, $\mathbf{x}_1 \notin \widehat{\mathsf{E}}_{\mathcal{E}^-}(\mathbf{x}_2)$
- $\mathbf{x}_2 \in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_1)$, $\mathbf{x}_2 \notin \widehat{\mathsf{E}}_{\mathcal{E}^-}(\mathbf{x}_1)$

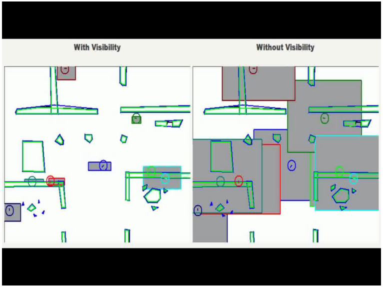
→ r_1 does not see $r_2 \Rightarrow (\mathbf{x}_1 \overline{\mathsf{V}} \mathbf{x}_2)_{\mathcal{E}^+}$

- $\mathbf{x}_1 \in \widehat{\mathsf{E}}_{\mathcal{E}^+}(\mathbf{x}_2)$, $\mathbf{x}_1 \notin \mathsf{E}_{\mathcal{E}^+}(\mathbf{x}_2)$
- $\mathbf{x}_2 \in \widehat{\mathsf{E}}_{\mathcal{E}^+}(\mathbf{x}_1)$, $\mathbf{x}_2 \notin \mathsf{E}_{\mathcal{E}^+}(\mathbf{x}_1)$



