Building your own contractor with IBEX

Intervals & Geometry Workshop

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Outline

Introduction

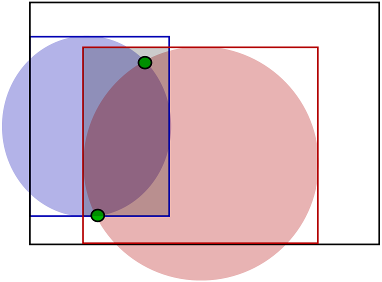
The goal of this talk is to show :

- a situation where a specific contractor can be introduced is complement to the generic ones that are built-in in IBEX.
- how to implement a specific contractor and make it collaborate with the generic ones.

This will be based on the simple example of the intersection of two circles.

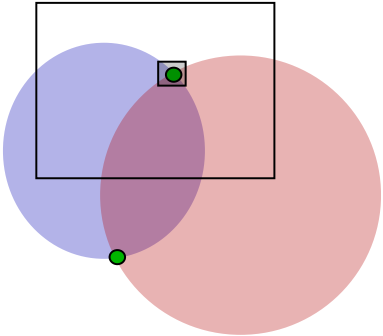
http://www.emn.fr/z-info/ibex/

The forward-backward contractor



Contract in almost any cases and optimaly with respect to each circle... but not with respect to the intersection.

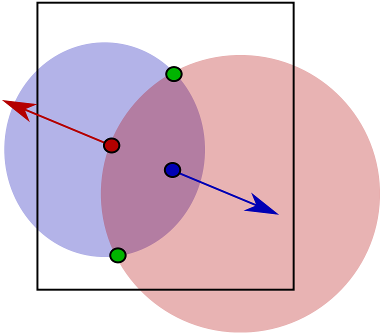
The interval Newton contractor



If the box is enclosing one solution, the interval Newton can give an optimal contraction.

Introduction

The interval Newton contractor



But it may not contract at all if a singularity occurs (especially in the case of a box enclosing two solutions).

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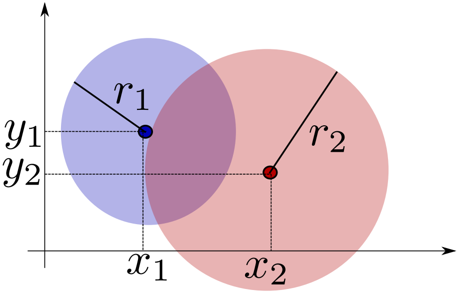
The two intersection point of two circles can actually be formally obtained.

Consider two circles with origin points (x_1, y_1) and (x_2, y_2) and radii r_1 and r_2.



Introduction

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Let d be the distance between the origins of the two circles.

- If $d > r_1 + r_2$ then the two circles are distant from each others (no intersection)
- If $d < r_1 - r_2$ or $d < r_2 - r_1$, then one circle is included in the other (no intersection)
- Otherwise, the two solutions are given by :

$$x = \frac{x_2 + x_1}{2} + \frac{(x_2 - x_1)(r_1^2 - r_2^2)}{2d^2} \pm \frac{y_2 - y_1}{2d^2} \sqrt{((r_1 + r_2)^2 - d^2)(d^2 - (r_1 - r_2)}$$

$$y = \frac{y_2 + y_1}{2} + \frac{(y_2 - y_1)(r_1^2 - r_2^2)}{2d^2} \pm \frac{x_2 - x_1}{2d^2} \sqrt{((r_1 + r_2)^2 - d^2)(d^2 - (r_1 - r_2)}$$

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The last formula characterizes explicitely the solutions but involves conditional operators (`if`) and disjunctions ($\pm$).

So it cannot be easily encoded as a usual function. It is rather an algorithm.

Fortunately, a contractor is a numerical object (it is a function on the set of intervals) and can embed any algorithm. So we can create a contractor with our formula.

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The formula is exact so the resulting contractor is optimal for any box if every parameter (circle position or radius) is fixed.

However, the multi-occurrence of the parameters make the contractor pessimistic in case of uncertainty.

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