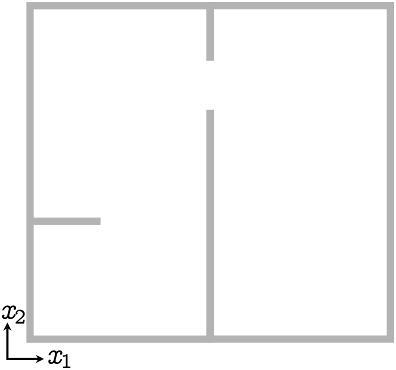
Results

- Simulator

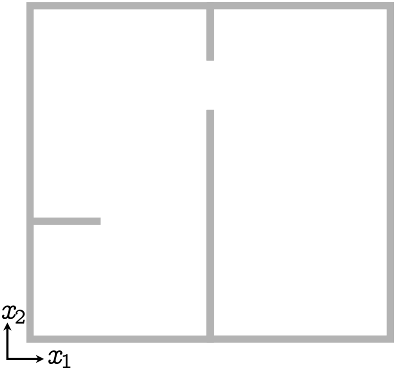


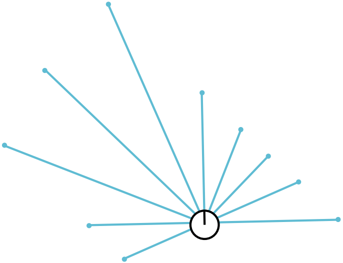
Outlines

Considered problem

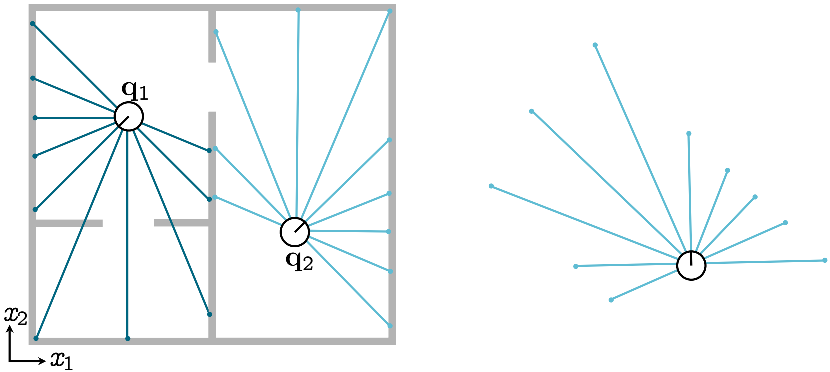


Considered problem



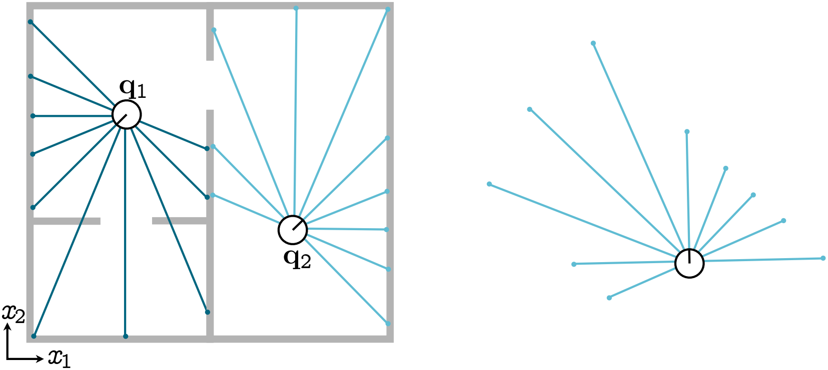


Considered problem



Two poses are consistent with the constraints

Considered problem



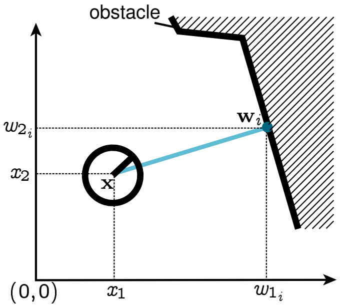
How to process the measurement intersection constraint ?

Considering an original constraint

- Constraint formalisation

Considering an original constraint

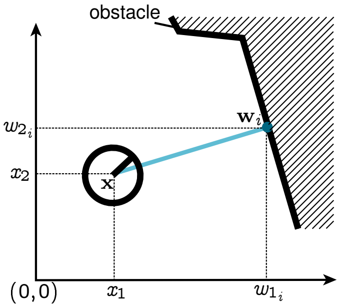
- Constraint formalisation



$\rightarrow$ Visibility relations between the robot and all the detected obstacles

Considering an original constraint

- Constraint formalisation



→ Visibility relations between the robot and all the detected obstacles

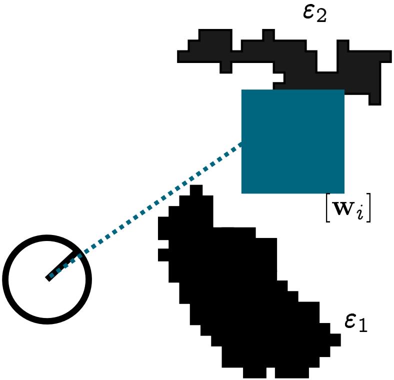
→ $\forall \varepsilon_j \in \mathcal{E}, \forall i, (\mathbf{x} \mathrm{V} \mathbf{w}_i)_{\varepsilon_j}$

Environment characterization

- Contraction over a visibility information
- Inner characterisation

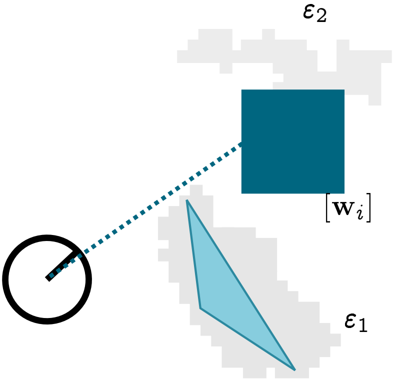
Environment characterization

- Contraction over a visibility information
- Inner characterisation



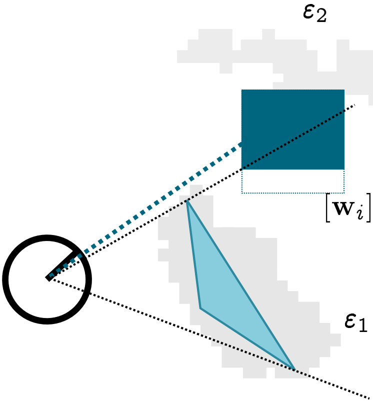
Environment characterization

- Contraction over a visibility information
- Inner characterisation



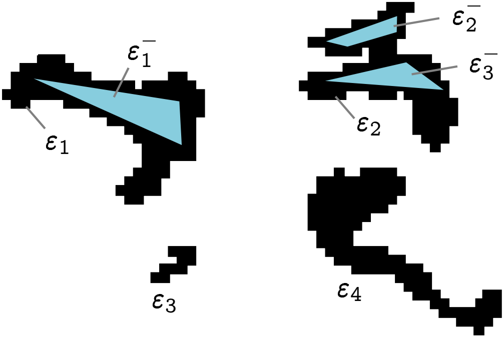
Environment characterization

- Contraction over a visibility information
- Inner characterisation

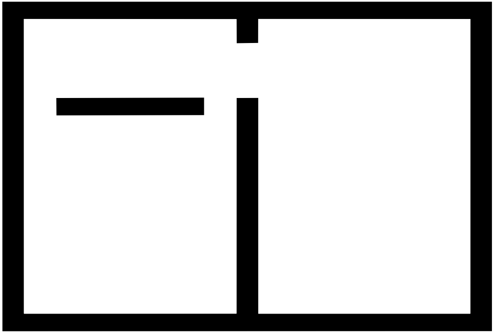


Environment characterization

- Contraction over a visibility information
- Inner characterisation
- Environment characterisation

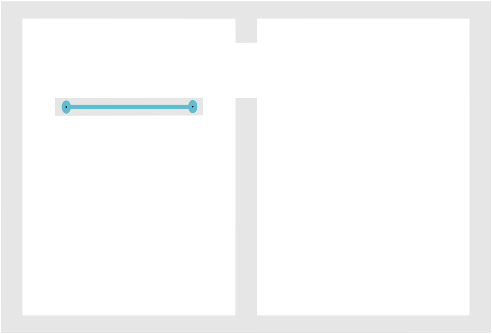


Example



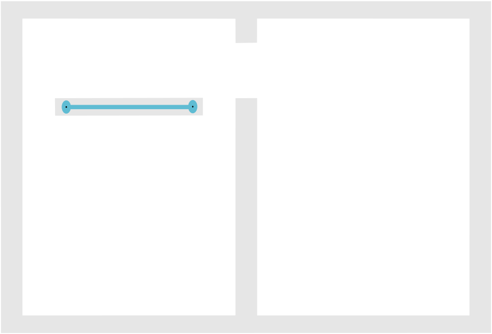
Considered environment

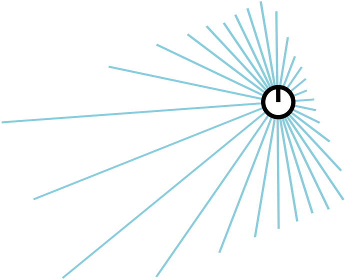
Example



Inner characterisation

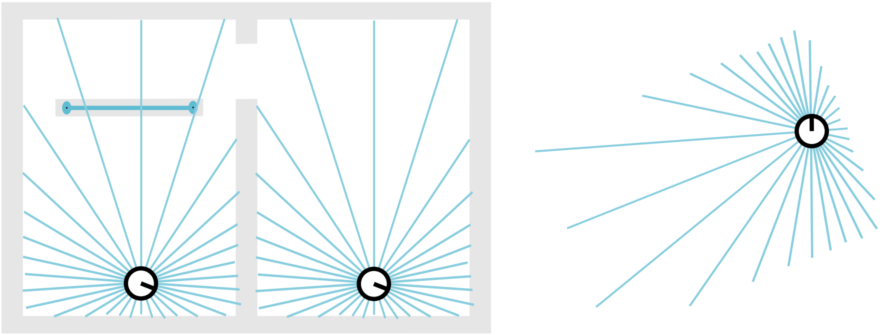
Example





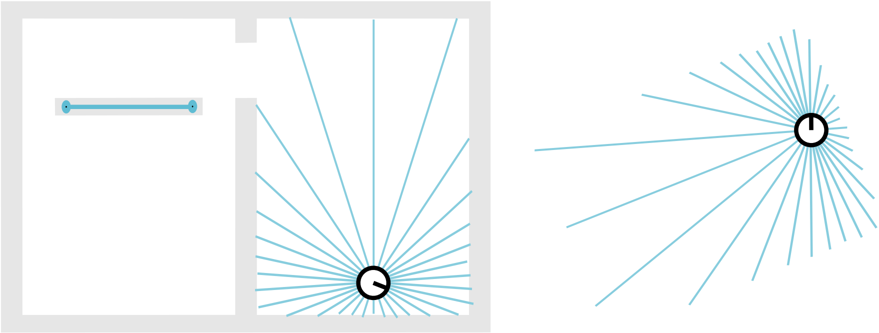
Considered measurements

Example



Result without visibility

Example



Result with visibility

Conclusion

→ Original formalisation of the visibility information

→ Optimal visibility contractors

→ Two applications to mobile robot localization

- Multi-robot localization
  - Avoid the drifting of the robots
  - Results depend of the number of robots and the environment topology
  - The contractors can be added to classical localization approach
- Original measurement intersection constraint
  - Improvement of the localization results
  - Avoidance of symmetries
  - Those contractors could be useful to other application